Weakly vibrated granular packing

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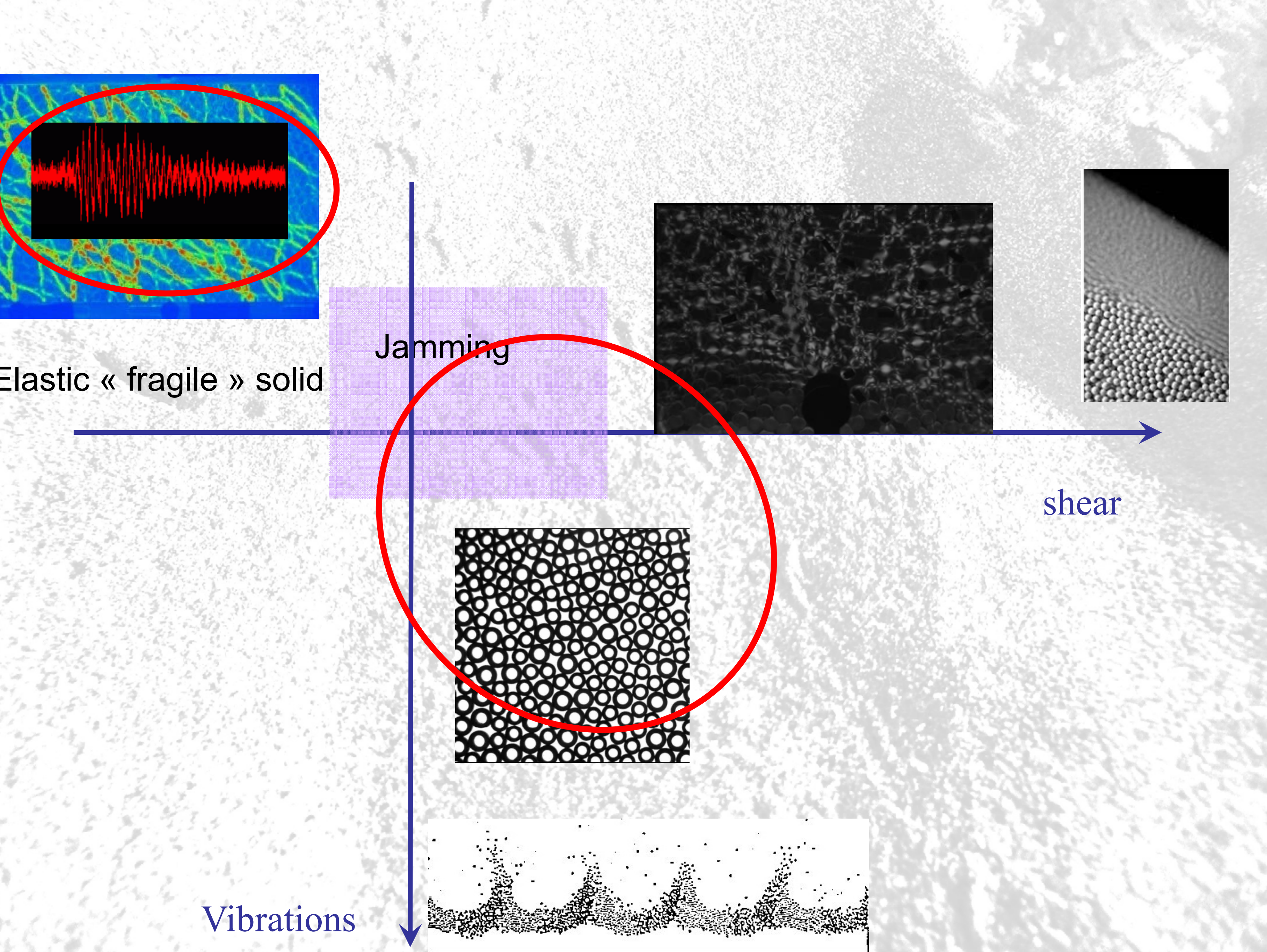
ESPCI-CNRS - Univ.Paris 6 & 7


Jamming
Elastic « fragile » solid
shear
Vibrations

I Introduction

II Acoustics in granular matter

NL elasticity
Mean field failure
Rigidity transition

III Surface waves acoustics

Model
Surface wave experiments

IV Acoustic activation

Intruder thread rheology
Simple model
Intruder bead mobility

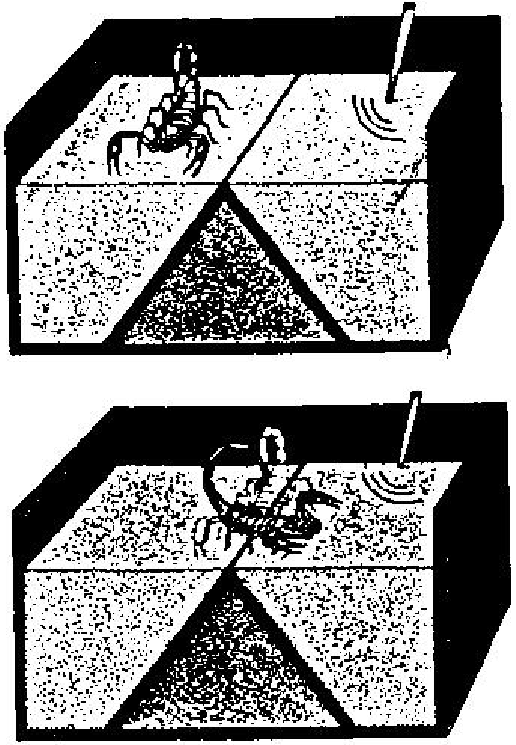
V Friction activated rheology

Droplet spreading
Inertia tribometer

Scorpion attack

Ph. Brownell, Science **197** 479 (1977)

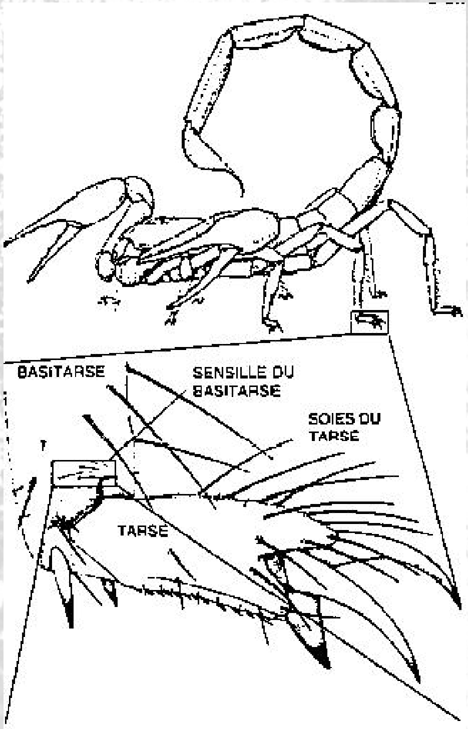
Ph. Brownell, Science **197** 479 (1977)

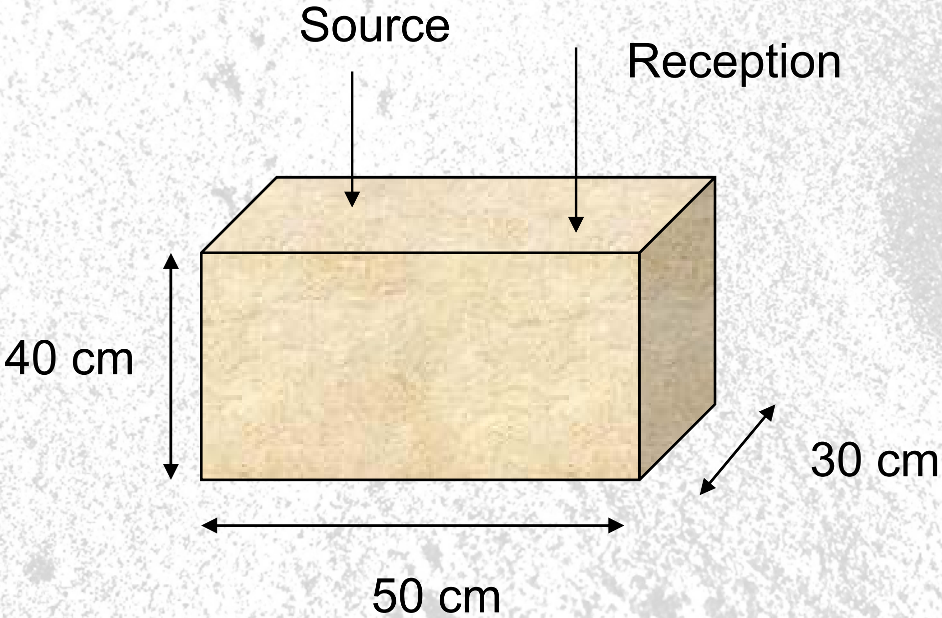


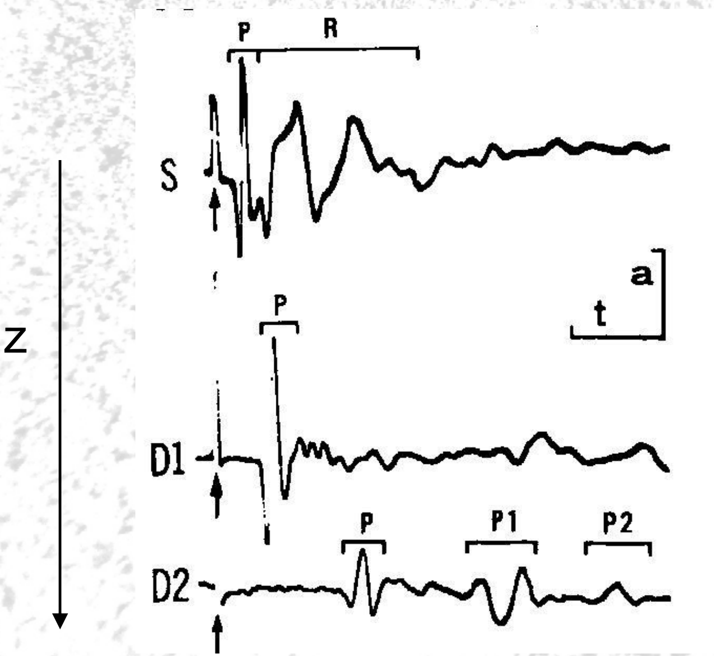
Paruroctonus mesaensis use sand surface wave to orient and strike their prey

Slit sensilla detectors

Sensitivity : 10^{-10} *m* !!







