International School on Complexity
Grains, Friction, and Faults

Power Laws and Scaling Laws in Earthquake Occurrence

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Erice, July 2009

1. **Size Distributions, Power Laws & SOC:**
2. **Waiting-Time Distributions & Scaling Laws:**

Traditional Reductionist Way of Doing

Case of physics:

- Matter is complex

$\Rightarrow$ Find its ultimate constituents

Case of earthquakes:

- An earthquake is a very complex phenomenon whose physics is largely unknown

$\Rightarrow$ Study specific parts of the problem

Great 2004 Sumatra-Andaman earthquake: more than 100 papers! (by title)
15 in *Nature* or *Science*!

Complementary Approach: Complex-Systems Philosophy

- Can we learn something from collective properties?

$\Rightarrow$ Study emergent statistical properties of (relatively) large areas:
Concentrate on the whole rather than on the parts

Hundreds of earthquakes are needed for a single paper!

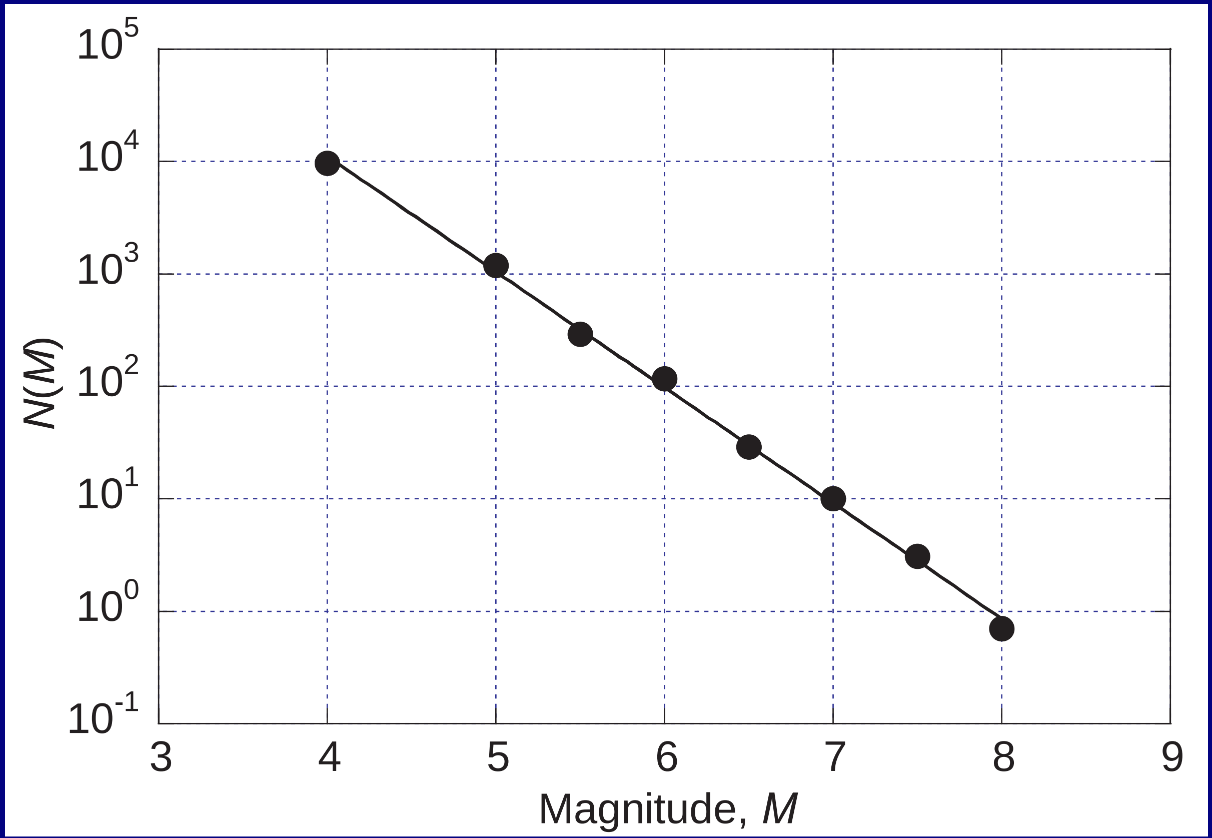
Gutenberg-Richter Law: most important law for the statistics of seismicity

- For each earthquake with magnitude $M \geq 8$ there are about
  - ⋆ 10 with $M \geq 7$
  - ⋆ 100 with $M \geq 6$, etc...