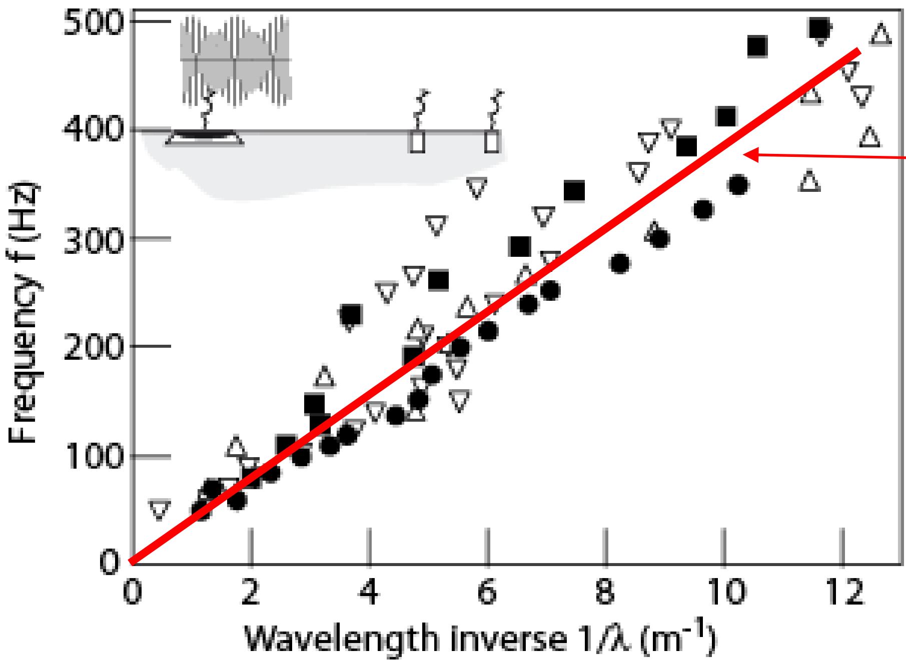
Slow « R » waves : 40-50 m/s

Fast « P » waves : 90-120 m/s

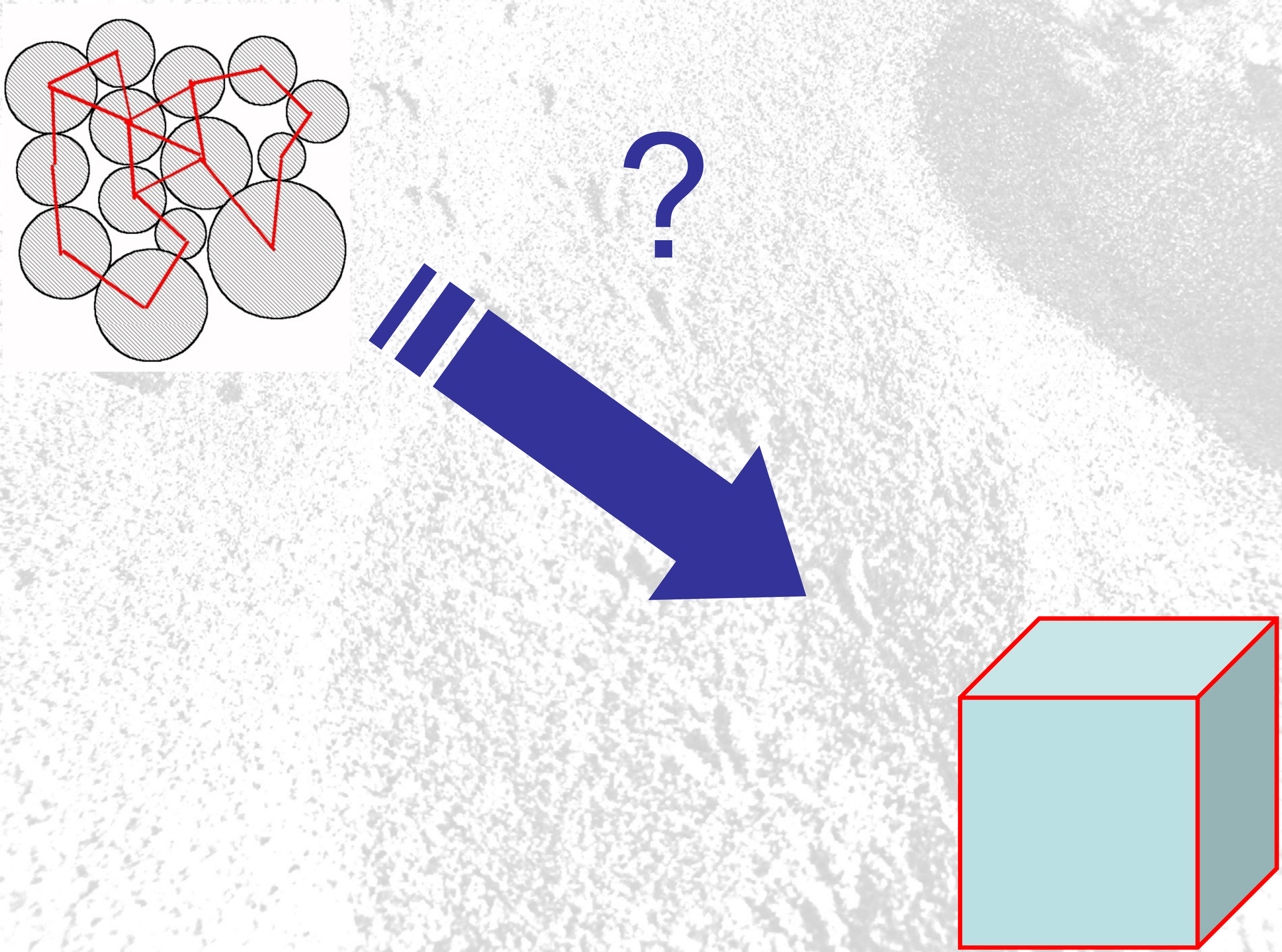
Field measurements

B. Andreotti, Phys. Rev. Lett. **93,** 238001 (2004)

Dispersion relation

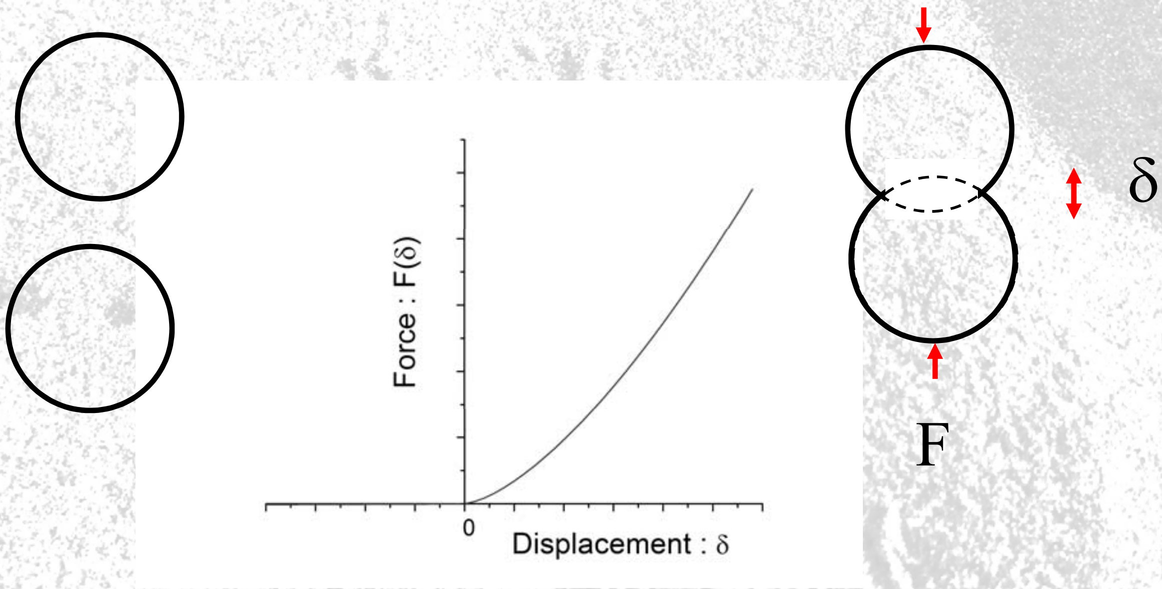


$$c \cong 40m/s$$



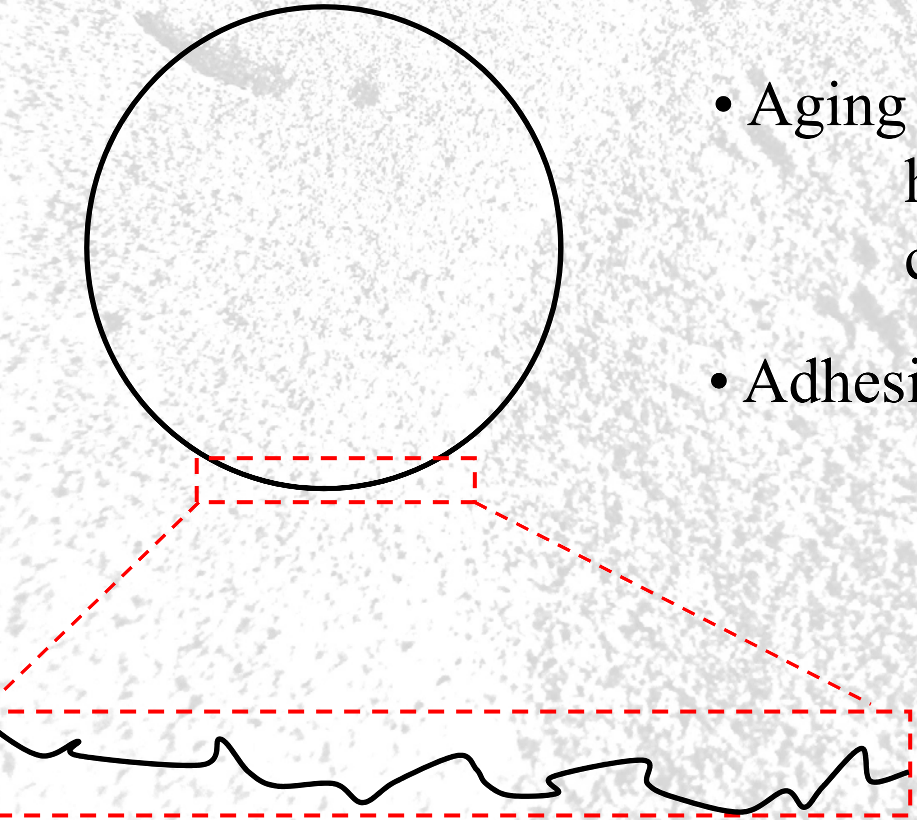
Non linear contact law

- Hertz contact laws

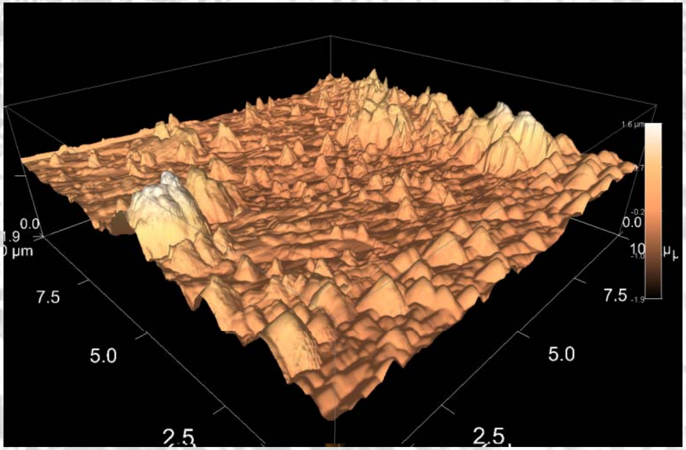


$$F \approx \mathrm{E}\,\mathrm{d}^2 \left(\frac{\delta}{d} \right)^{3/2}$$

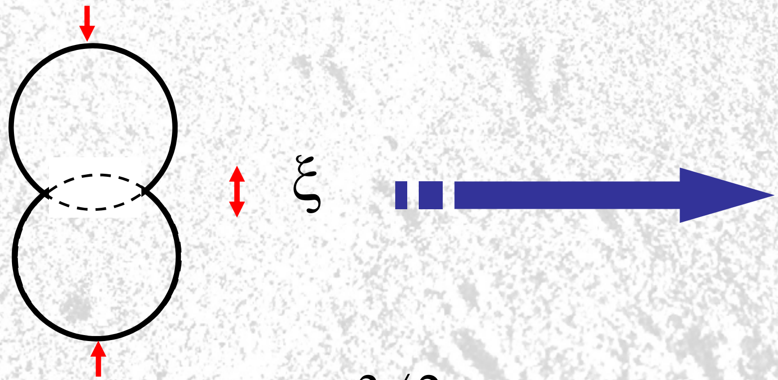
Roughness scale



- Solid friction
- Aging
  - humidity
  - contact plasticity
- Adhesion

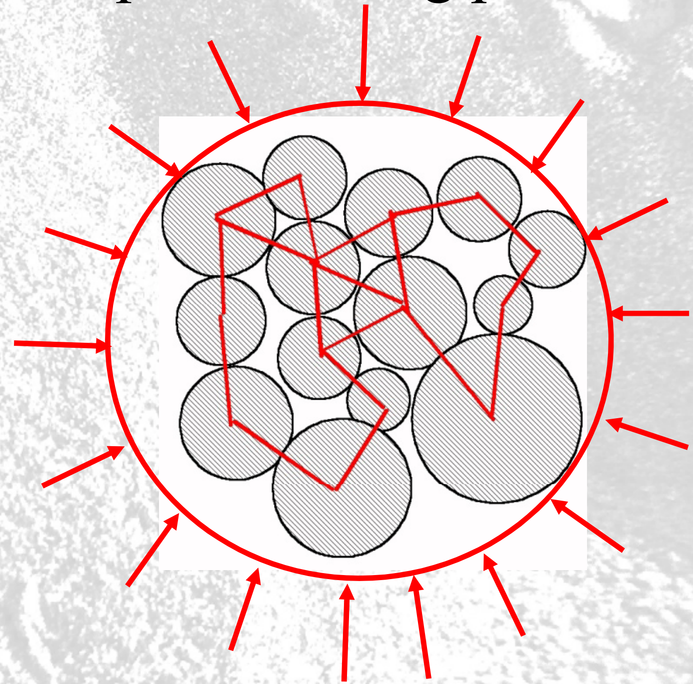


Hertz contact law model



$$F \approx \mathrm{E}R^2\left(\frac{\xi}{R}\right)^{3/2}$$

P_0 : isotropic confining pressure



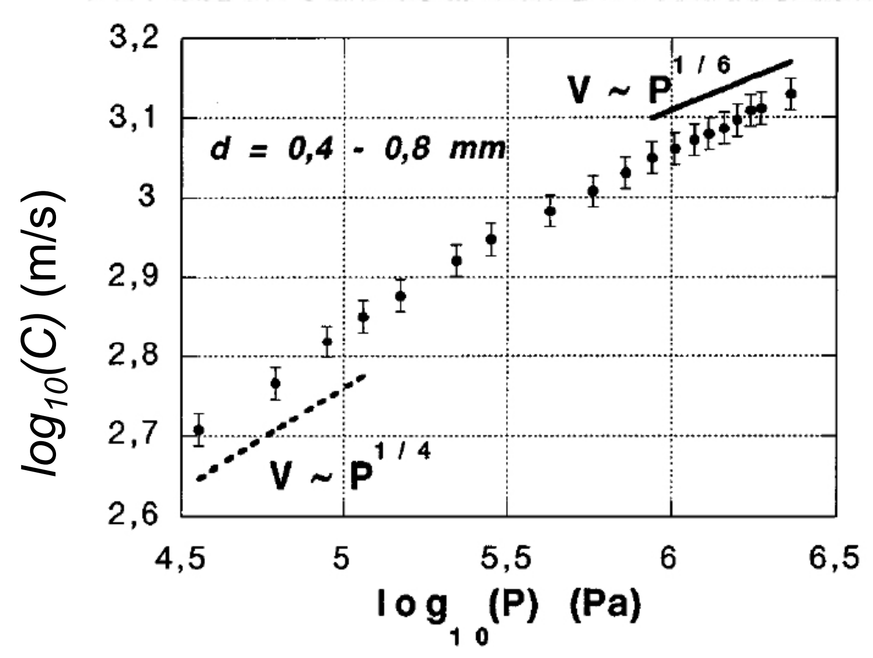
Macroscopic packing stiffness

$$E_{\mathrm{eff}} \approx \mathrm{E}\ \left(\phi Z\right)^{2/3}\ \left(\frac{P_0}{E}\right)^{1/3}$$

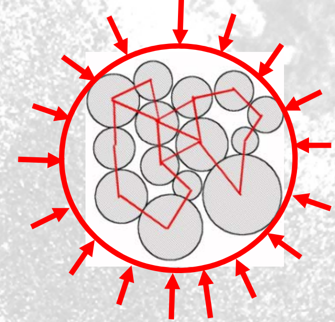
ϕ : packing fraction

Z : average number of contact/grains

Confined packing



Jia et al. PRL **82**, 1886 (1999)



$P_0 = 10^5 - 10^6$ Pa

Hertz model predicts:

$$C \approx C_0 \left(\phi Z\right)^{1/3} \left(\frac{P_0}{E}\right)^{1/6}$$

A large P_0 evolution of the contact network with confining pressure

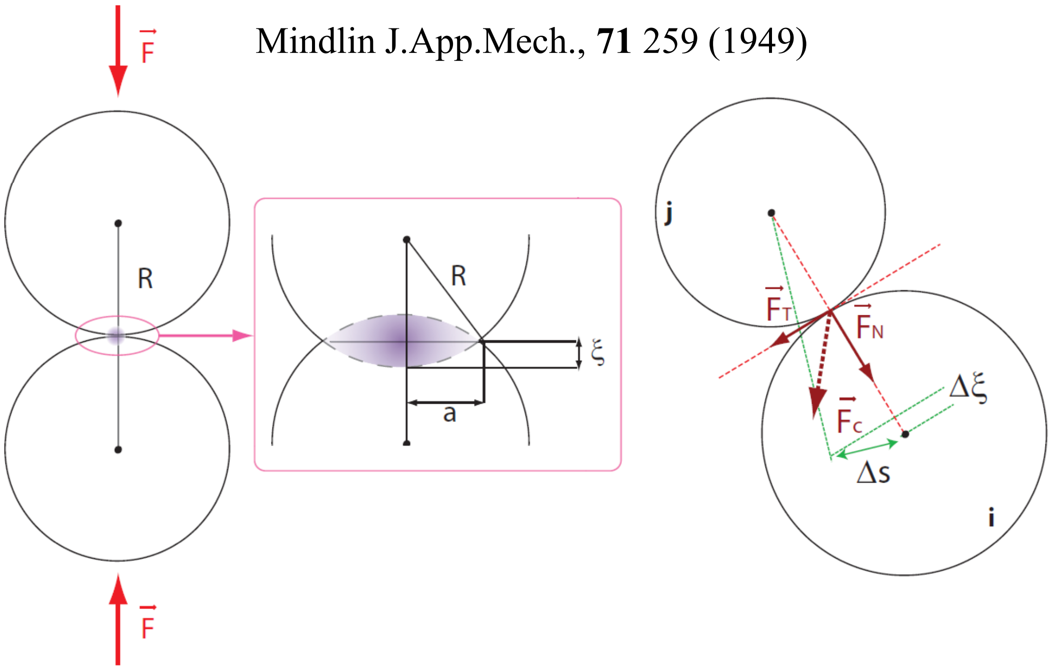
$Z \uparrow$ when $P_0 \uparrow$

But not only !

Goddard, Proc.Royal Soc **430** 105 (1990)
de Gennes Europhys. Lett., **35** 145 (1996)
Velický & Caroli Phys. Rev. E **65**, 021307 (2002)

Contact model

Mindlin J.App.Mech., **71** 259 (1949)



$$F_n = \frac{8}{3}\frac{G_g}{1-\nu_g}R\left(\frac{\xi}{R}\right)^{3/2}$$

Normal contact force

$$\Delta F_t = \frac{8\,Gg}{2-\nu_g}R\left(\frac{\xi}{R}\right)^{1/2}\Delta s$$

Tangential contact force

$$\mu_g = \left|\frac{F_t}{F_n}\right|$$

Coulomb yield criterion

Mean field calculation

Walton J. Mech. and Phys. Sol. **35** 213 (1987)
Johnson & Norris J. Appl. Mech. **64** 39 (1997)

Bulk modulus

$$K_{\mathrm{MF}} = \frac{1}{3\pi}\frac{G_g}{(1-\nu_g)}(\Phi z)^{2/3}\left(\frac{3\pi(1-\nu_g)}{2\,G_g}P\right)^{1/3}$$

Shear modulus

$$G_{\mathrm{MF}} = \left(\frac{1}{1-\nu_g} + \alpha\frac{3}{2-\nu_g}\right)\frac{G_g}{5\pi}(\Phi z)^{2/3}\left(\frac{3\pi(1-\nu_g)}{2\,G_g}P\right)^{1/3}$$

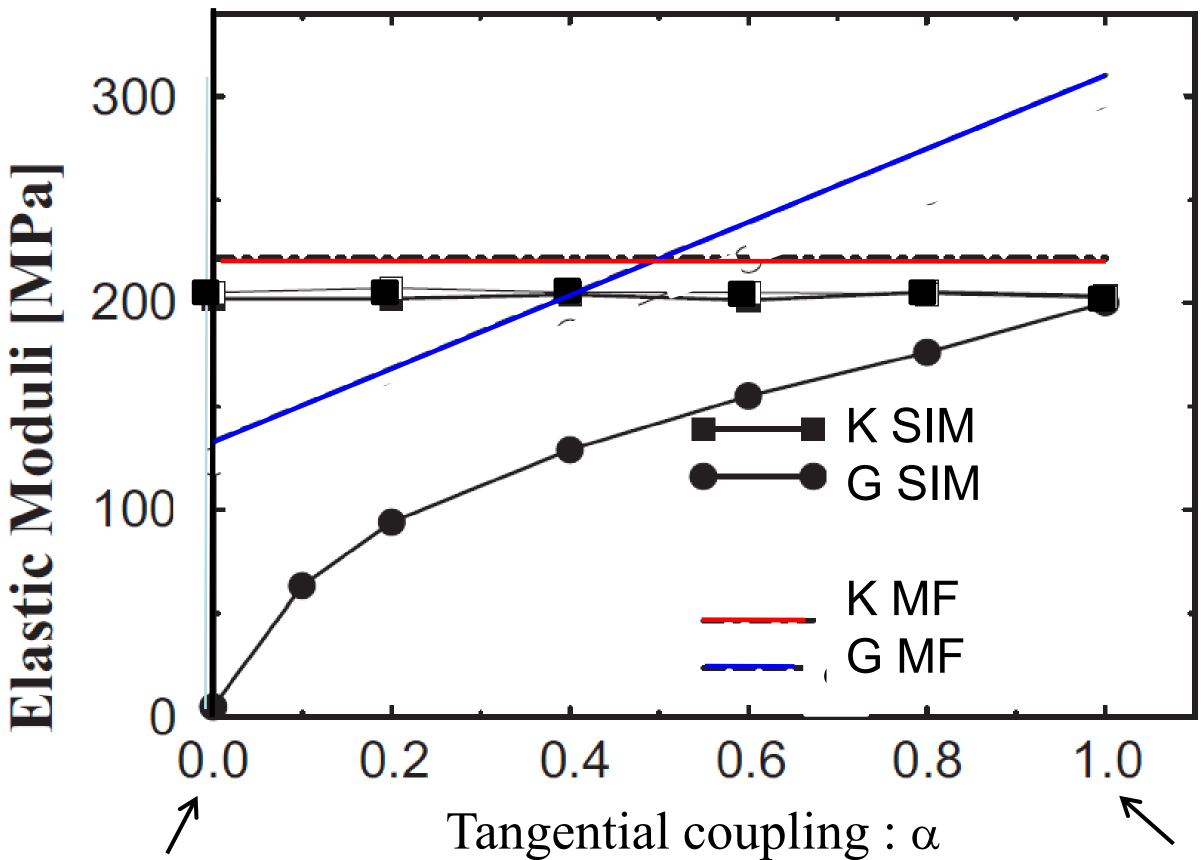
$\alpha=0$ no friction

$\alpha=1$ infinite friction

Mean field failure

Numerical simulations

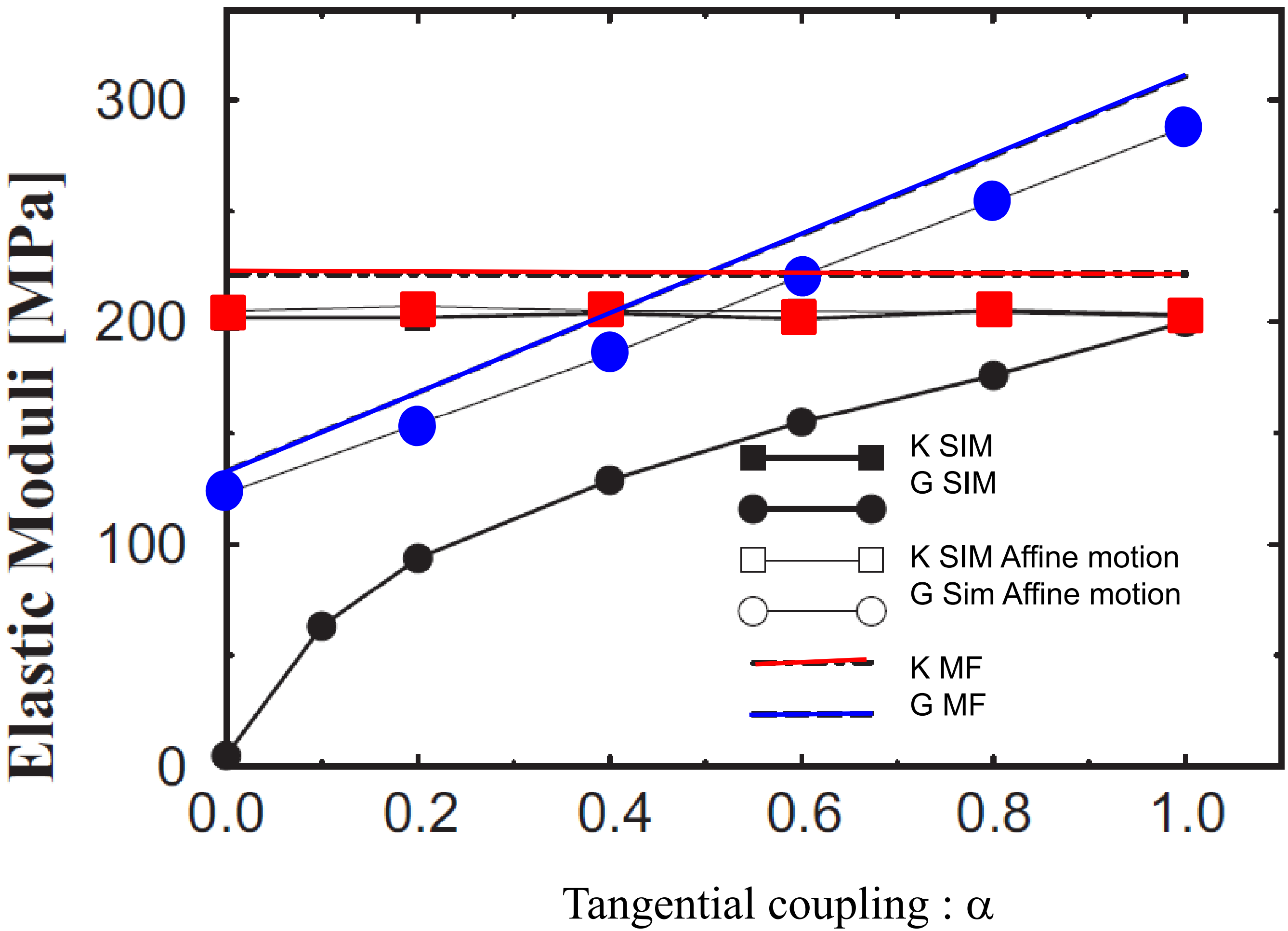
H.A. Makse et al.Phys. Rev. Lett. **83,** 5070 (1999)
Phys. Rev. E **70,** 061302 (2004)



$\alpha=0$ no friction

α=1 infinite friction

Major failure of shear modulus *G*



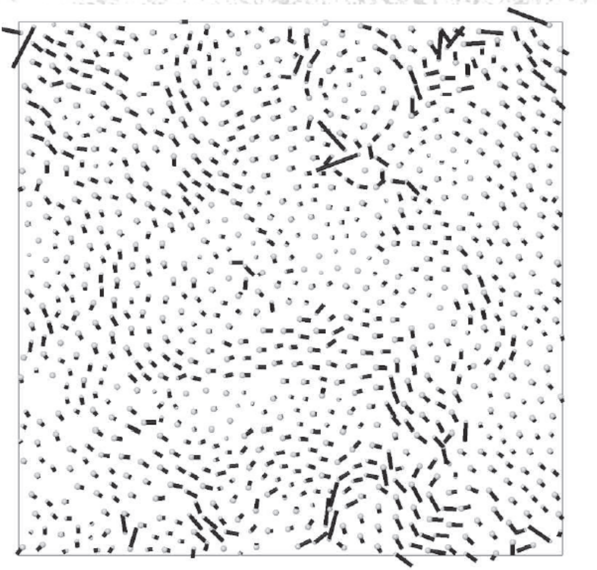
- Affine motion hypothesis breaks down at the granular level
- Granular packing reorganizes

Elastic anomalies at the rigidity transition

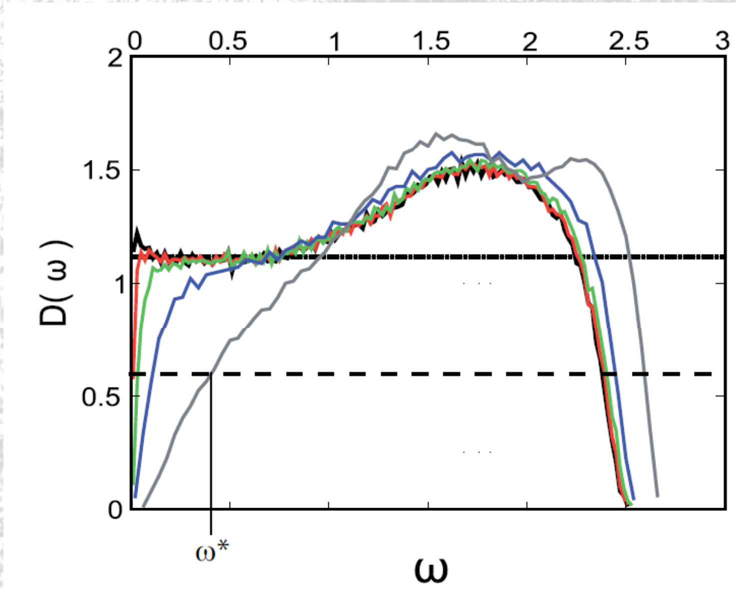
Isostatic critical point ($\alpha = 0$)

O'Hern et al. Phys. Rev. E **68,** 011306 (2003)
Wyart et al., Phys.Rev. E **72,** 051306 (2005)
Agnolin & Roux Phys.Rev. E **76,** :061304(2007)

$$\mu = 0 \qquad Z_c = 2d$$



No affine collective motion



Soft-modes anomalies

$$Z - Z_c \sim (\phi - \phi_c)^{1/2}$$
$$P \sim (\phi - \phi_c)^{3/2}$$

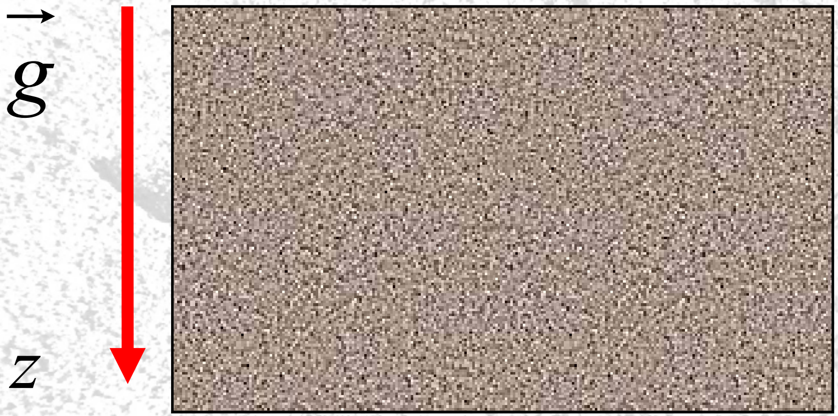
$$\Longrightarrow \quad \frac{G}{K} \propto P^{1/3}$$