⇒ Number of earthquakes decays exponentially

$$N(M) \propto 10^{-bM}$$

(with $b \simeq 1$)



Kanamori & Brodsky, Rep. Prog. Phys. 2004

⇒ Many small earthquakes, few big, good news!

Distribution of magnitudes

- We use the concept of probability density, defined as

$$D(M) \equiv \frac{\text{Prob } [M \leq \text{magnitude} < M + dM]}{dM}$$

and estimated as

$$D(M) = \frac{\text{number of earthquakes with } M \leq \text{magnitude} < M + dM}{\text{total number of earthquakes} \times dM}$$

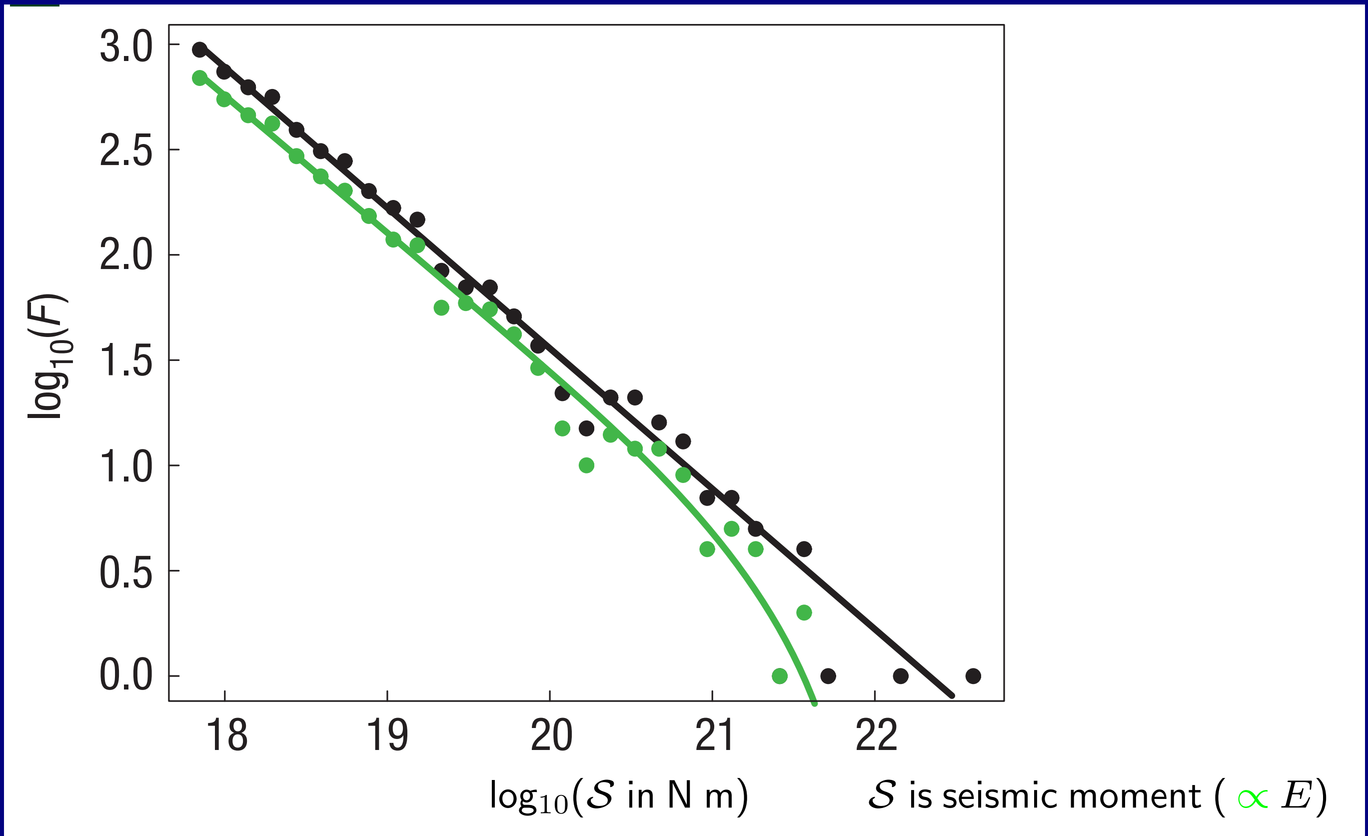
$\Rightarrow$ $D(M) \propto dN(M)/dM$

- The Gutenberg-Richter law yields the same function for $D(M)$

$$D(M) \propto 10^{-bM} \propto e^{-b \ln 10 \, M}$$

- Earthquake radiated energy:
  energy is (roughly) an exponential function of magnitude, $E \propto 10^{1.5M}$
  As $D(E)dE = D(M)dM \Rightarrow D(E) = D(M)dM/dE$

$\Rightarrow$ Energy follows a power-law distribution: $D(E) \propto 1/E^{1+0.67b}$



Main *et al.* Nature Geosci. 2008

Power Laws and Scale Invariance

What is special about power laws?