Surface wave acoustics of granular packing under gravity

With Lénaïc BONNEAU and Bruno ANDREOTTI
PMMH
ESPCI-CNRS-PARIS 6- PARIS 7

Refs
Bonneau et al. PRE **75** 016602 (2007)
Bonneau et al. PRL **101**, 118001 (2008)

Surface wave propagation



Under gravity

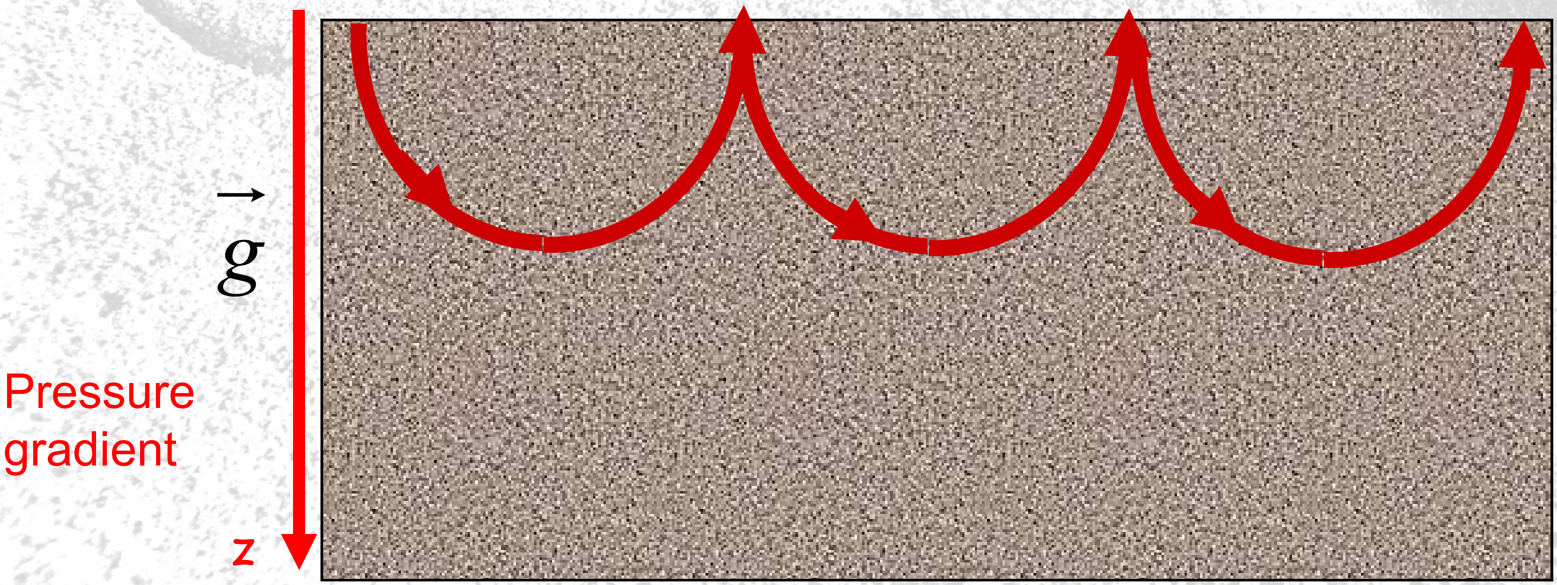
$$P = \rho g z$$

$$C \approx C_0 \, (\phi Z)^{1/3} \left(\frac{P}{E}\right)^{1/6} \Rightarrow c \propto z^{1/6}$$

A mirage effect

$$c \propto z^{1/6}$$

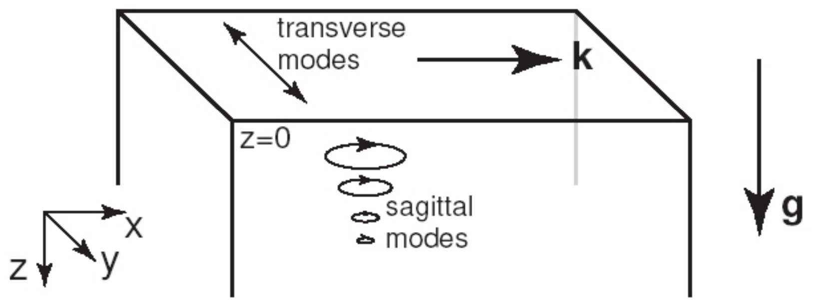
The wave is refracted toward the surface



- No bulk wave propagation !
- Sand packing behave as a index gradient waveguide

A theoretical calculation

Bonneau et al. PRE **75** 016602 (2007)



$$F_{el} = E\delta^{1/2}\left(\frac{2}{5}\mathcal{B}u_{ll}\delta^2 + \mathcal{A}u_s^2\right)$$

Elastic free energy

Jiang and Liu PRL ***91,****144301* (2003)

$$\delta = -Tr(u_{ij})$$

Volumic compression

$$u_s^2 = u_{ij}^0 u_{ij}^0$$

Shearing strain

E : Material Young's modulus

$\mathcal{B}$ and $\mathcal{A}$ dimensionless constants

Stress/strain relation

$$\sigma_{ij} = E\sqrt{\delta}\left(\mathcal{B}\delta\delta_{ij} - 2\mathcal{A}u^0_{ij} + \frac{\mathcal{A}u_s^2\delta_{ij}}{2\delta}\right)$$

Boussinesq (1873)

new

- B coupling with bulk compression
- A coupling with shear strain

Mean-field values of the model parameters

$$E = \frac{8}{3}\frac{G_g}{1-\nu_g}$$

Gg grain's shear modulus
νg grain's Poisson ratio
E grain's Young's modulus

$$A_{MF} = \frac{z\Phi}{3^{3/2}5\pi\left(1+\nu_g\right)}\left(\frac{1}{1-\nu_g}+\frac{3\alpha}{2-\nu_g}\right)$$

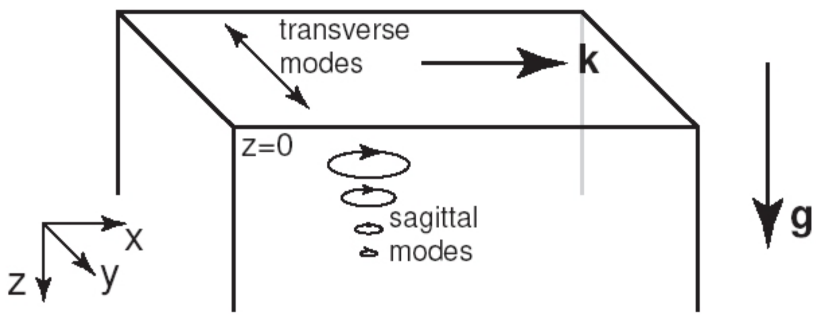
Frictionless grains $\alpha=0$

$$B_{MF} = \frac{5}{3}A_{MF}$$

Infinite friction grains $\alpha=1$

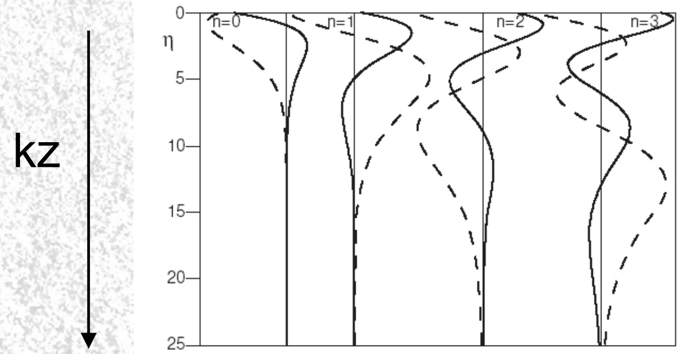
$$B_{MF} = \frac{5}{7}A_{MF}$$

Surface modes

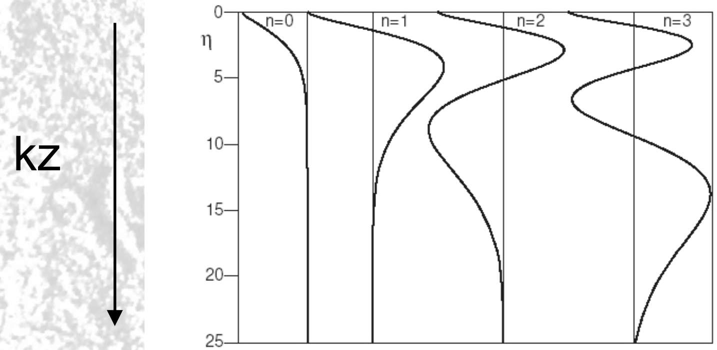


• Modes localized in depth

Saggital modes



Transverses modes



$$l_p \propto n\lambda$$

Penetration length

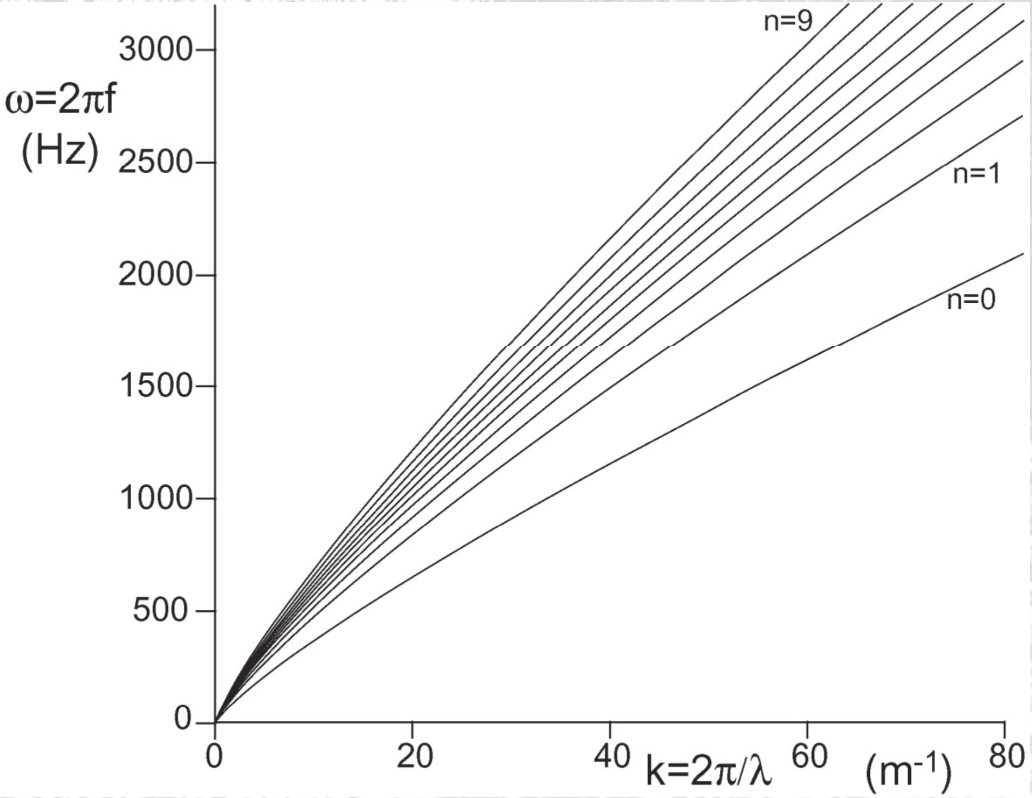
Dispersion relations

Dispersion relation

$$\omega \propto \left(\frac{E}{\rho}\right)^{1/3} g^{1/6} k^{5/6}$$

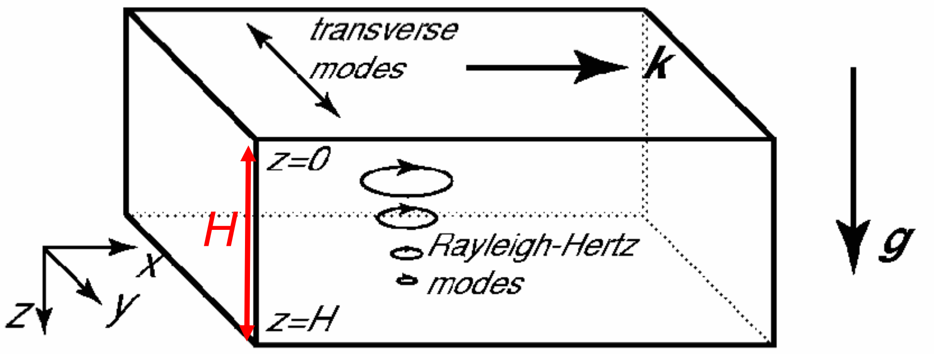
$$P = \rho g (n\lambda)$$

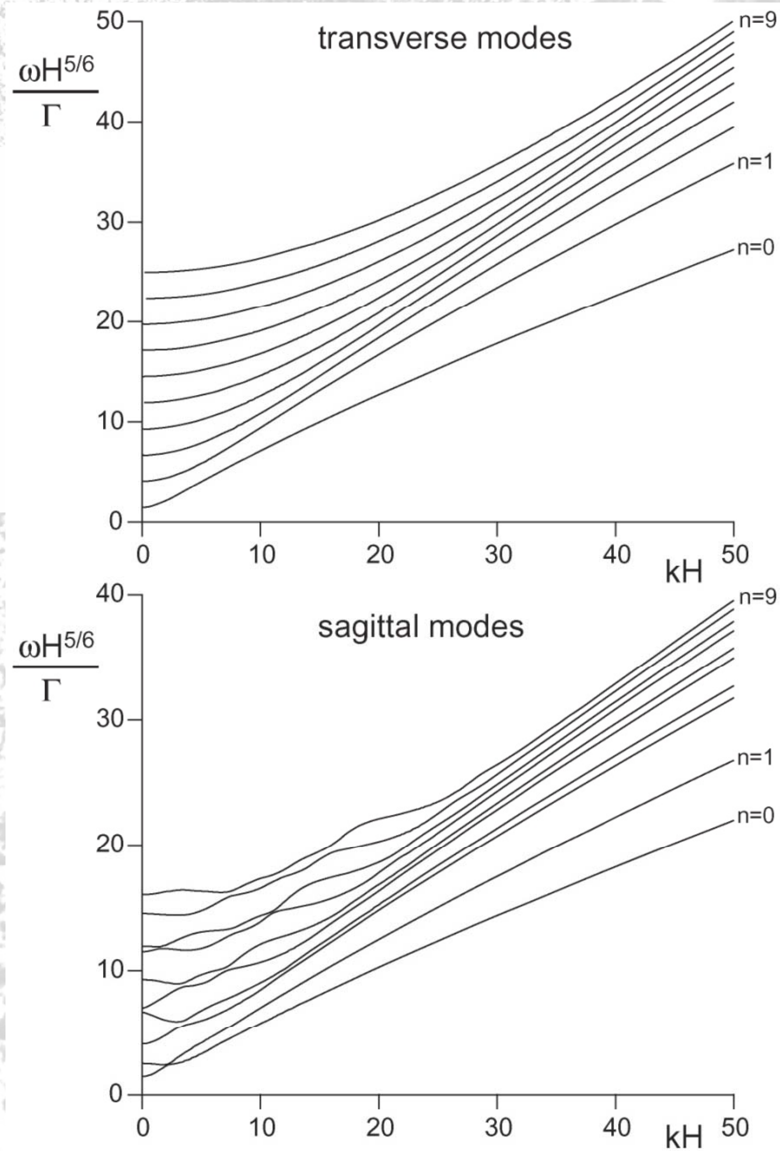
$$c \propto (n\lambda)^{1/6}$$



Finite size effects

Finite depth : H

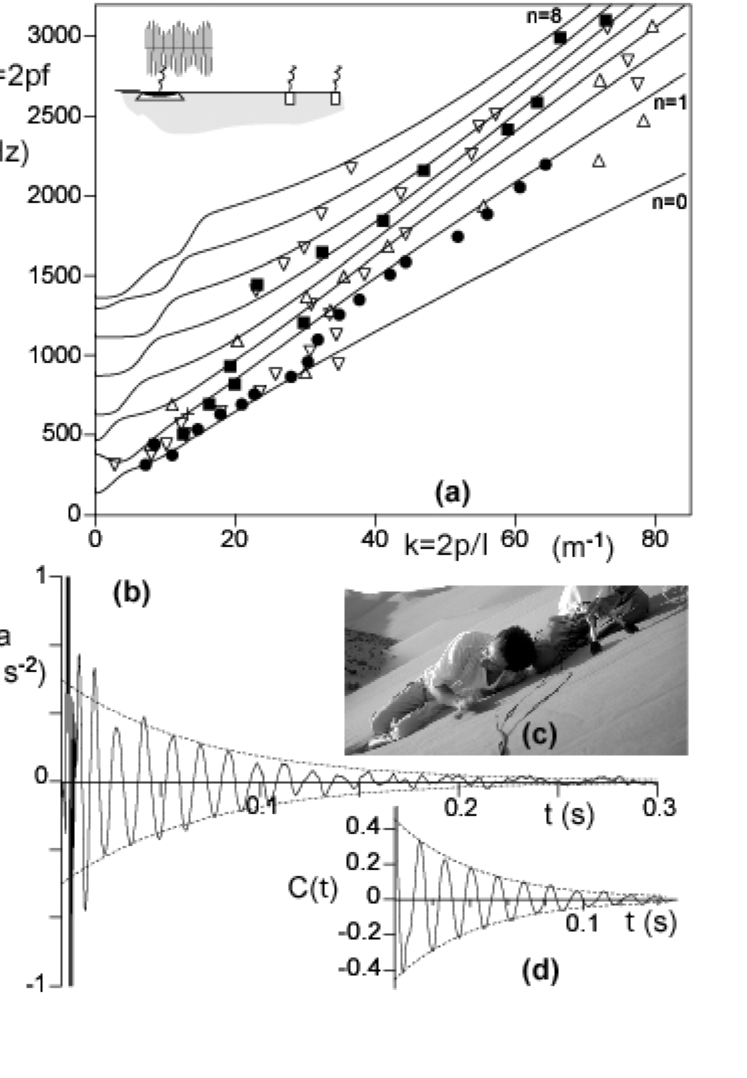




Wave guide cut-off

$$\omega_c \propto H^{-5/6}$$

Reinterprétation of field measurements



$\Gamma = 50\ s^{-1}m^{5/6}$ ($\Gamma_{MF} = 106\ s^{-1}m^{5/6}$)

$$\omega_c = 2\pi\ 73\ Hz$$

→ Wet sand layer at H_0=50cm

- Multiplicity of modes !!