- Let us perform a scale transformation on a function $y = F(x)$,

$$x \to x' \equiv ax,$$
$$y \to y' \equiv cy.$$

  In the new axes, the function $F(x)$ transforms into

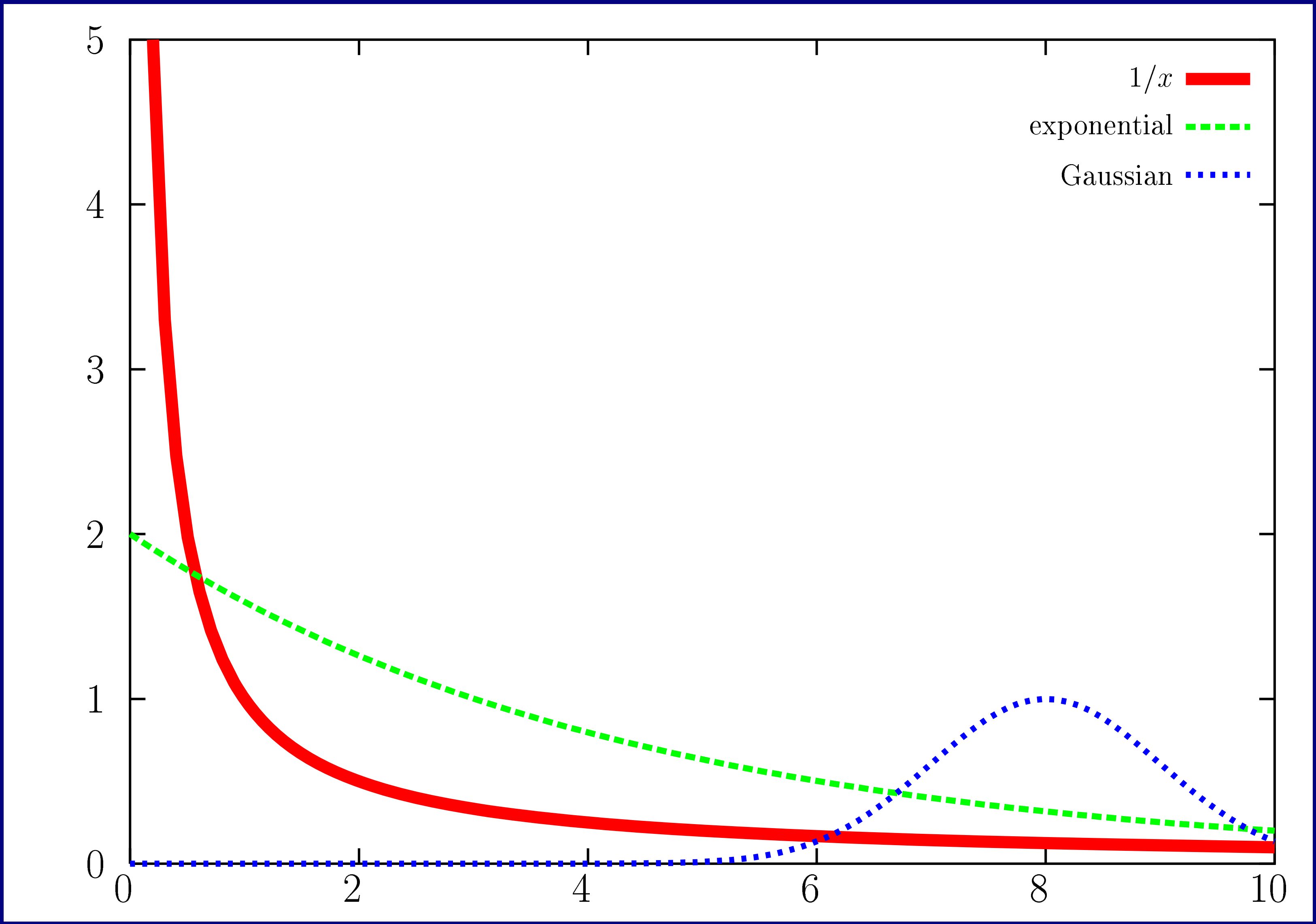
$$F(x) \to cF(x'/a)$$

- Scale invariance means that the new function looks the same, $F(x) = cF(x/a)$

  The solution is given by a power law:

$$F(x) = Ax^{\alpha} \qquad \text{with } c = a^{\alpha} \qquad A \text{ arbitrary,}$$

- Invariance of power laws under scale transformations



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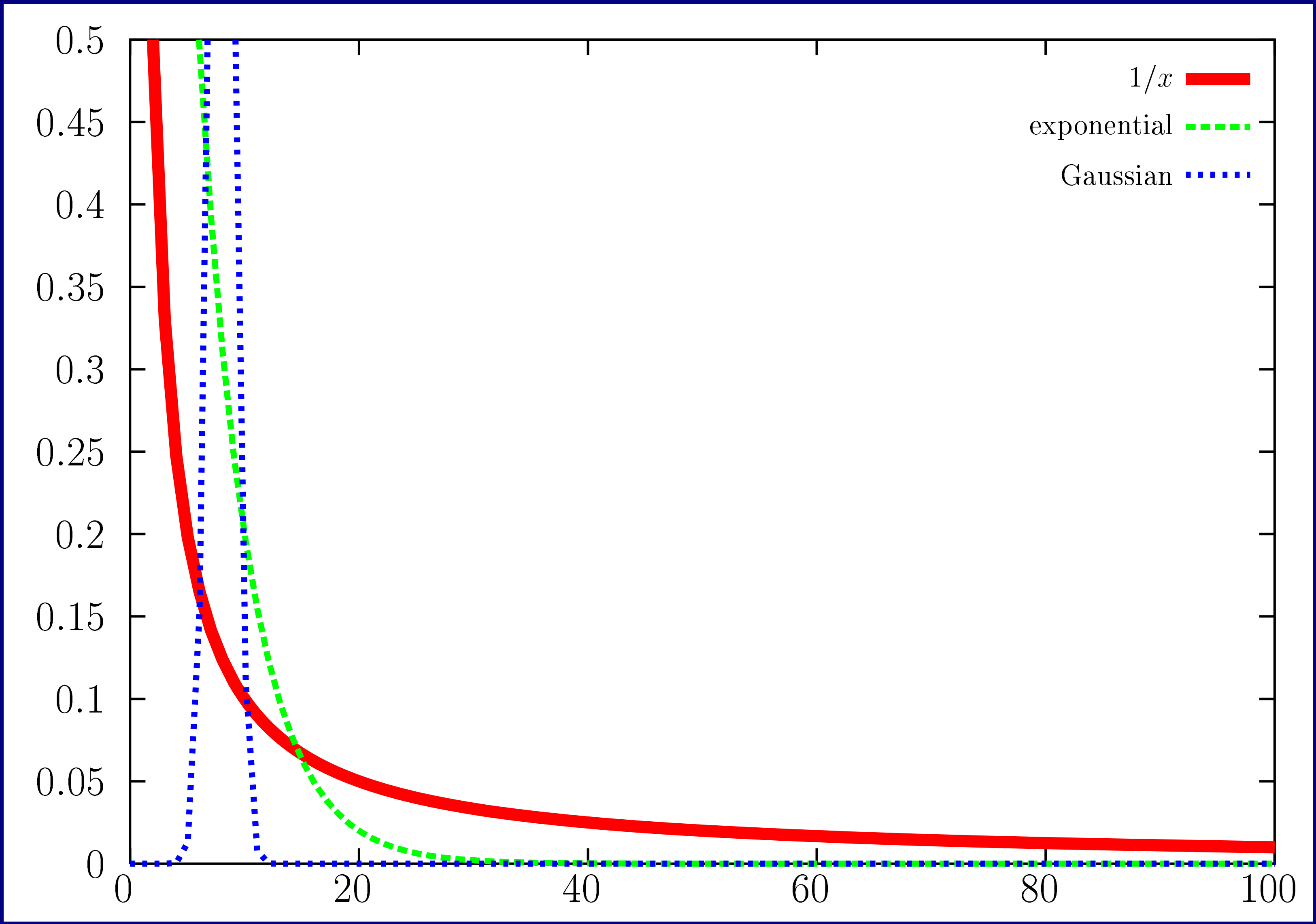
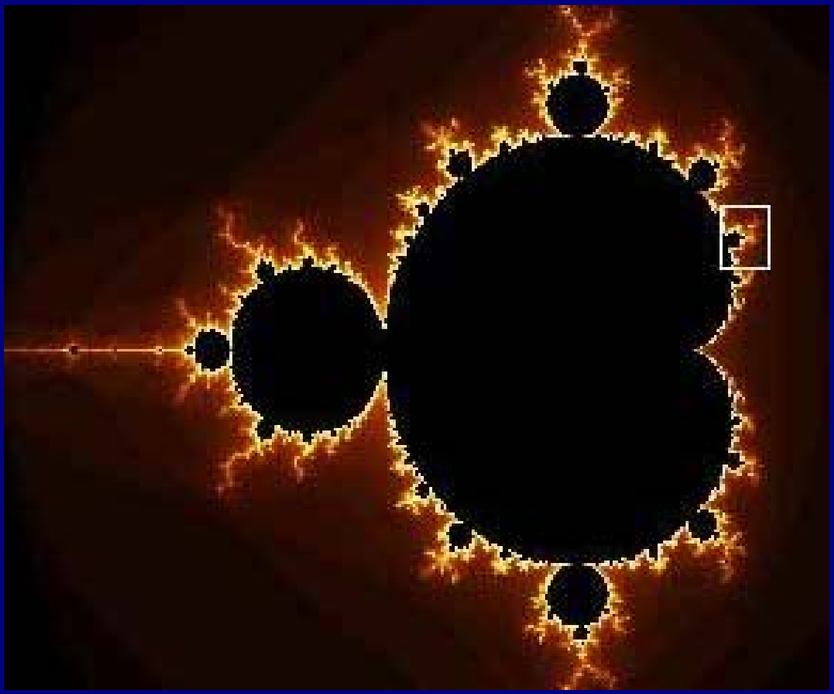


- Example of scale invariance: fractals



Fractal: an object that shows the same structure at all scales

No characteristic scale $\Rightarrow$ Power-law distribution of structure sizes

Scale Invariance of Earthquake Sizes

- Power law or "fractal" distribution of earthquake sizes (energy)

  $\Rightarrow$ There is no characteristic size for earthquakes

  $\Rightarrow$ It is not possible to answer this simple question:

  "How big are earthquakes in a given region?"

Mean Energy of Earthquakes

- Using the Gutenberg-Richter law, the mean energy:

$$\langle E \rangle = \int_{min}^{\infty} E D(E) dE \propto \int_{min}^{\infty} \frac{dE}{E^{0.66}} = \infty$$

The mean radiated energy is infinite!

How can it be? The Earth has a finite energy content...