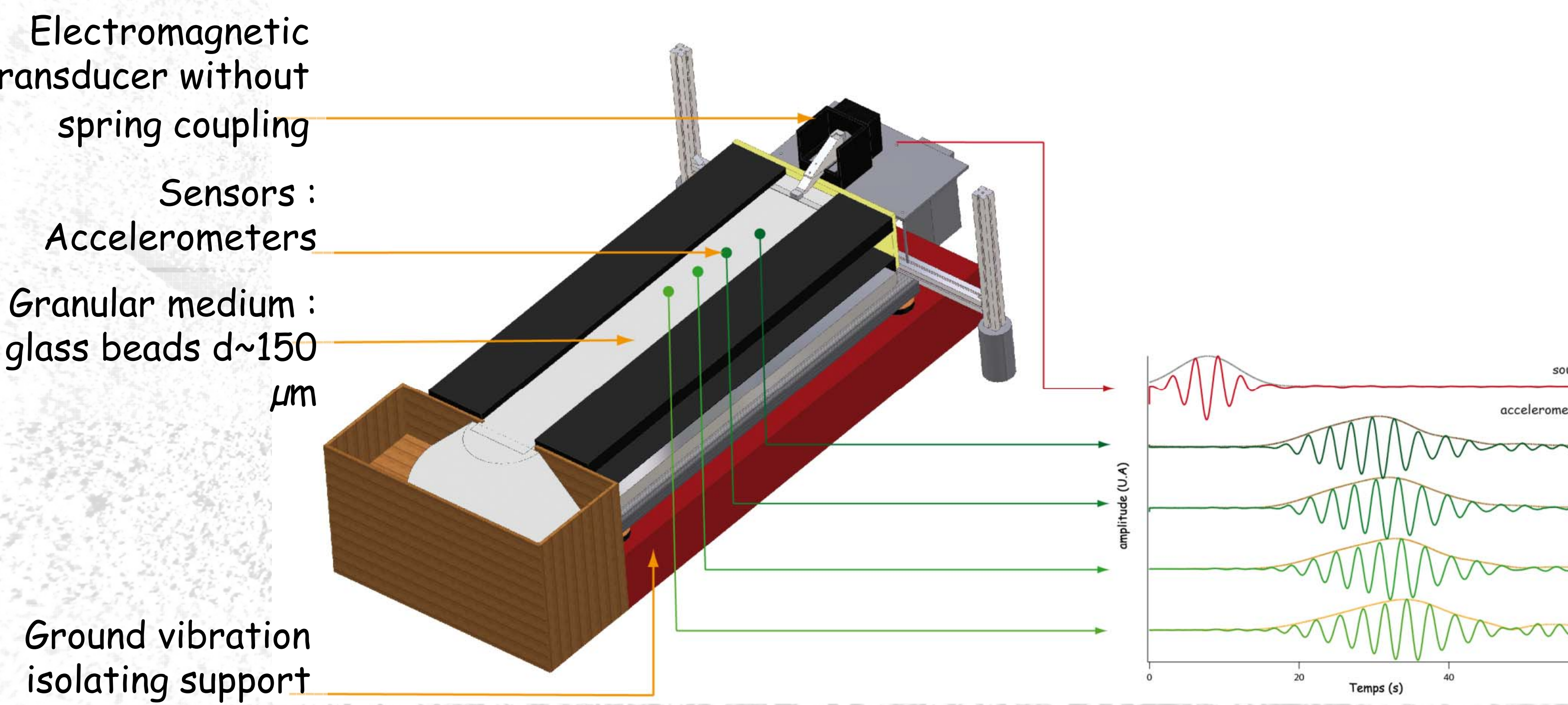
Laboratory scale surface wave propagation

Bonneau et al. PRL **101**, 118001 (2008)

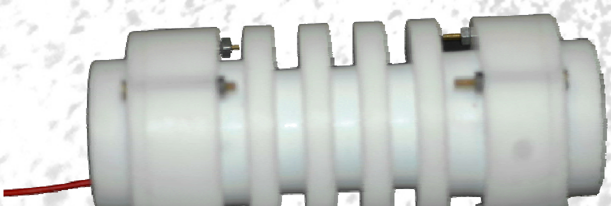
- Can we observe surface wave propagation in the lab ?
- Difficulties (multiplicity of modes – packing preparation)
- Exploration of the vanishing pressure limit (jamming)

Experimental set-up

aggital waves generation

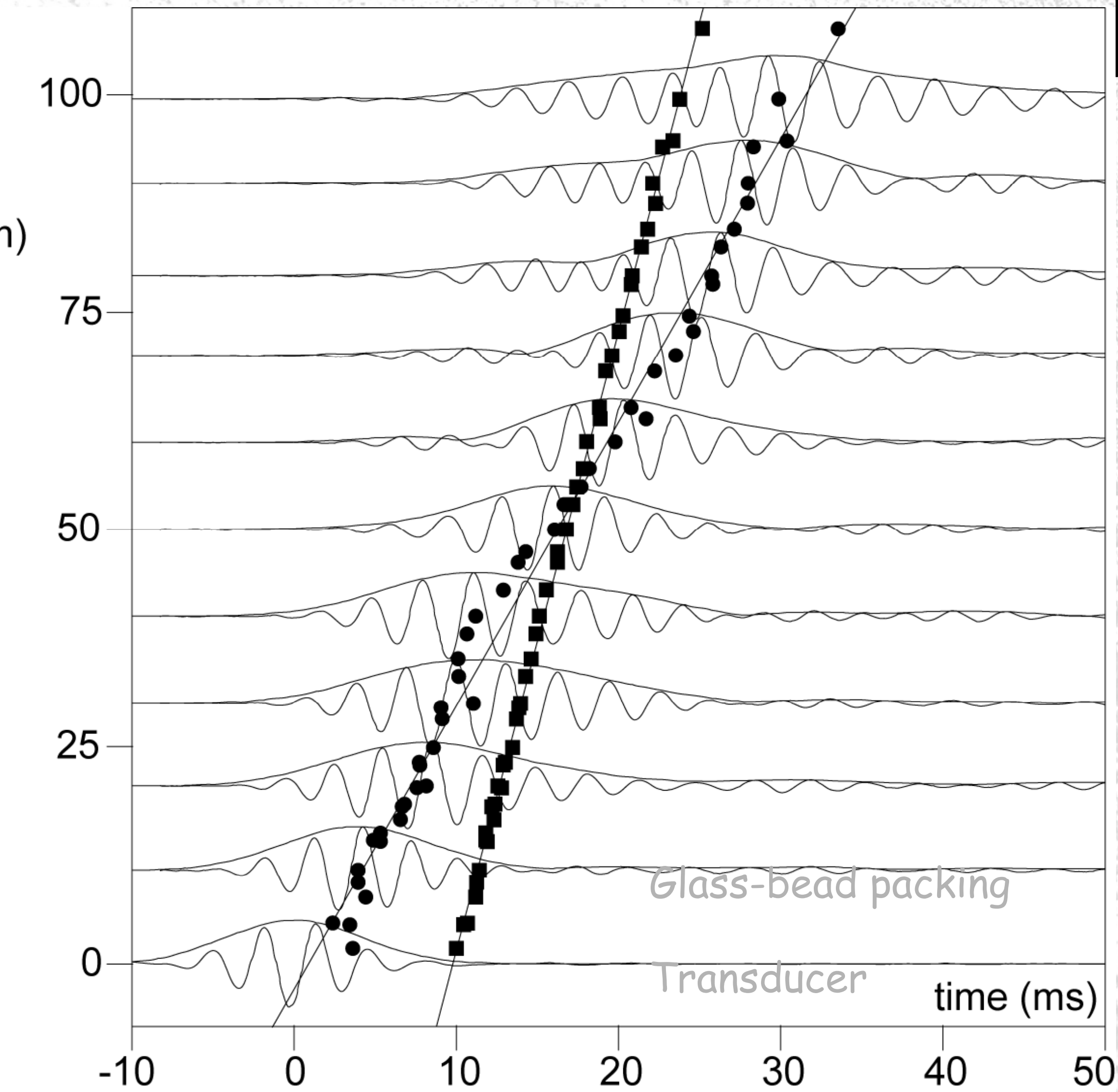


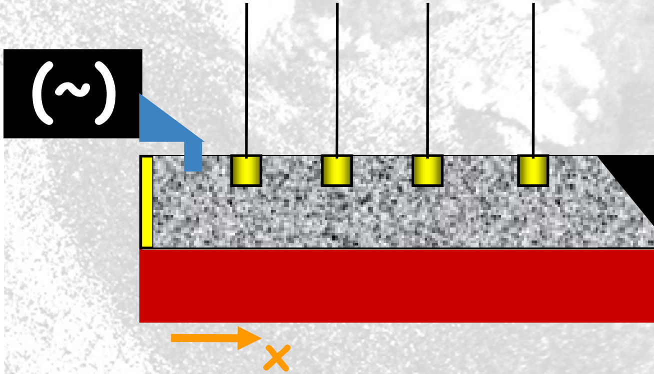
ransverse waves generation



To avoid grain scale heterogeneities :
grain size << sensors size

Wave propagation





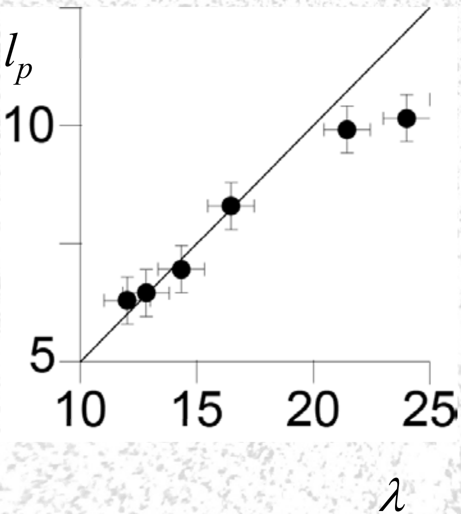
Sagittal waves : f= 315Hz :

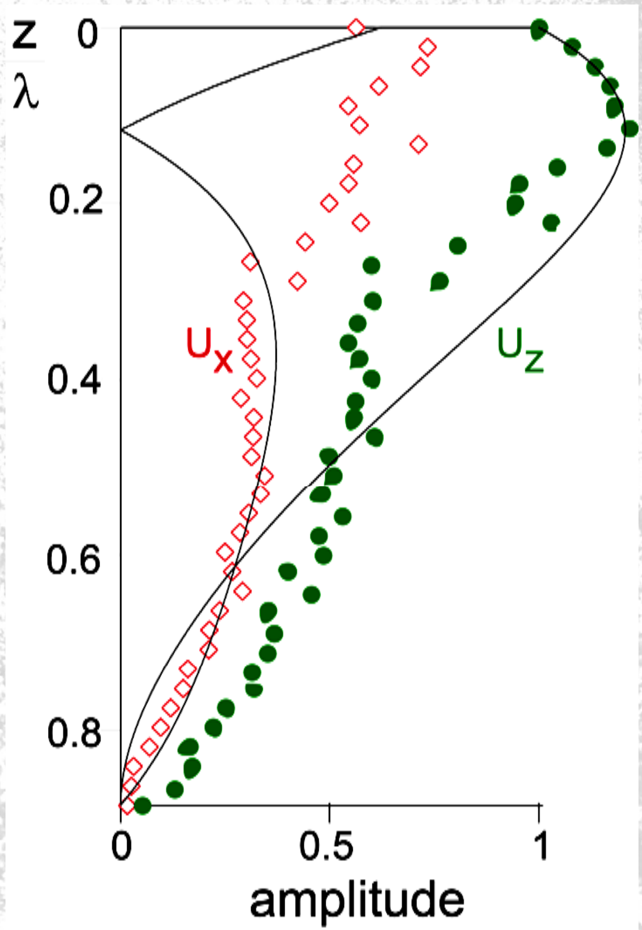
■Phase velocity : 72.9 m/s

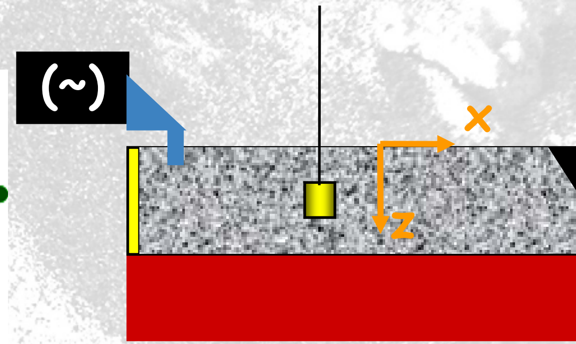
●Group velocity : 31,2 m/s

Surface waves

$$l_p \cong \lambda / 2$$







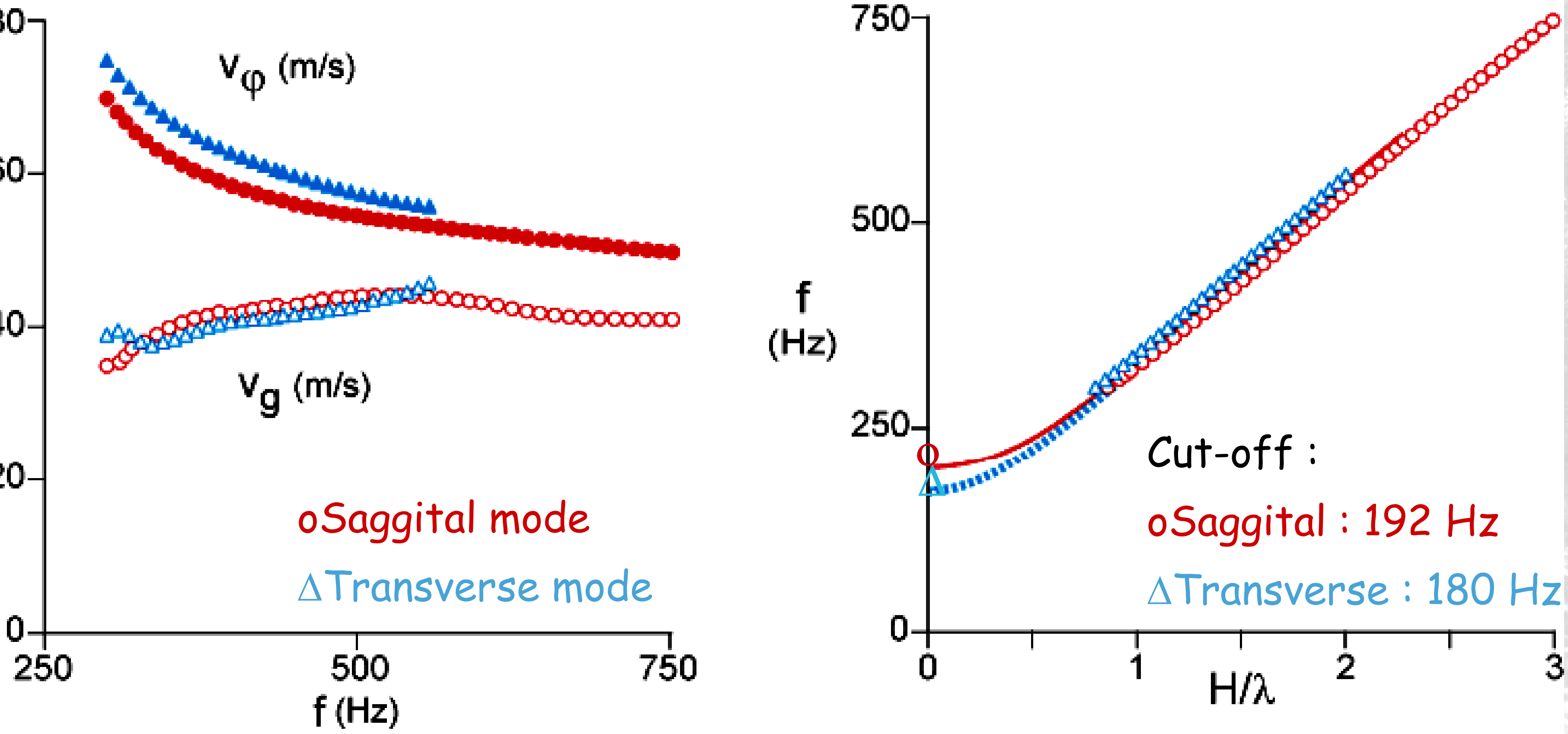
Saggital waves :

◇ Axial component

• Vertical component

- Waves localized at the surface
- Channel geometry selects the lowest propagation mode

Experimental dispersion relations

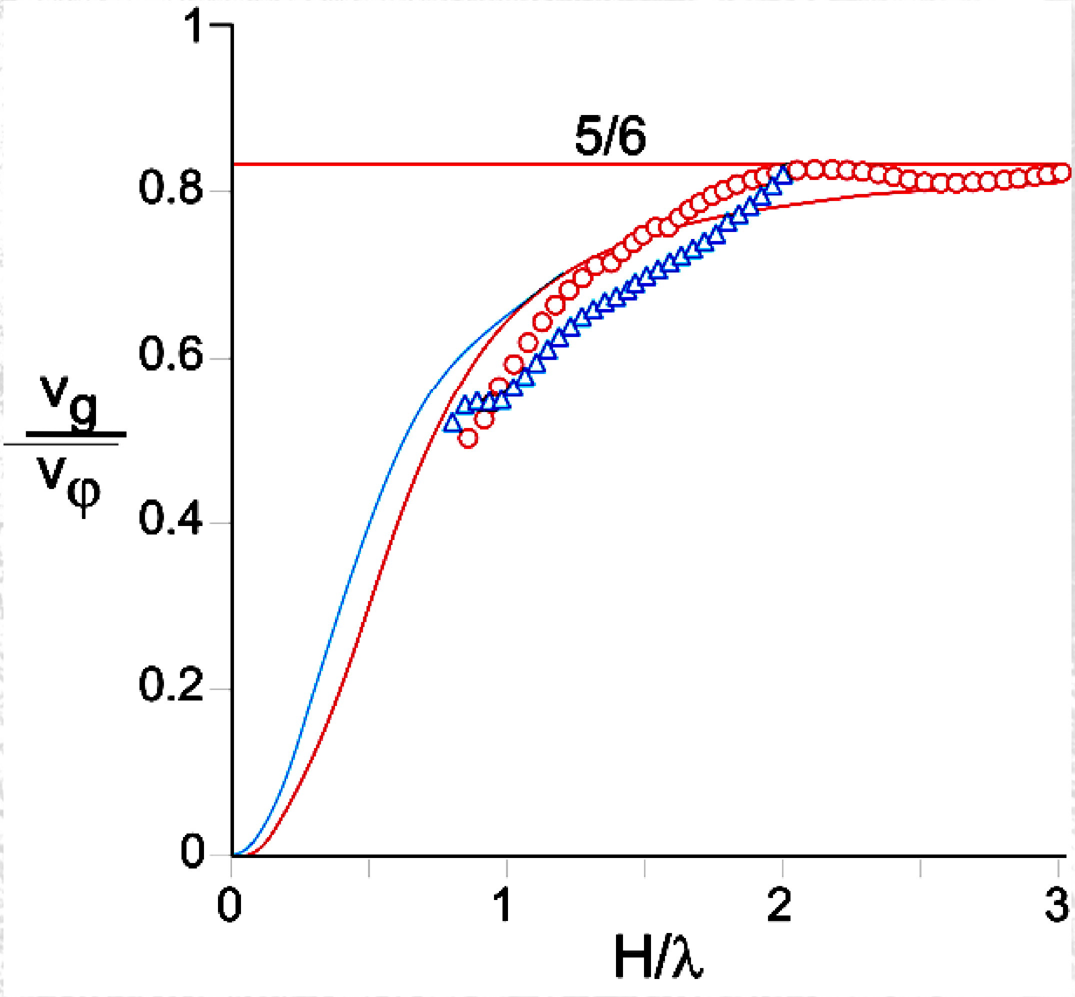


- Transverse and saggital dispersion relations are matching !

A test of Hertz scaling

$$\omega \propto k^{\alpha} \Rightarrow \frac{V_g}{V_\varphi} = \alpha$$

Hertz $\quad \frac{V_g}{V_\varphi} = \frac{5}{6}$



• Hertz contact law relevant down to very low confining pressure ($P_0 \cong 10^2 \rho g d \cong 2 10^2 Pa$), however…

Mean-field failure

Dispersion relations

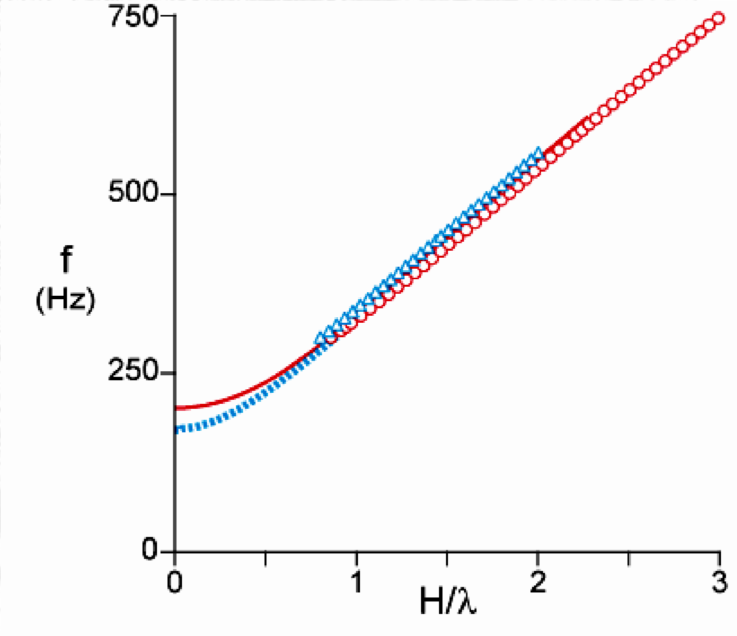
Transverse mode

$$\omega = F_{trans}(A,B,k)$$

Saggital mode

$$\omega = F_{sagg}(A,B,k)$$

$$A < \frac{B}{5} \Rightarrow F_{trans} \approx F_{sagg}$$



Experiment

$$A^{1/2}B^{-1/6} \approx 0.23$$

Mean-field theories

Frictionless grains

$$A^{1/2}B^{-1/6} \approx 0.44$$

Infinite friction

$$A^{1/2}B^{-1/6} \approx 0.61$$

- Shear stiffness 3.5 to 7 times smaller than mean-field predictions !

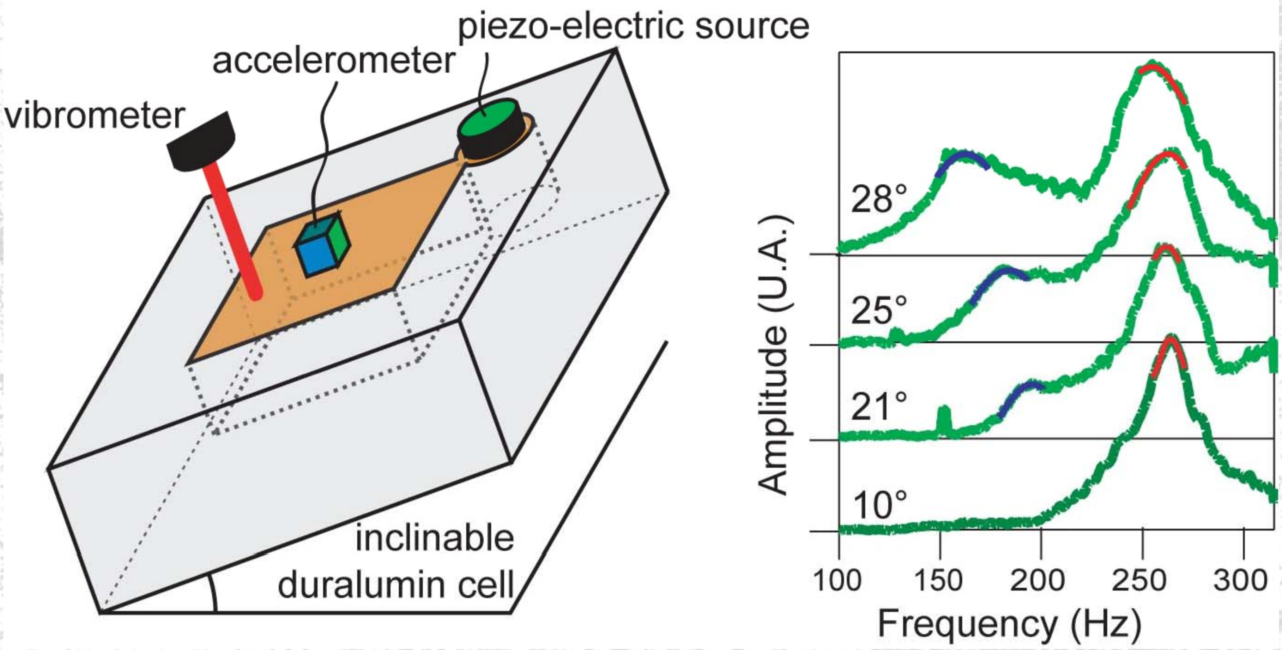
Summary and Conclusions

- In a granular packing under gravity
  - Acoustic perturbations travel as surface waves
  - Waveguide feature of the channel setup selects the fundamental mode
  - Obtained transverse and saggital fundamental mode dispersion relations for a glass-beads packing
  - Hertz non-linear scaling is consistent with our measurements
  - Mean-field fails for determining the elastic constants

- New issues
  - Shear modulus weakness ?
  - How to measure independently A and B (new experimental configuration)
  - The non-linear high amplitude regime ?

Approach to unjamming

Acoustic resonance study (preliminary results)



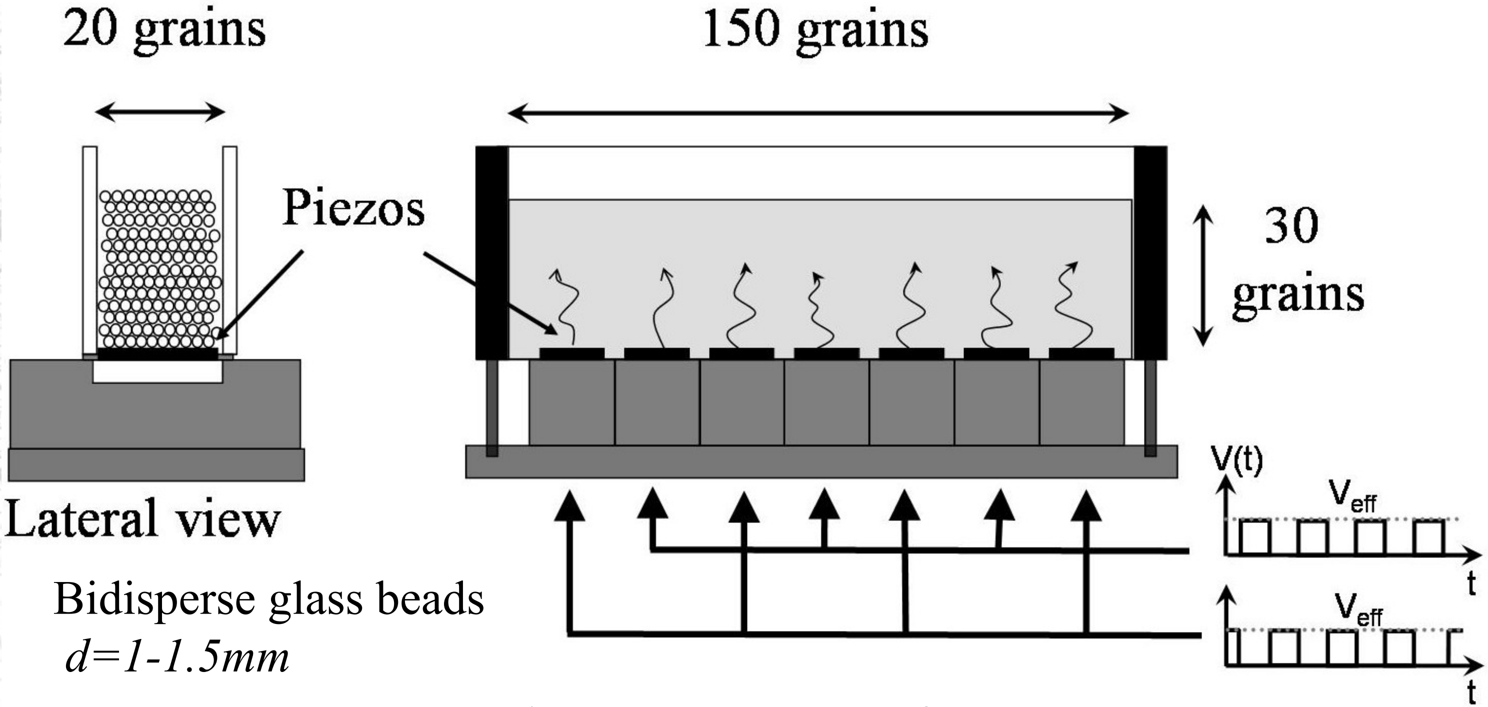
Near avalanche onset

- Stiffness weakening
- Dissipation increases
- « Soft-modes » signature?

Sono-fluidization of granular packing

With Gabriel Caballero C.I.M.A. Monterey Mexico

Ref. Caballero, Clément *preprint* Cond-Mat 0907.0317v2 (2009)
Caballero et al. Powder and Grains,p 339 (2005)



Bidisperse glass beads
d=1-1.5mm

Frequencies : *300 Hz < f < 1000 Hz*

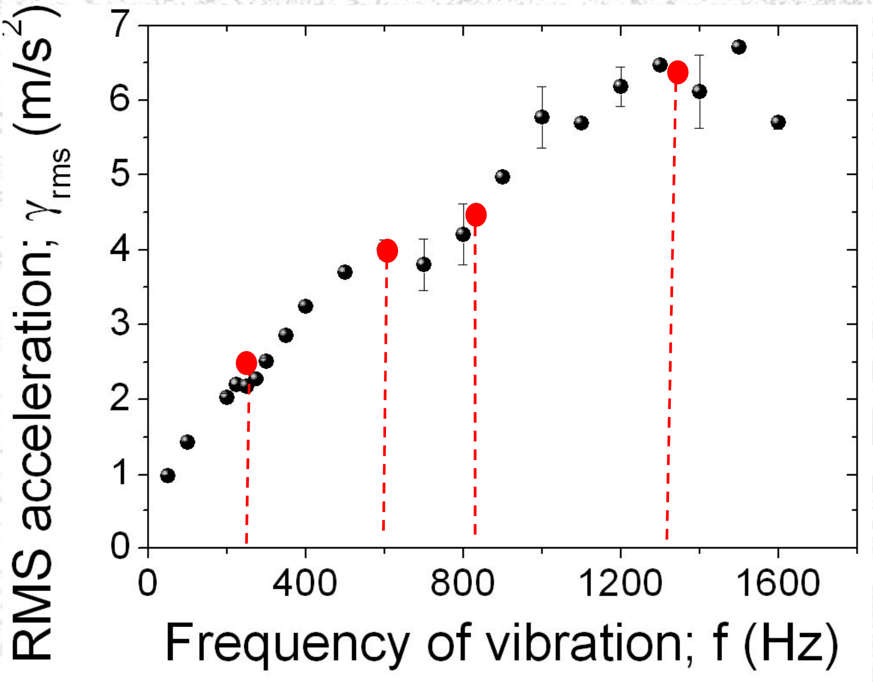
Weak agitation

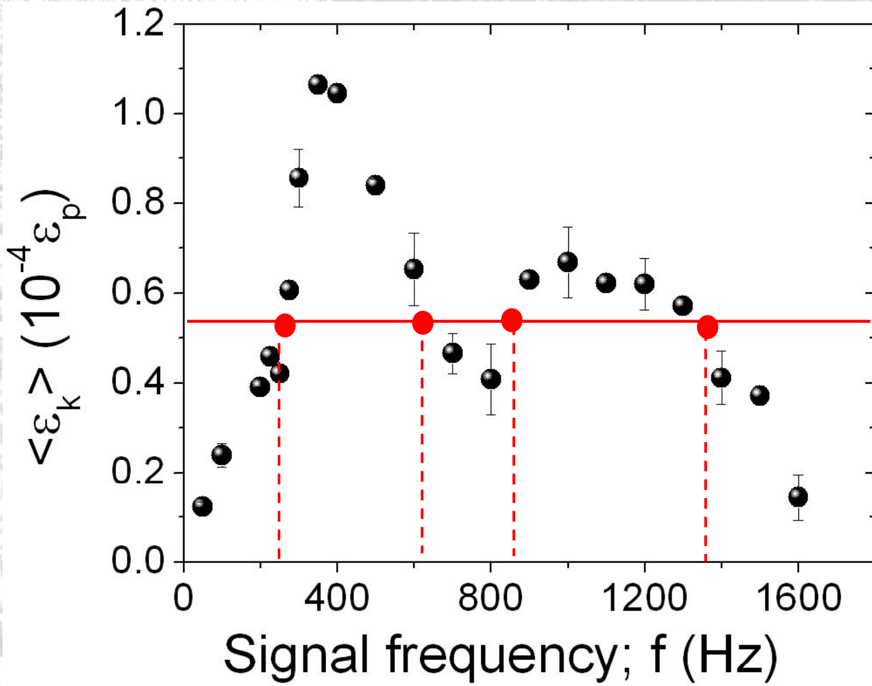
$$\langle \varepsilon_k \rangle < 10^{-4} \rho g d$$

$$\gamma < g$$

Acceleration and energy vs driving frequencies

Constant $V_o = 10V$

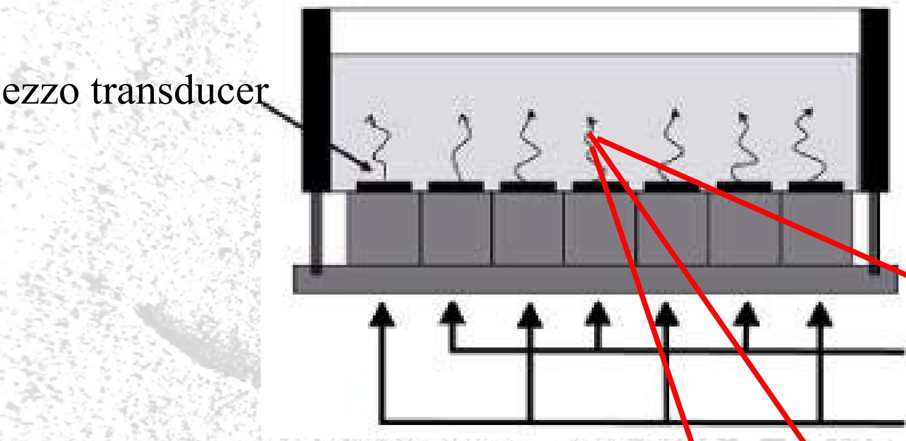




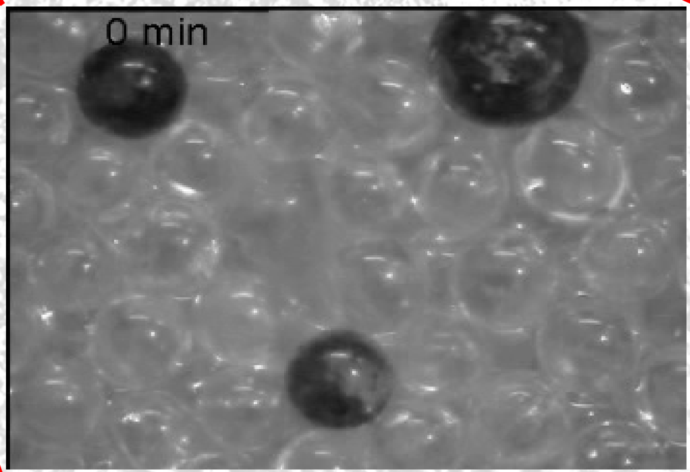
$\gamma_{rms} < g$

$<\varepsilon_k> << \varepsilon_p$

Sono-fluidization



Sound injection (400Hz)