What does it mean?

$$E(\text{Joules}) \simeq 60000 \cdot 10^{1.5M} \quad \text{(roughly)}$$

Magnitude	Energy (Joules)	Number	Total energy
5	2×10^{12}	1000	2×10^{15}
6	6×10^{13}	100	6×10^{15}
7	2×10^{15}	10	2×10^{16}
8	6×10^{16}	1	6×10^{16}

In practice, the mean does not converge

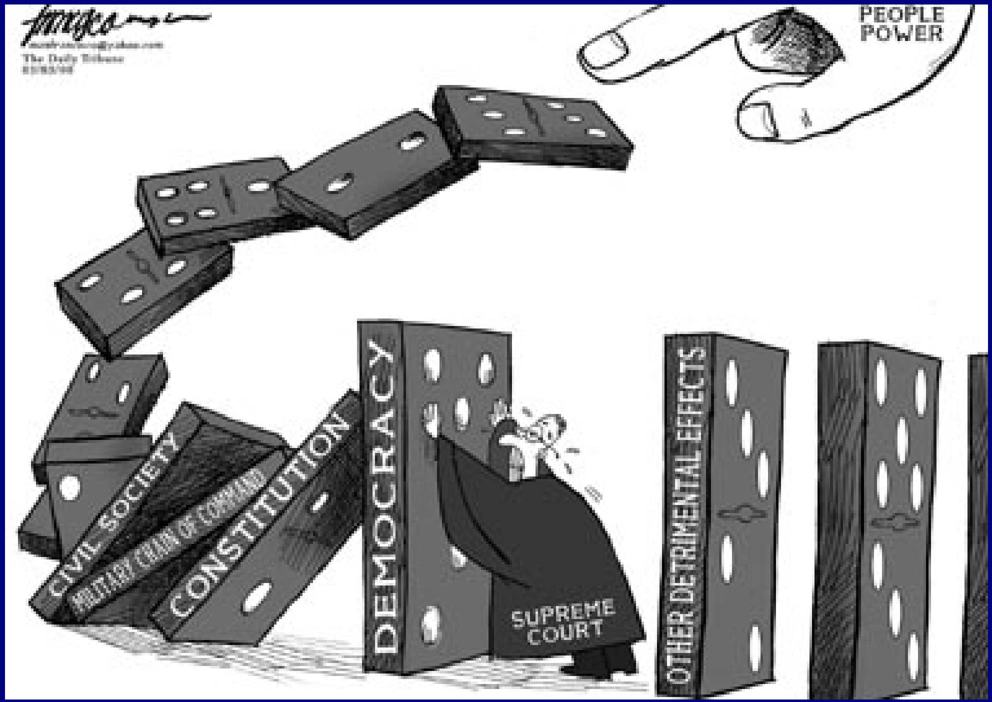
$\Rightarrow$ This means that extreme (rare) events determine the dissipation

$\Rightarrow$ Big earthquakes are responsible of the energy release!

$\Rightarrow$ **Bad news**!!

The GR law is telling us something about the physics of earthquakes
But what?

- **"Domino theory"**

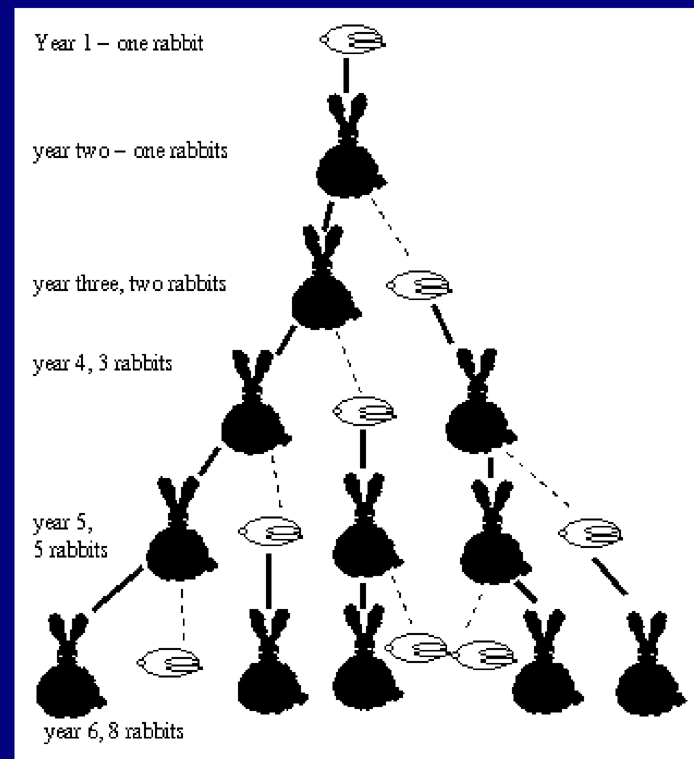


M. A. Francisco

Tectonic fault: analogous to a domino-like network
Earthquake: chain reaction of topplings or avalanche