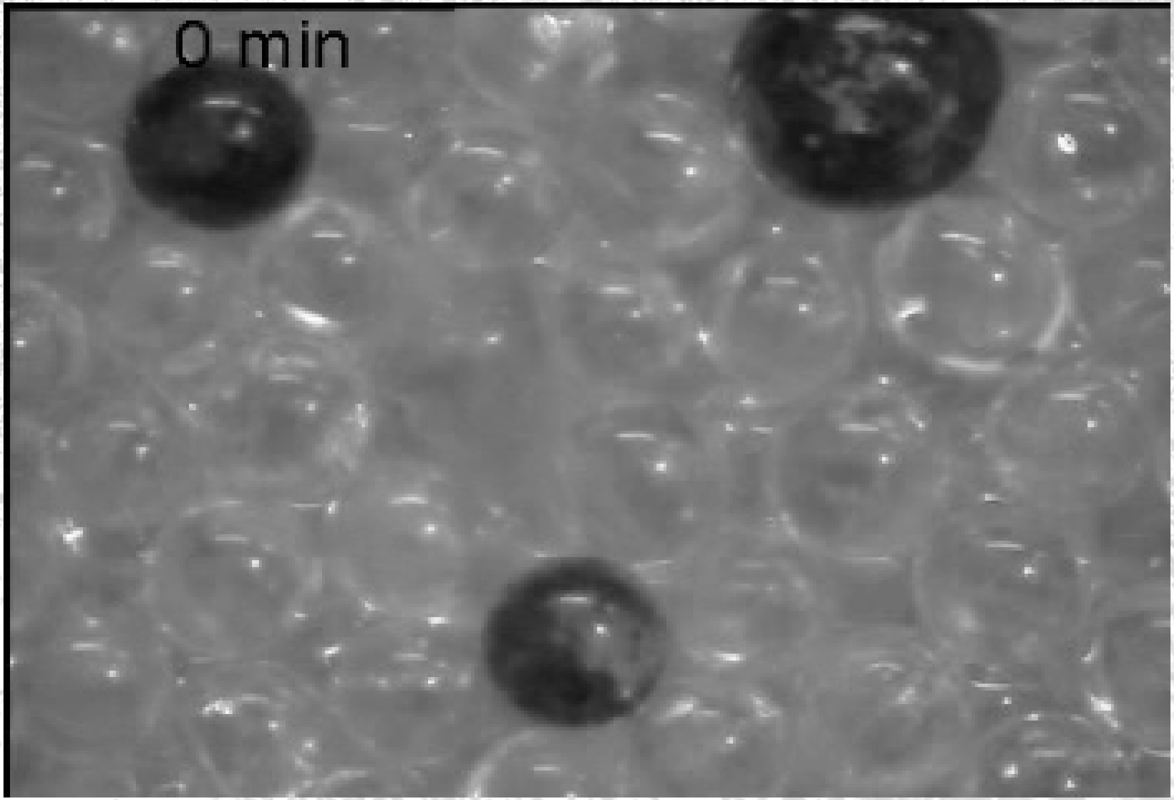


1 image/min

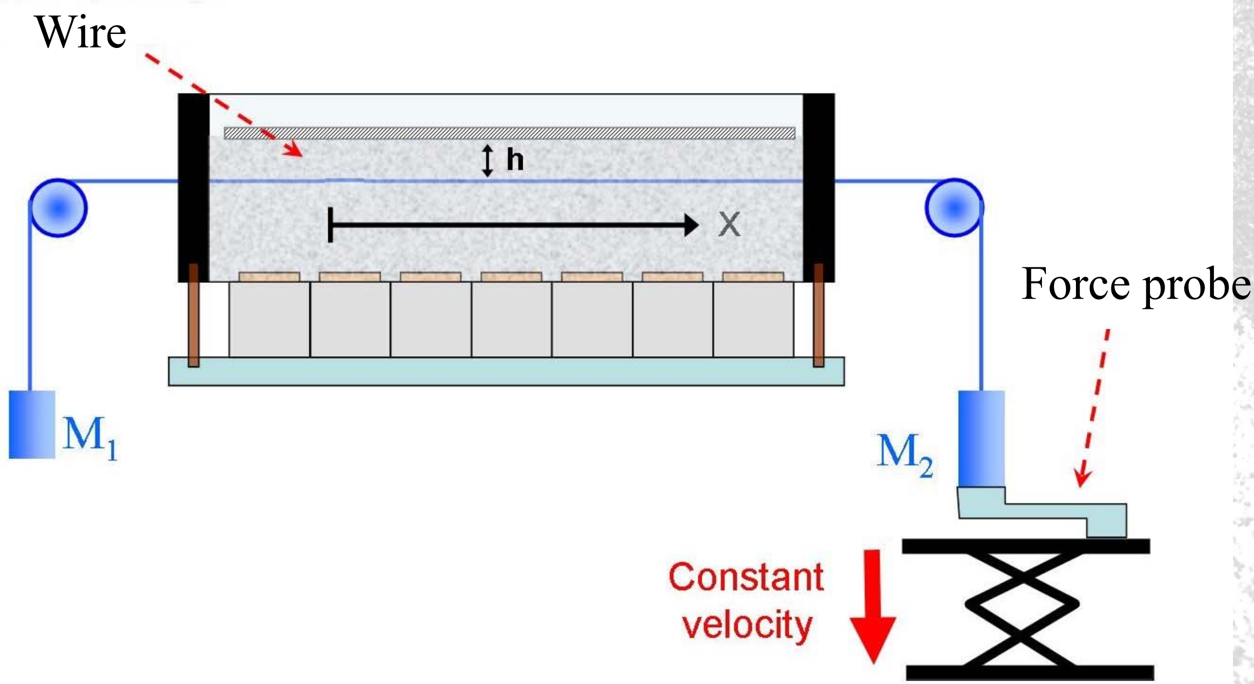
Microscopic dynamics

$F = 400Hz$
$\gamma_{rms}=g/3$

One image each 3 minutes

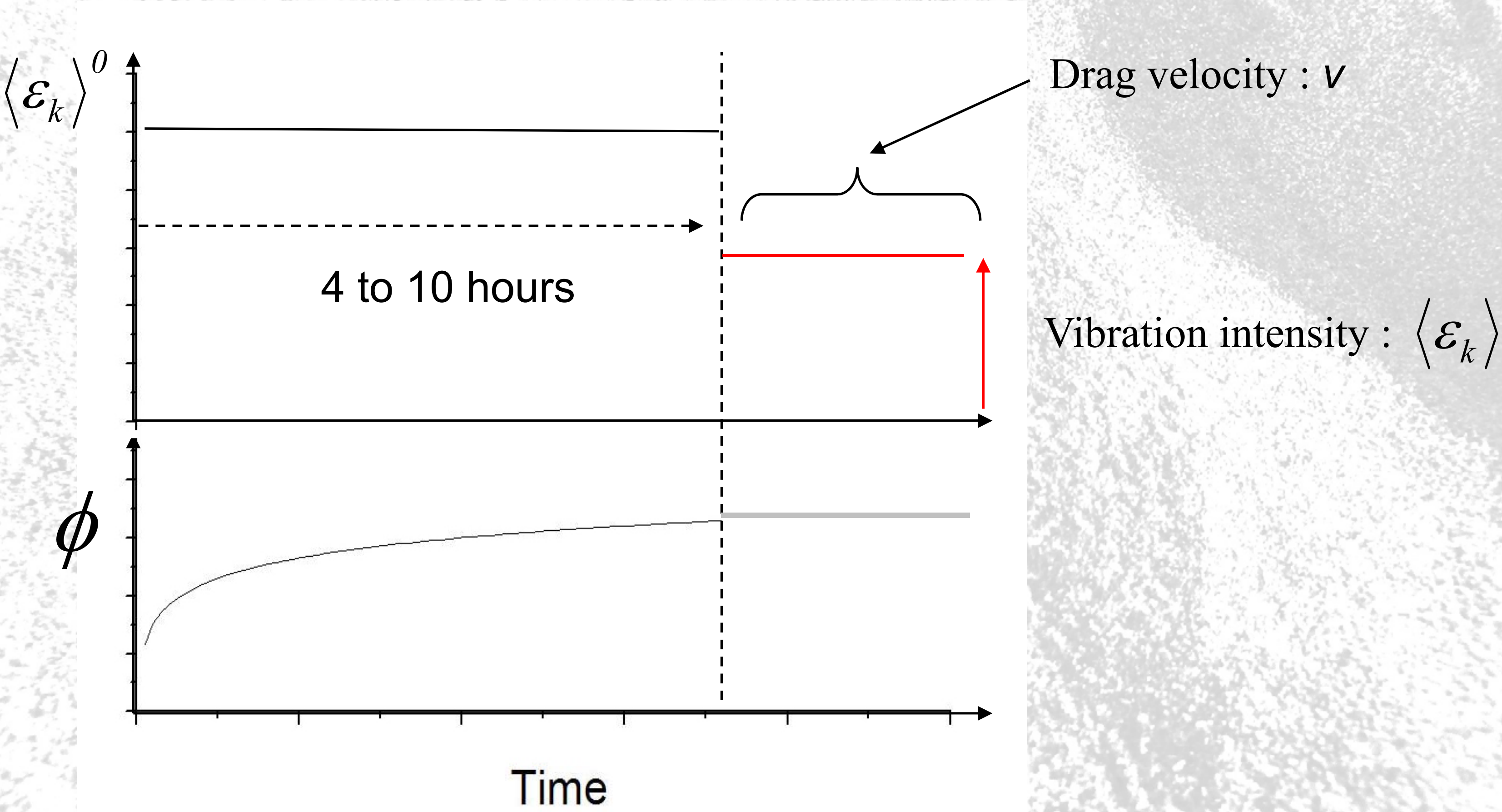


Drag force experiments

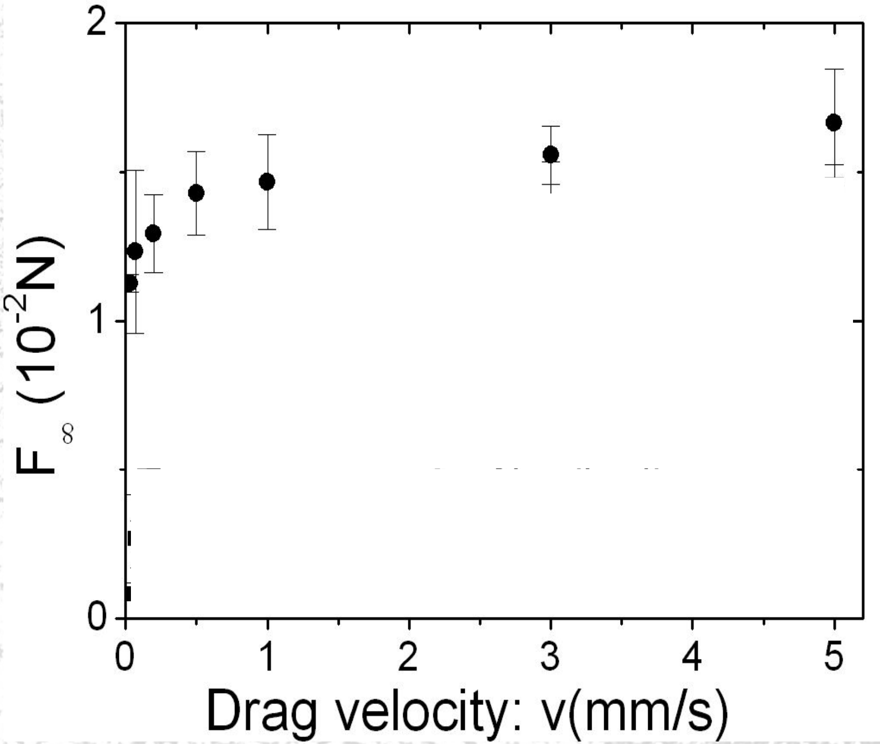


Metallic wire : $e=0.1mm < d=1mm$

Protocol and control parameters



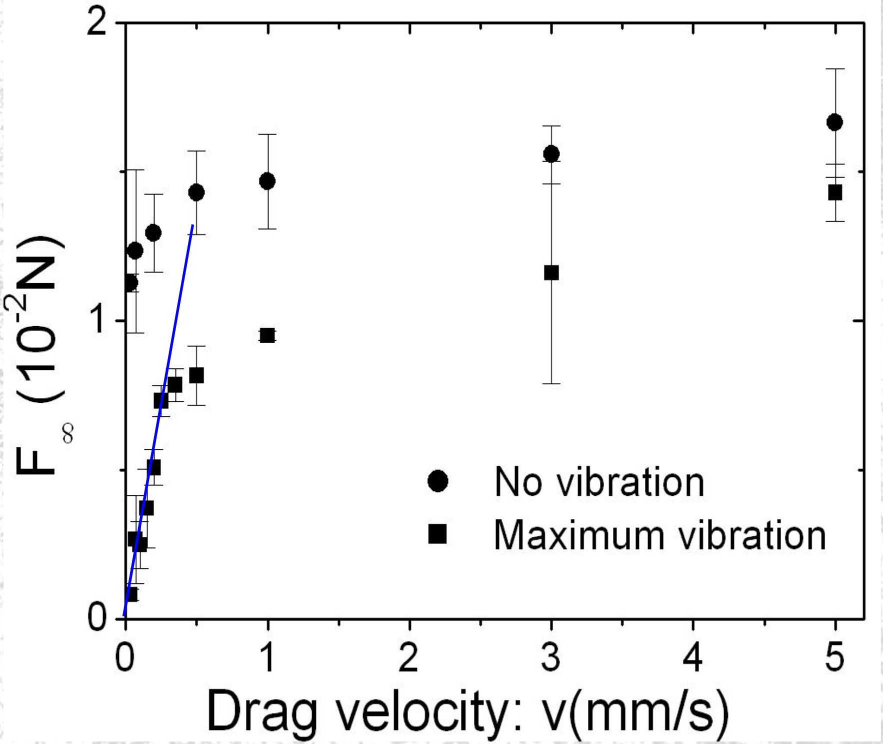
Drag force on the thread

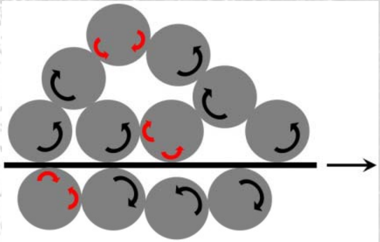


• No vibration

$$F \cong F_0 + A\,ln\left(V / V_0\right)$$

Drag force on the thread



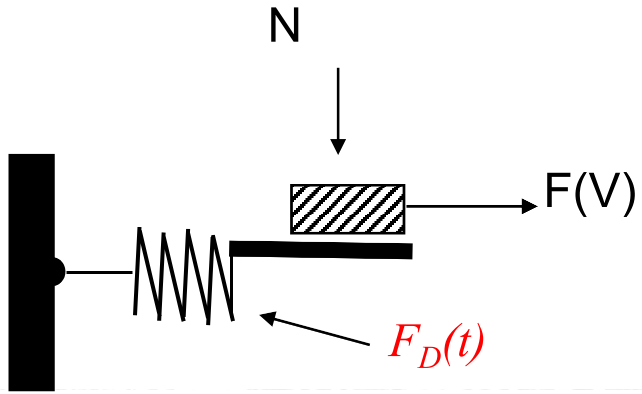


- Low driving velocity

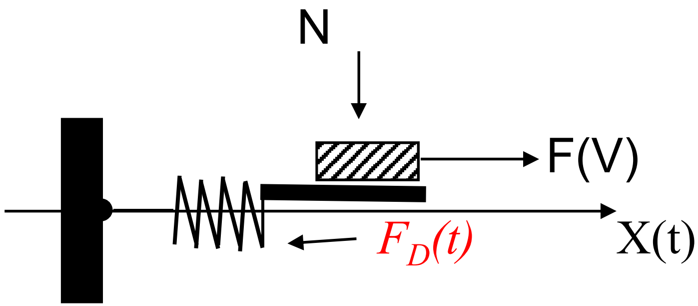
A linear regime : $F \propto V$ Viscosity ?

A simple heuristic model

Elasticity/friction



- Explain dynamic stress threshold vanishing ?
- Explain $F \propto V$?



Constant V block driving
μ friction
ω_0 spring resonance pulsation
M spring mass

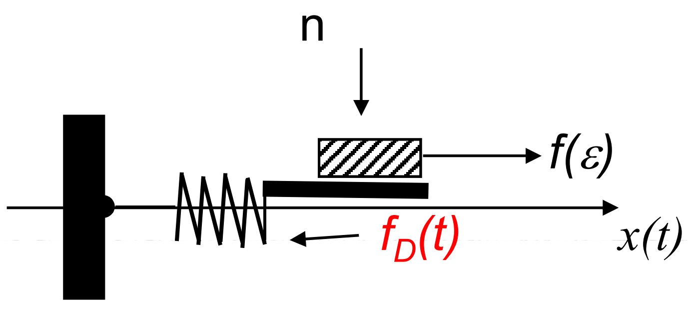
$F_D = M\,G(t)$ driving force on the spring

$G(t) = G_0 \sin 2\pi f_D t$ harmonic driving

Adimensionalisation parameters

$$F_0 = \mu N \qquad t_0 = 1/\omega_0$$

$$V_0 = \frac{\mu N}{M\omega_0} \qquad X_0 = \frac{\mu N}{M\omega_0^2}$$



Dimensionless driving velocity

$$\varepsilon = \frac{V}{V_0}$$

Eq. of motion $\ddot{x} = -x + \tilde{f} - \tilde{f}_D$

Sliding $\tilde{f} = sgn(\varepsilon - \dot{x})$

Sticking $\dot{x} = \varepsilon, \ \tilde{f} = x - \tilde{f}_D$

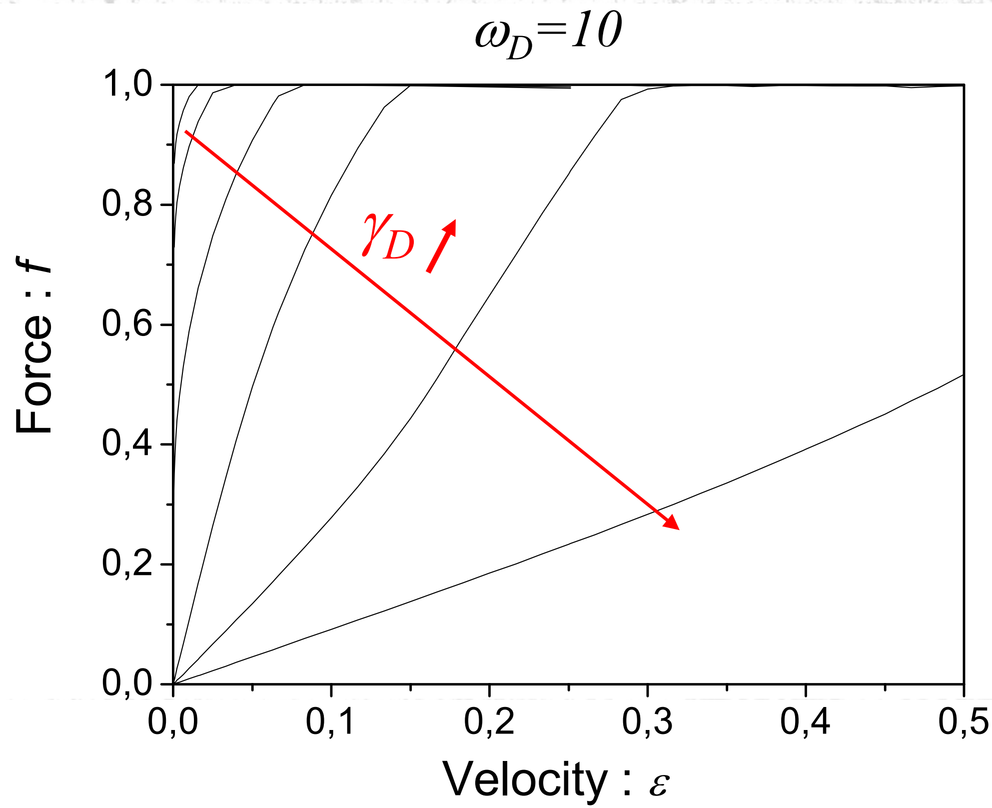
$$\tilde{f}_D = \gamma_D \sin \omega_D \tau$$

Harmonic driving

- Numerical solution : $\bar{f}(\varepsilon)$ for varying γ_D the driving amplitude
- Exact results when $\varepsilon >> 1$ $\bar{f}(\varepsilon) \to 1$

when $\gamma_D >> 1$ $\bar{f}(\varepsilon) = \frac{2}{\pi} \frac{|\omega_D^2 - 1|}{\omega_D} \varepsilon \propto \varepsilon$

Mean drag force $f(\varepsilon)$



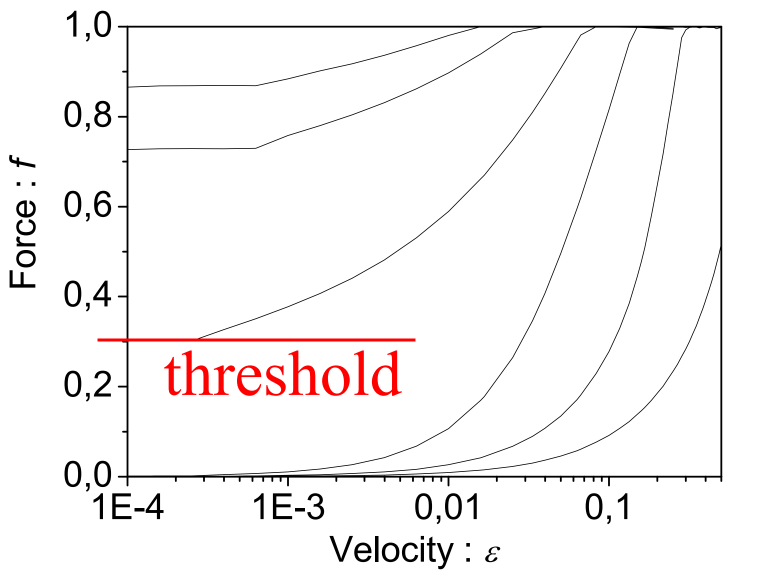
$$\gamma_D = \frac{MG_0}{\mu N}$$

$$\tilde{f}_D = \gamma_D \sin \omega_D \tau$$

- Weakening with driving amplitude $\quad \bar{f}(\varepsilon)\downarrow$ when $\gamma_D \uparrow$
- Hardening rheology $\quad \bar{f}(\varepsilon)\uparrow$ when $\varepsilon \uparrow$

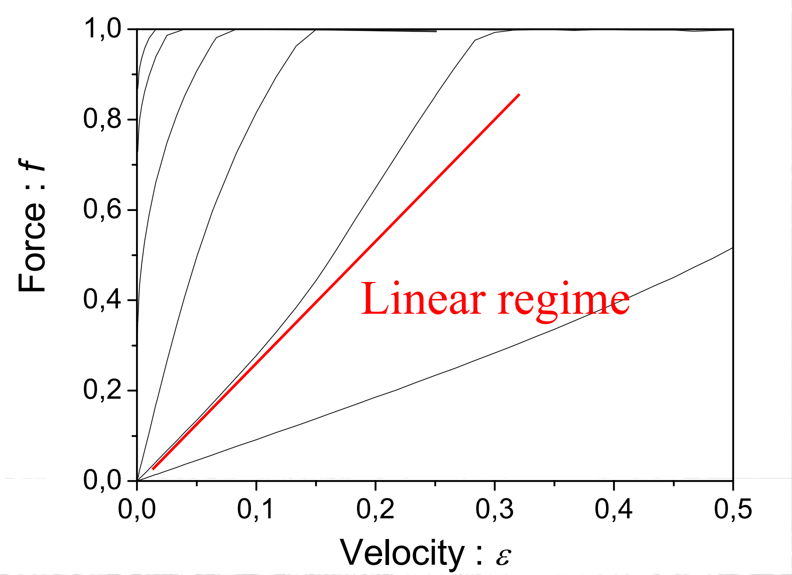
Mean drag force $f(\varepsilon)$

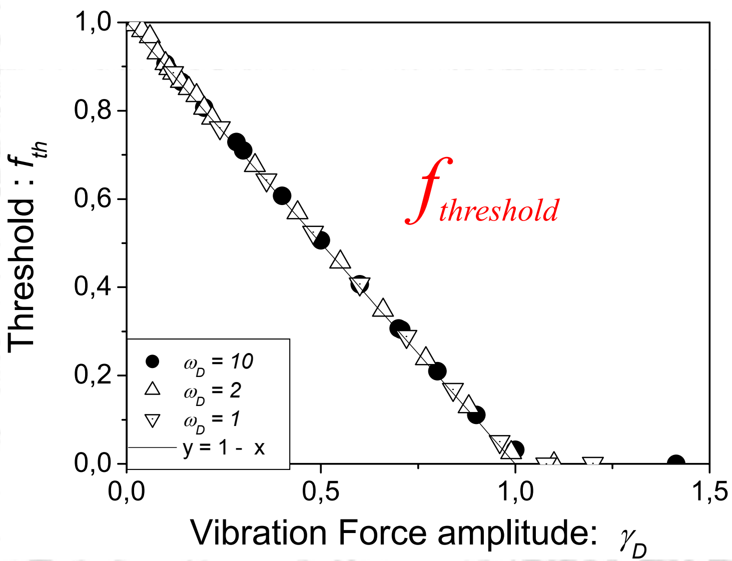
Low amplitude driving



$\omega_D = 10$

Large amplitude driving





- if $\gamma_D < 1$, threshold, $\mu_{th} = \mu(1 - \gamma_D)$
- if $\gamma_D \geq 1$, linear regime

$$F \propto \mu \frac{V}{\delta V} N$$

δV rms velocity fluctuations

Summary on the model outcome

- Simple heuristic model (internal elastic modes / solid friction)
- Drag resistance decreases with driving intensity
- Stress hardening (no lnV regime)
- Low driving intensity : threshold friction force at low V
- Large driving intensity : linear relation $F \approx \mu N \frac{V}{\delta V}$

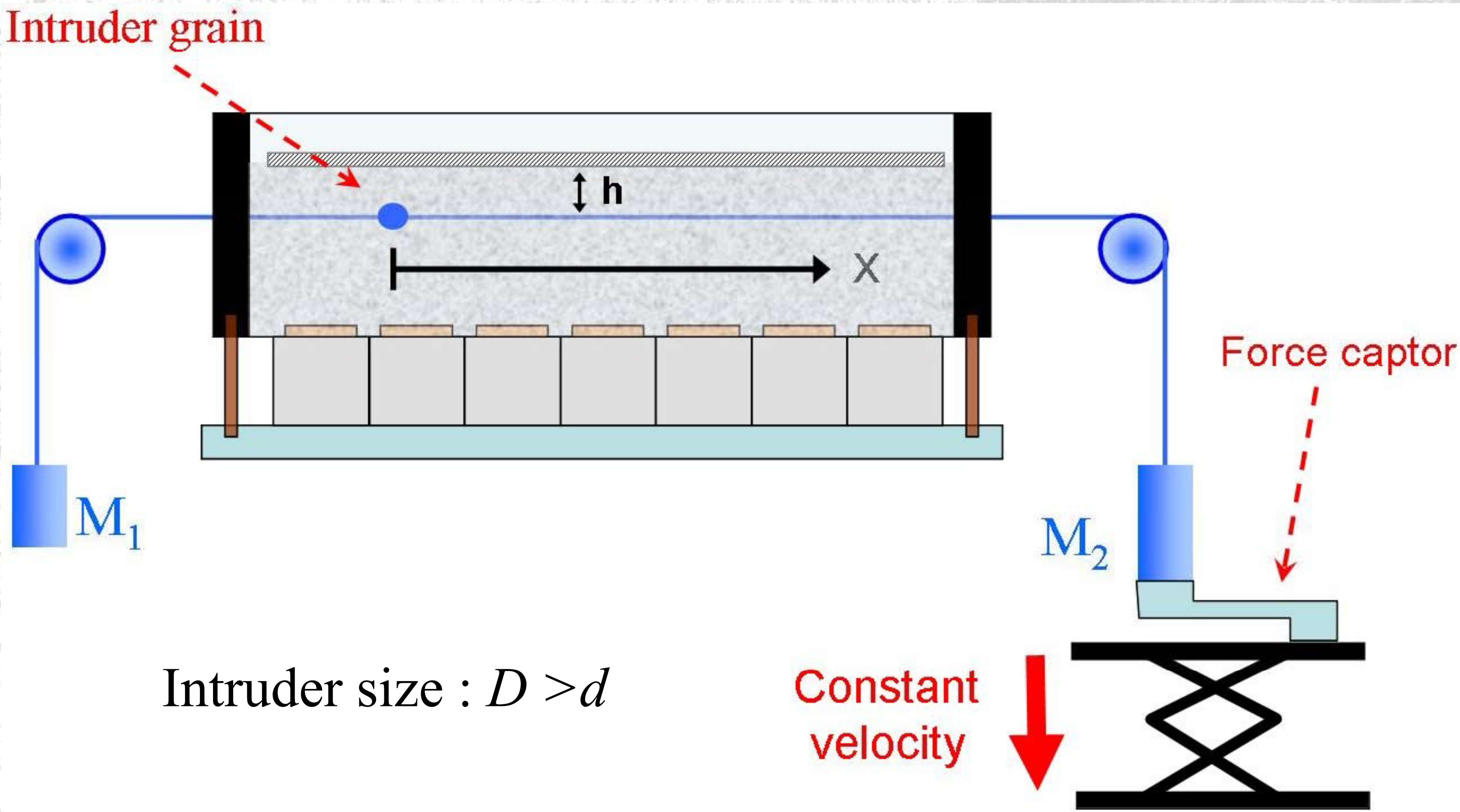
Experimentally

$$\mu_{eff} = \frac{F}{N} = \alpha \frac{V}{\delta V}$$

δV rms velocity fluctuation

$\delta V = 1.2\ 10^{-3}$ m/s
$\alpha = 1.3$

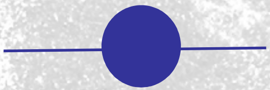
Drag force on a spherical intruder

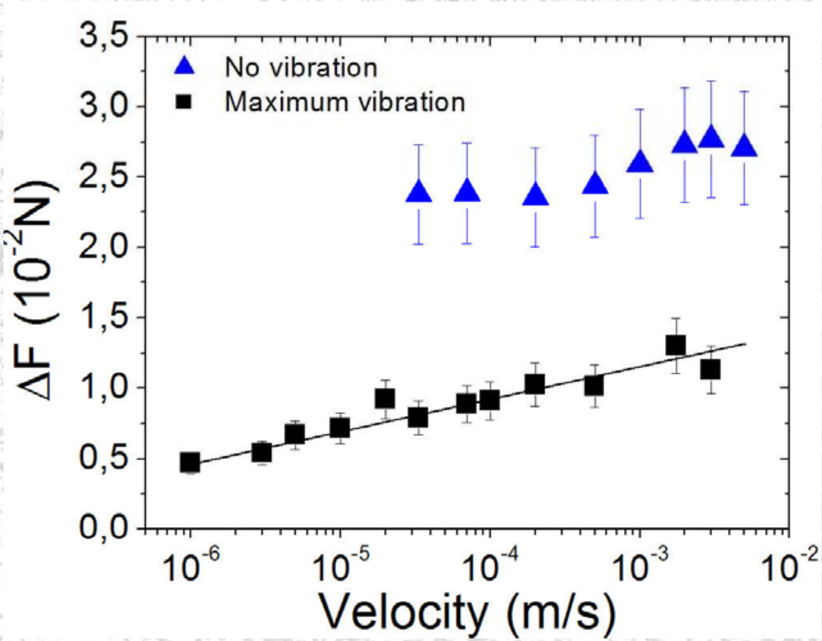


Intruder size : $D > d$

Drag force on thread + bead

Bead contribution : $\Delta F = F_{\infty} - F_{\infty}^{(thread)}$





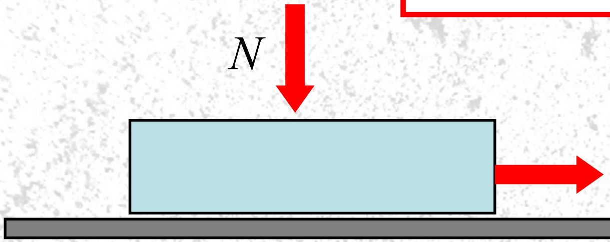
D=2mm

No vibration

Vibration

- Stress strengthening
- Threshold at low vibration intensity
- Non linear variation : $\Delta F \cong \Delta F_0 + B\, ln\left(V / V_0\right)$

Solid/solid friction : creeping motion



$$F = \mu N$$

$$\mu(\lambda, \dot{x}) \propto \mu_0 + B \ln(\frac{\lambda}{\lambda_0}) + A \ln(\frac{\dot{x}}{V_0})$$

$$\dot{\lambda} = 1 - \frac{\lambda \dot{x}}{D_0}$$

λ : age of the interface

(Dietrich – Ruina closure)

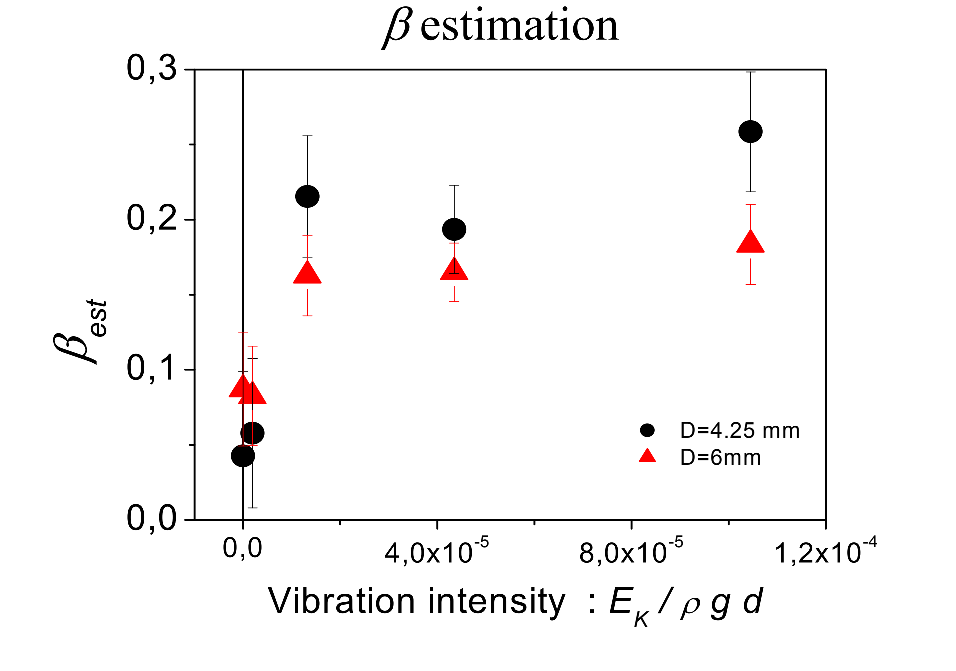
- Stress strenghtening/stress weaknening regimes
- Pinning/unpinning dynamics
- Thermal activation of the unpinning processes

$$A \propto \frac{k_B T}{\sigma_O \xi^3}$$

« nano bloc » size $\xi = 10^{-9}$m

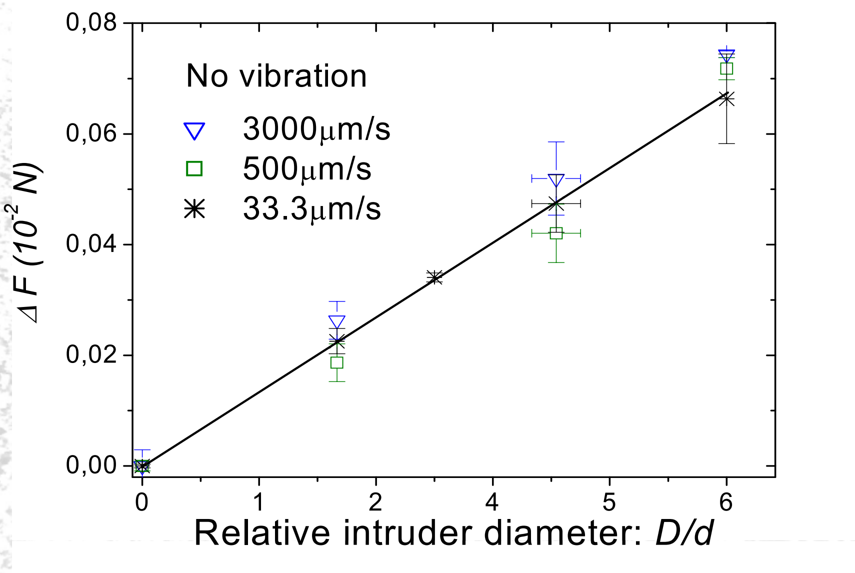
Ex: Baumberger et al. PRB (1999)

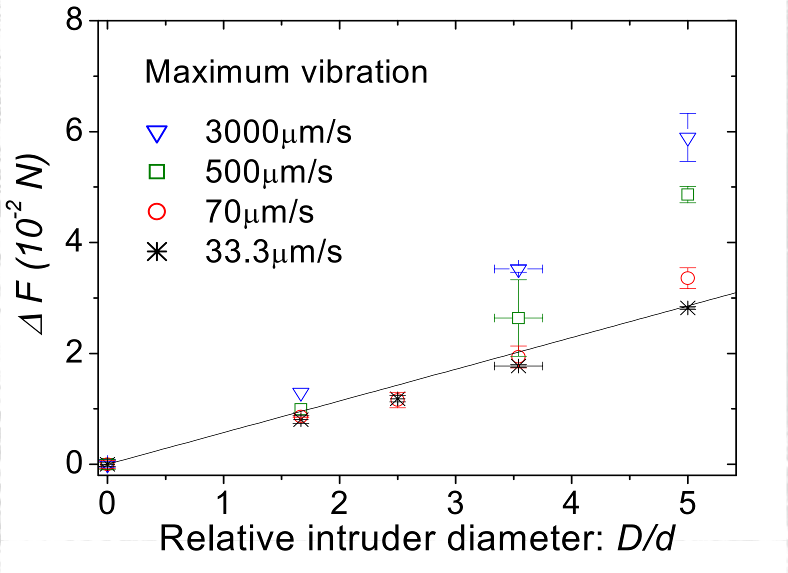
$$\mu(v) = \mu(v_{ref}) + \beta \ln(v/v_{ref})$$



- Weak dependence on kinetic energy

Intruder size dependence





Without vibration
With vibration at low V ⇨ $\Delta F \propto D$

• Anomalous « geometrical hardening » $\mu_{eff} \propto D^{-1}$

Summary

• Effective friction of a macroscopic intruder decreases with vibration intensity

•Effective rheology displays logarithmic hardening

$$\mu(\mathrm{v}) = \mu(\mathrm{v}_{ref}) + \beta\, ln(\mathrm{v}/\mathrm{v}_{ref})$$

• Effective friction is larger for smaller objects (geometrical hardening)

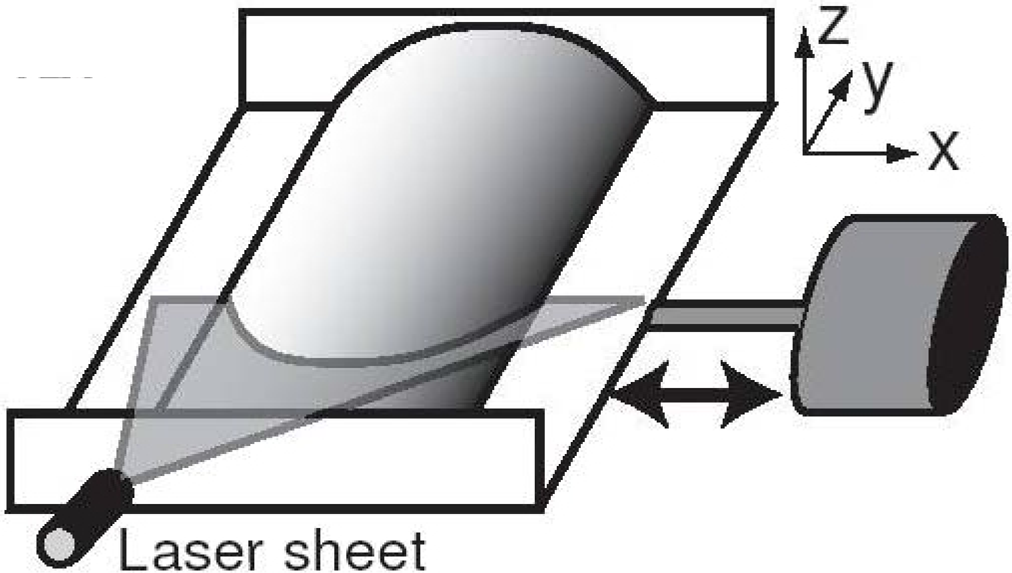
• Hardening weakly depends on vibration energy

Spreading of a granular droplet
An inertia tribometer

With

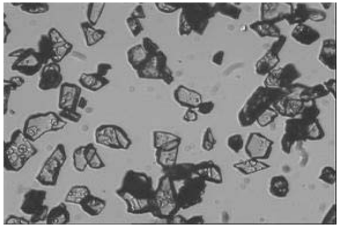
- ***Ivan SANCHEZ, Univ. Simon Bolivar-Venezuela***
- ***Franck RAYNAUD, José LANUZA, Bruno ANDREOTTI, ESPCI***
- ***Igor ARANSONArgonne National Lab***

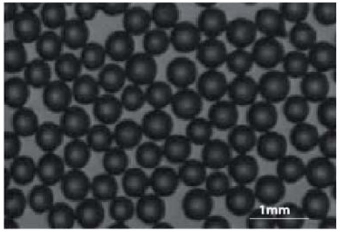
Ref : Sanchez et al, Phys.Rev.E **76**, 060301 (2007)



Horizontal vibrations

Grains
d=250μm



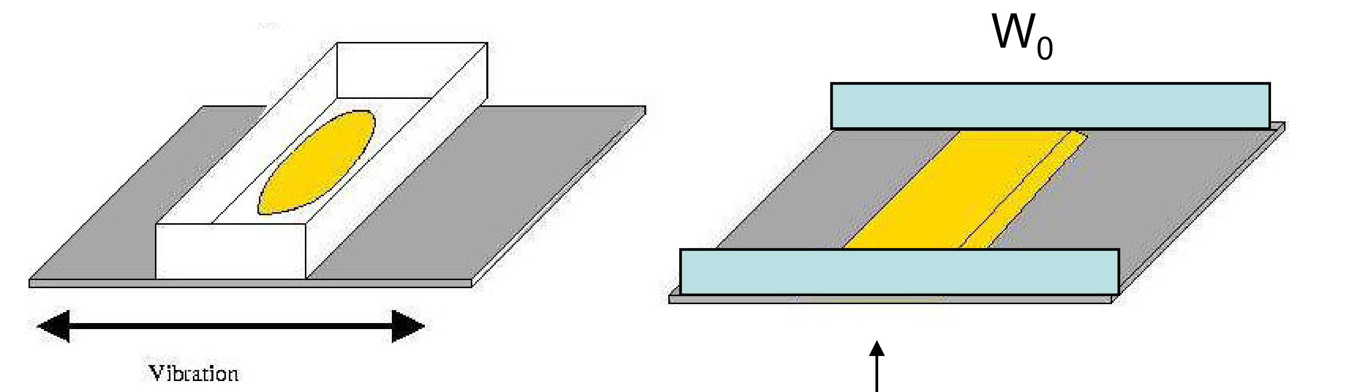


Fontainebleau Sand

Glass beads

Substrate roughness : *10μm<<d*

Granular film preparation procedure

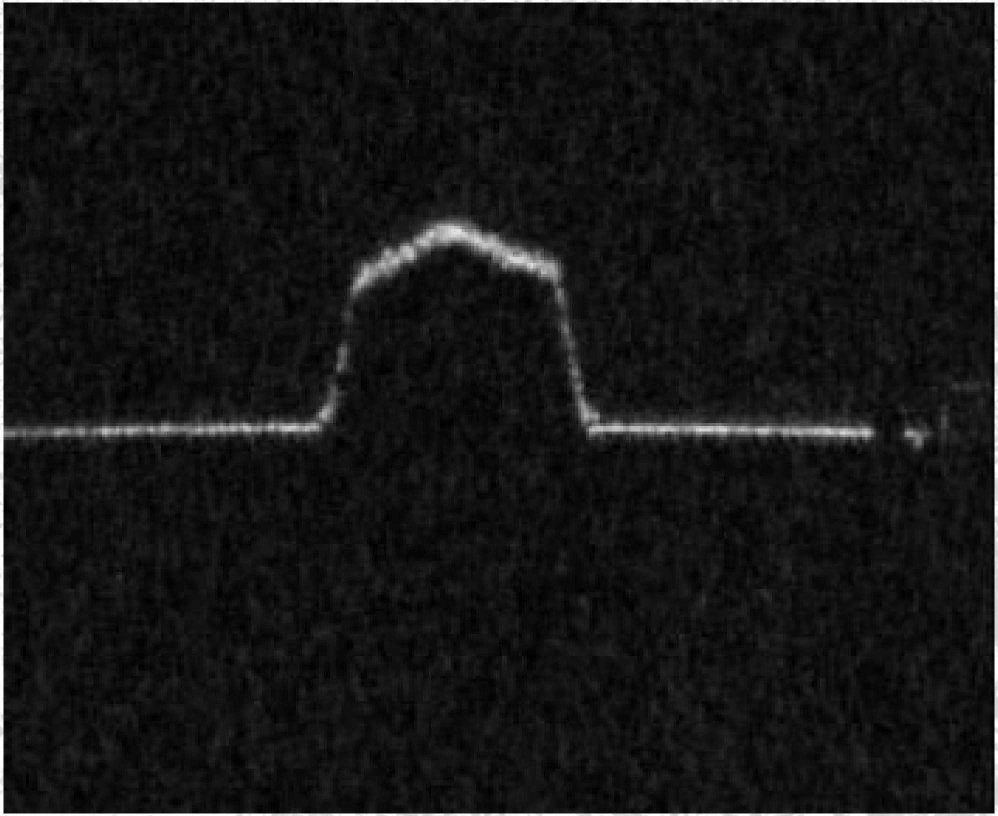


Mobile support

after vibration
at 40Hz during 1min

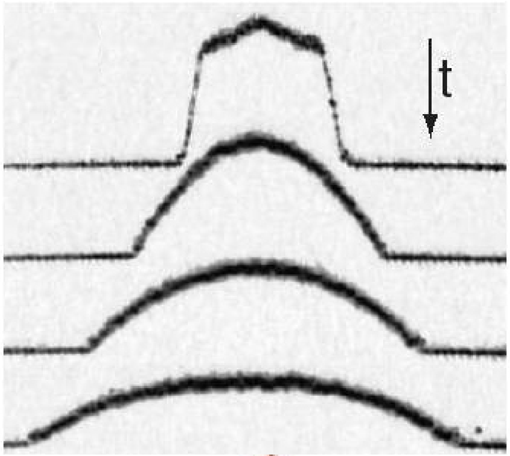
$h_0=3\text{-}6\ mm \ll W_0=2\text{-}5cm$

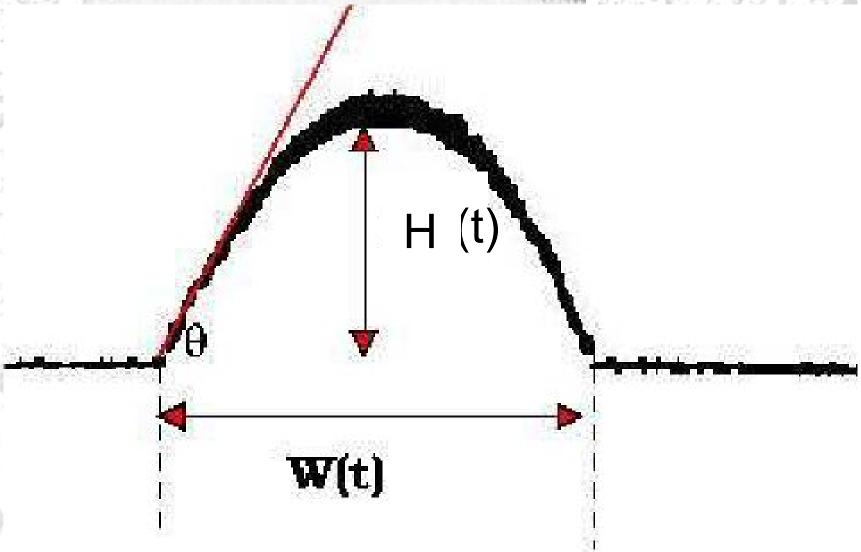
$f = 26\ Hz$



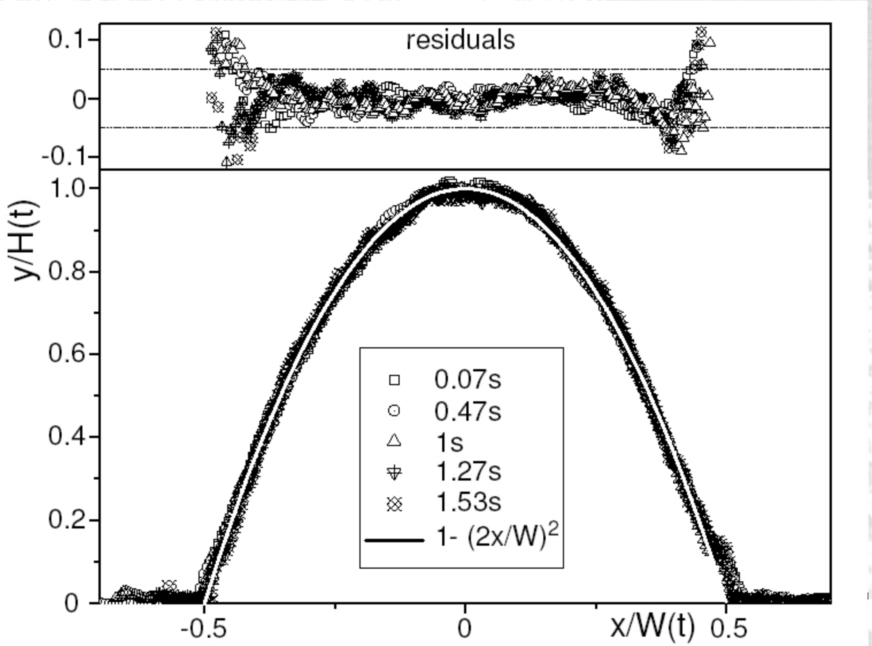
X10

Universal parabolic profile shape

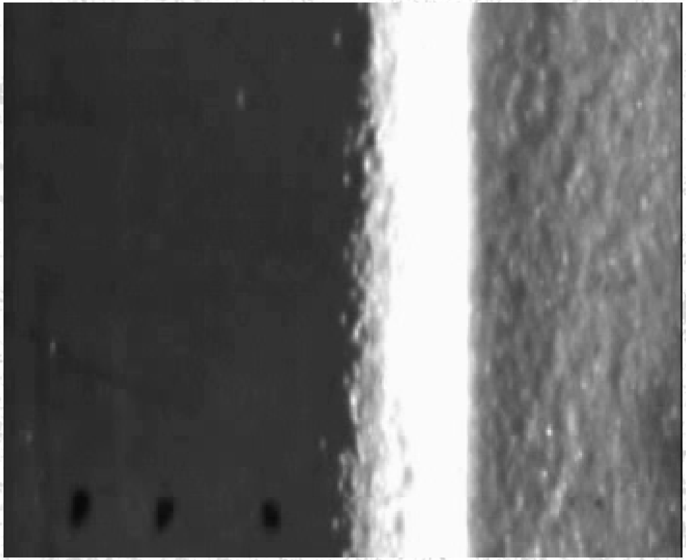


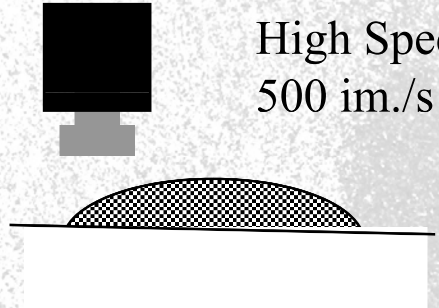


The droplet !

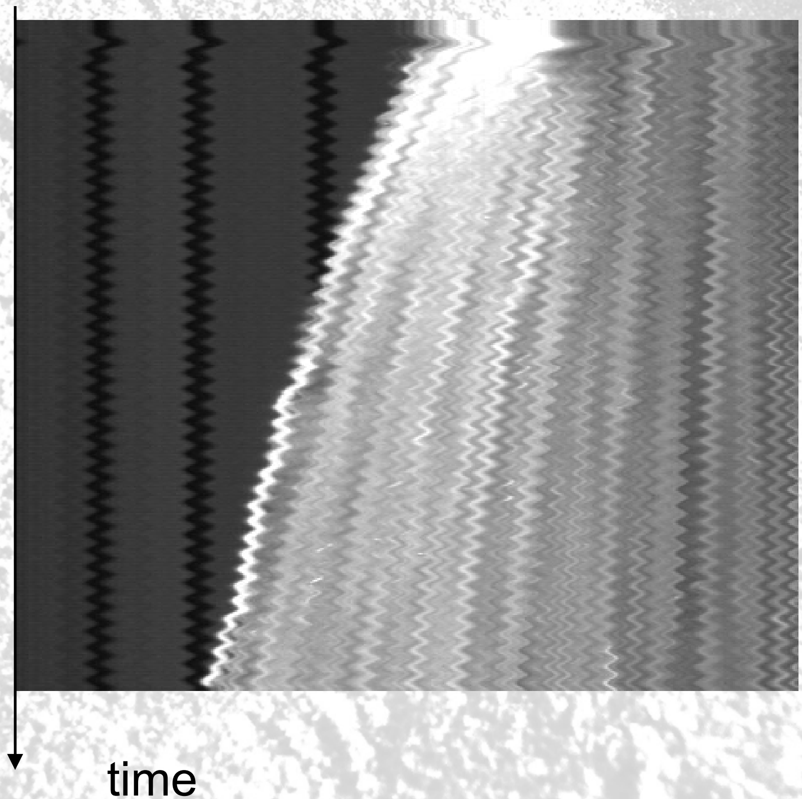


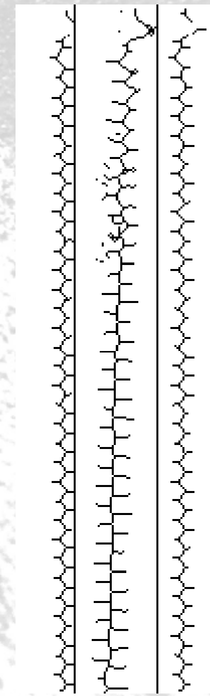
Parabolic profile



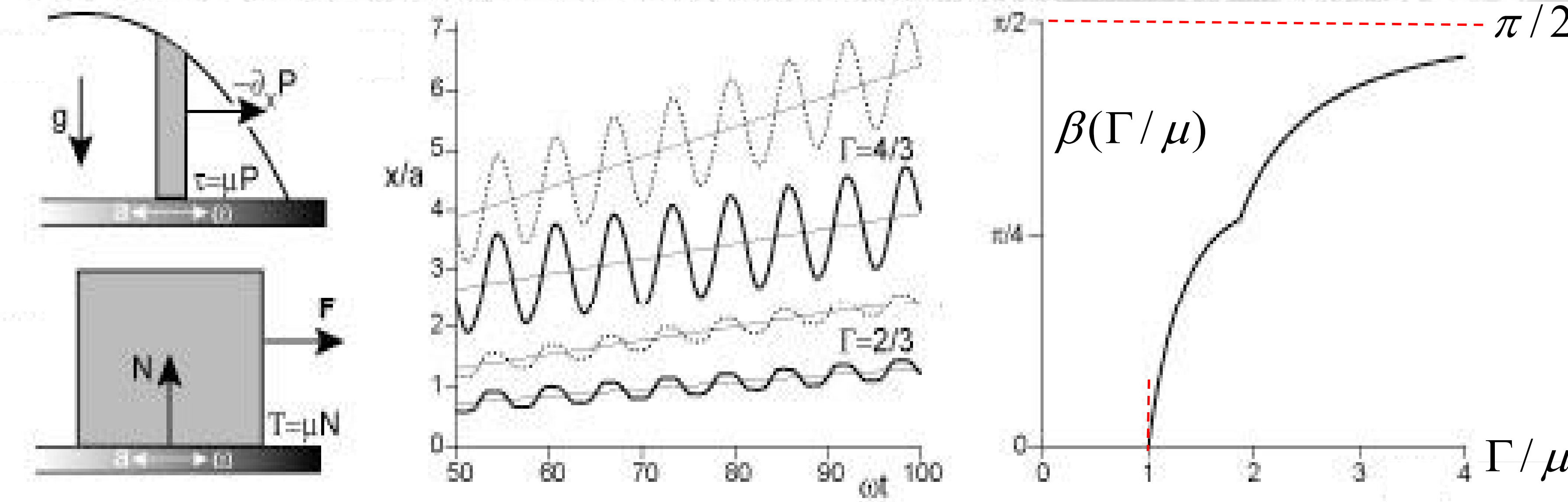


High Speed Camera
500 im./s





Sliding Block Model



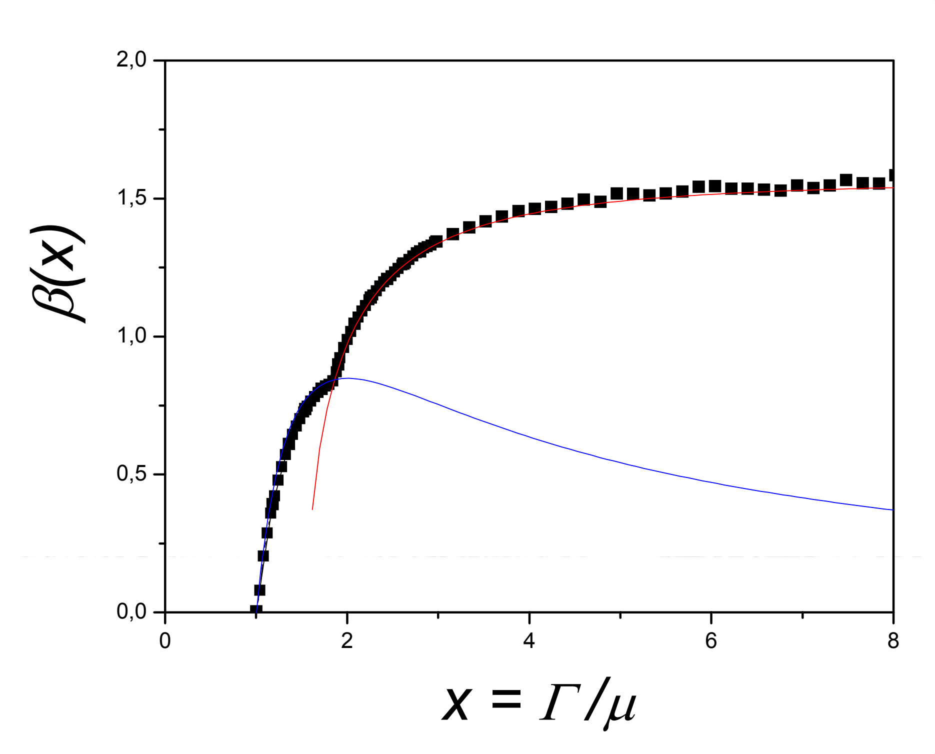
Model parameters

- Harmonic plate motion : $y_p = a \sin \omega t$
- Sliding friction : μ
- Relative acceleration : $\Gamma = a\omega^2/g$

Exact result

- Block sliding velocity : $V = \dfrac{\beta(\Gamma/\mu)}{\mu} a\omega \dfrac{F}{N}$

$$F = \frac{\mu}{\beta(\Gamma / \mu)} \frac{V}{a\omega} N$$



Continuous sliding

$$\beta\left(\frac{\Gamma}{\mu}\right) = \frac{\pi}{2}\sqrt{1-\left(\frac{\pi}{2}\frac{\mu}{\Gamma}\right)^2}$$

Stop/go dynamics

$$\beta\left(\frac{\Gamma}{\mu}\right) = \frac{(\phi - \arcsin(\mu/\Gamma))}{2\pi}\frac{\mu}{\Gamma}$$

$$\cos\phi - \sqrt{1-(\mu/\Gamma)^2} + \mu/\Gamma(\phi - \arcsin(\mu/\Gamma)) = 0$$

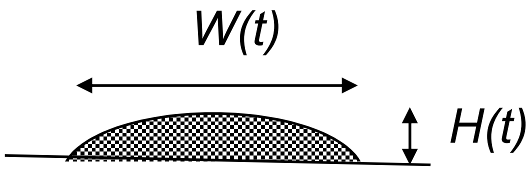
$$J = hV$$ Flux of matter

$$\partial_t h + \partial_x J = 0$$ Mass conservation

$$\boxed{\partial_t h = \partial_x U \partial_x h^2}$$ Non-linear diffusion equation

with $U = \frac{a\omega}{2\mu}\beta\left(\frac{\Gamma}{\mu}\right)$

$$S = 2W(t)H(t)/3$$

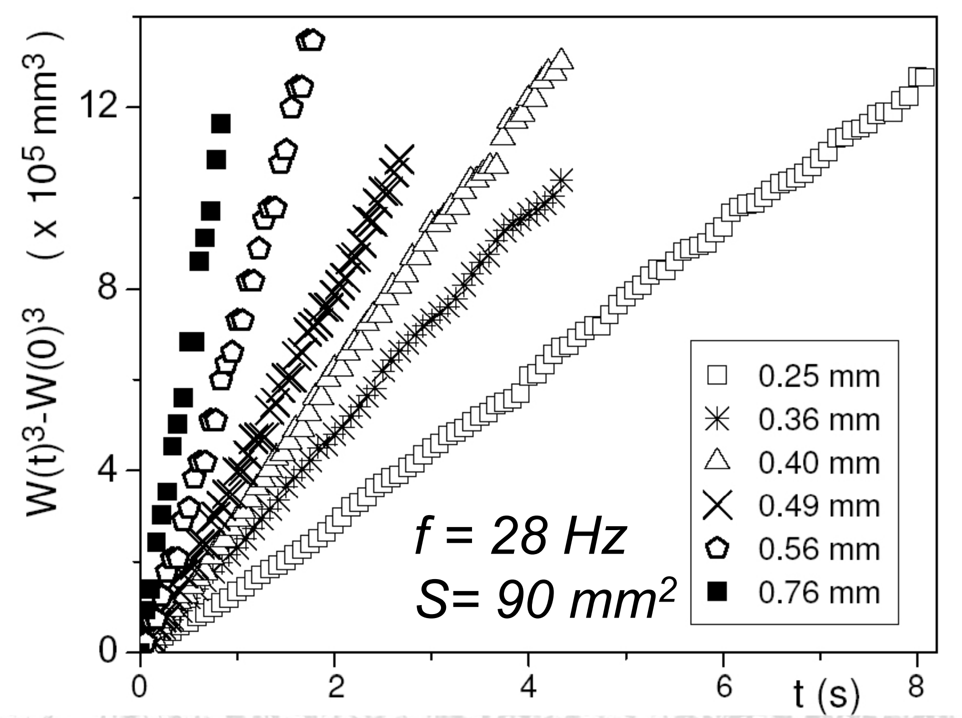


$$h(x,t) = \frac{3S}{2W(t)}\left[1 - \left(\frac{2x}{W(t)}\right)^2\right]$$ Parabolic profile

$$W^3(t) - W_0^3 \cong \frac{36\pi\, S\, a\omega}{\mu}\beta\left(\frac{\Gamma}{\mu}\right)t$$ Spreading law

$$\alpha = \frac{36\pi\, S\, a\omega}{\mu}\beta\left(\frac{\Gamma}{\mu}\right)$$

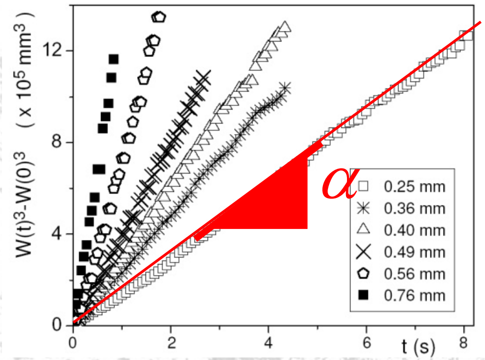
Spreading dynamics



Experimentally

$$W^3(t) - W_0^3 = \alpha t$$

An inertial tribometer !



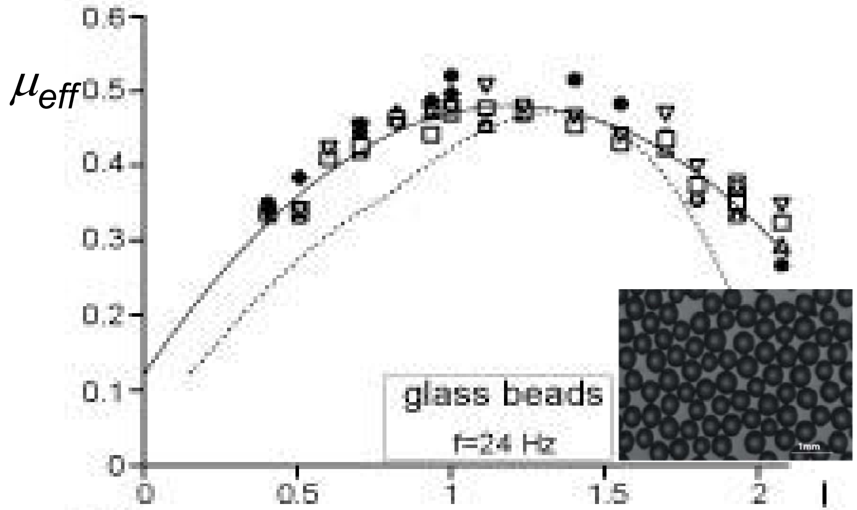
$$\alpha = \frac{dW^3}{dt} = \frac{18gS}{\pi f}\,\frac{\Gamma}{\mu}\beta\left(\frac{\Gamma}{\mu}\right)$$

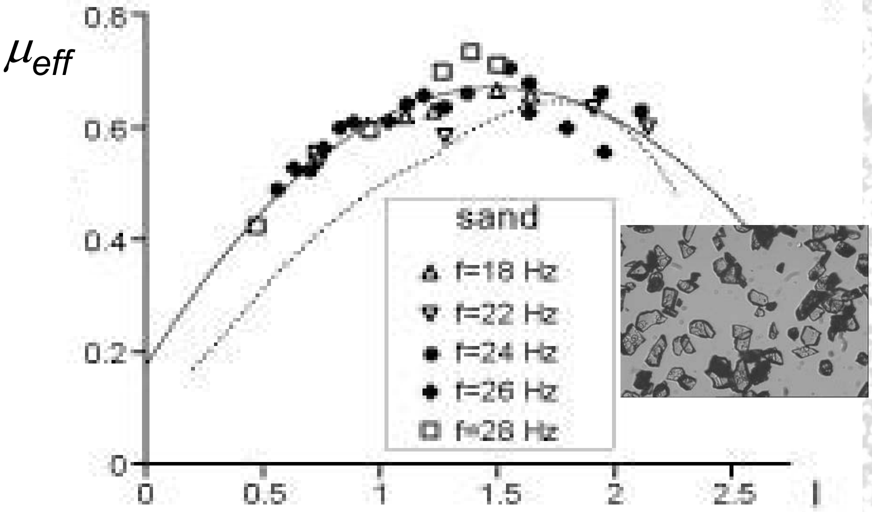
Invert the relation

$$\frac{\Gamma}{\mu}\beta\left(\frac{\Gamma}{\mu}\right) = \frac{18gS}{\alpha\,\pi\,f}$$

- Get the effective basal friction : μ_{eff}

Effective basal friction under vibration



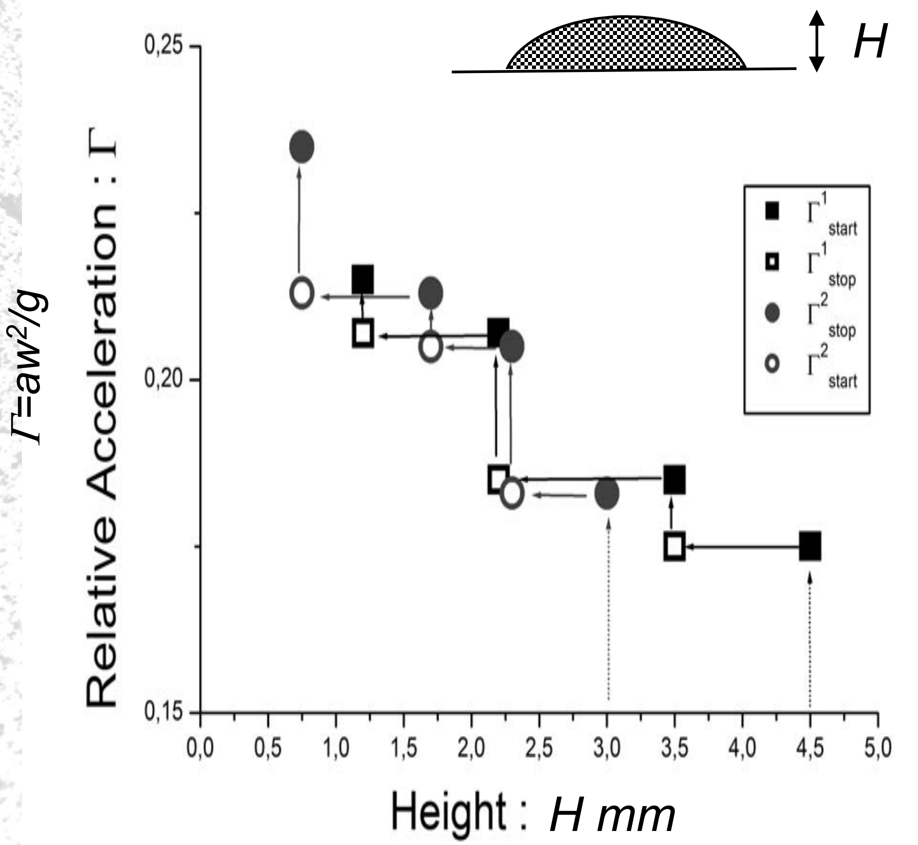


- Measure the effective sliding friction under vibration

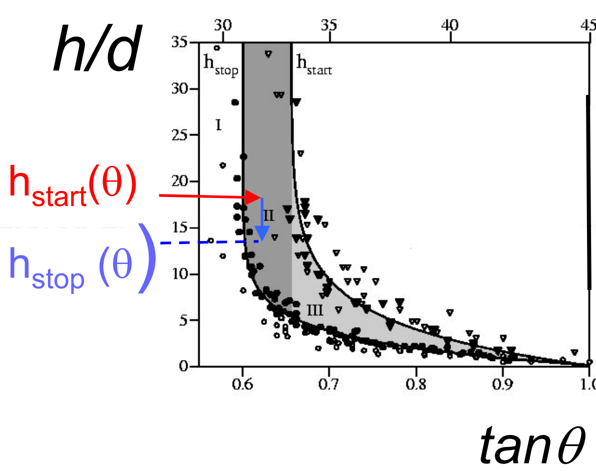
$$\mu_{eff}(I) \quad with \quad I = \frac{a\omega}{\sqrt{gd}}$$

- Hardening (low I) and weakening (high I)

Dynamics in the metastable domain?



Granular layer on a slope



$\mu_s(H)$ and $\mu_d(H)$!

Conclusions

- A granular film spreads when shaken horizontally

- Parabolic universal shape :
- Spreading law : $W^3(t) - W_0^3 = \alpha t$
- From theory one obtains the effective basal friction : $\mu_{eff}(a\omega/(gd)^{1/2})$

Non monotonous variation !
- Hardening at low vibration
-Weakening at higher vibration