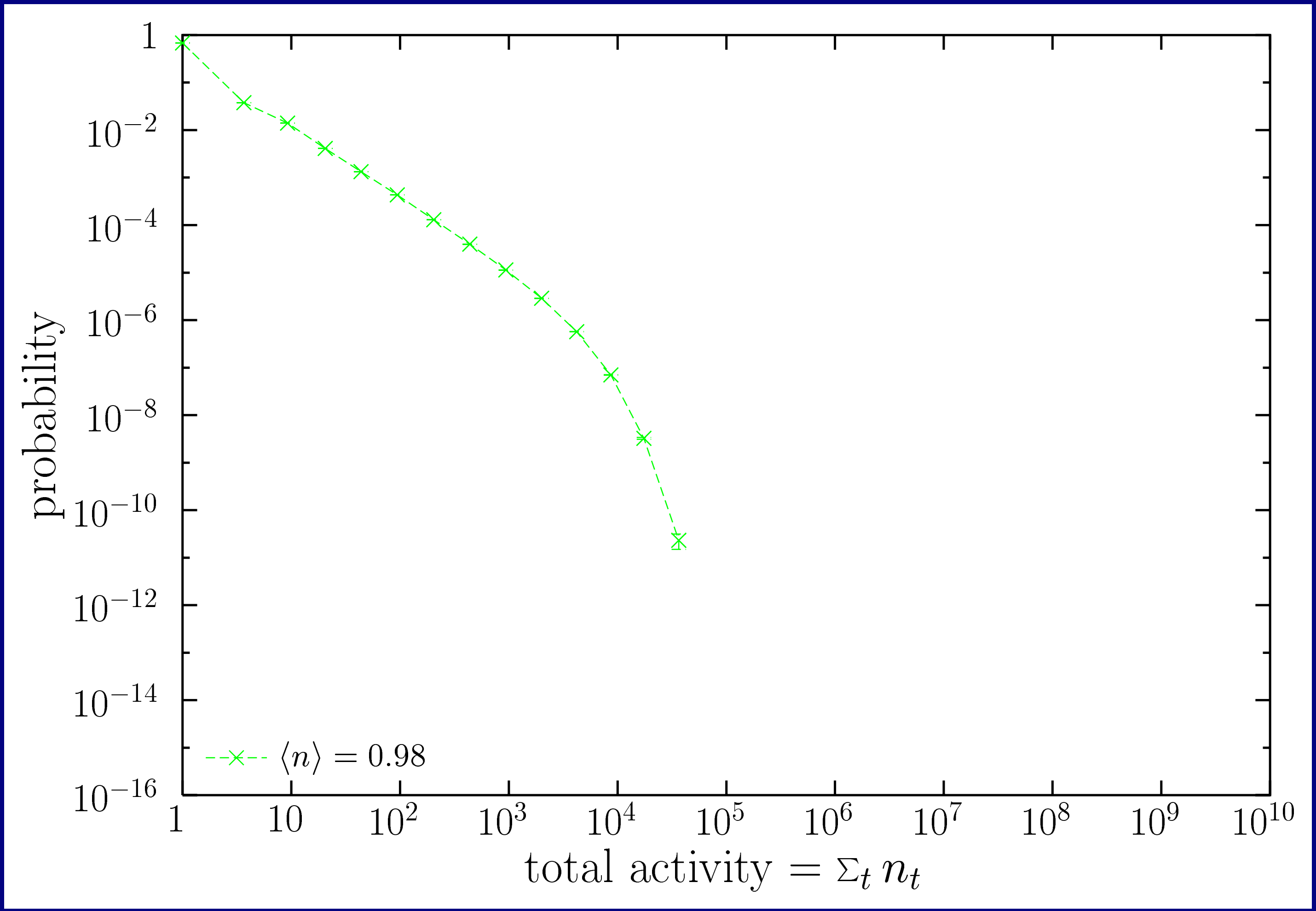
- **Branching process**

  Each mother leaves a random number n of daughters



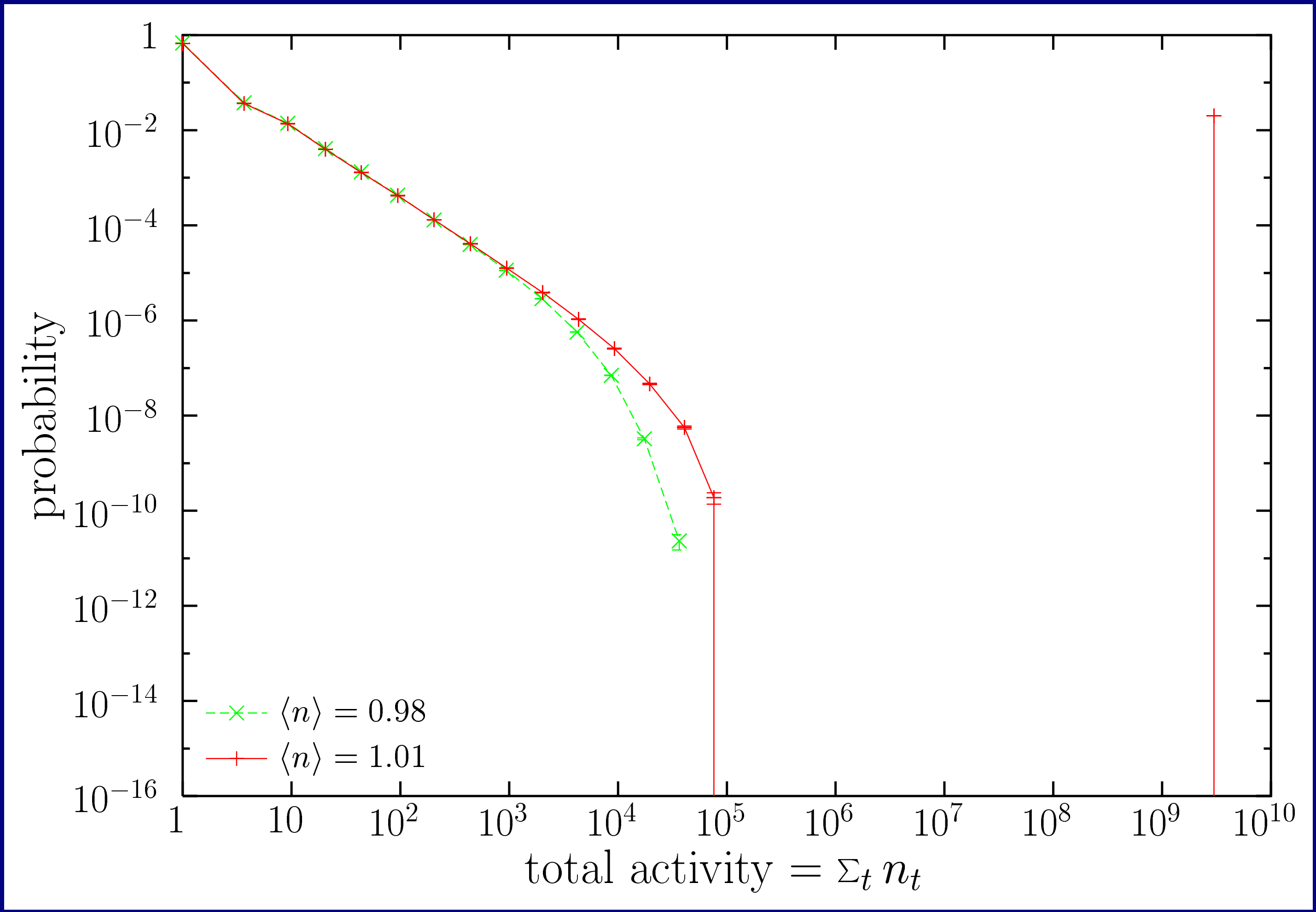
Which will be the total number of offsprings?
i.e., domino topplings $\equiv$ "activity", proportional to energy

- Probability distribution of total activity, $\langle n \rangle < 1$



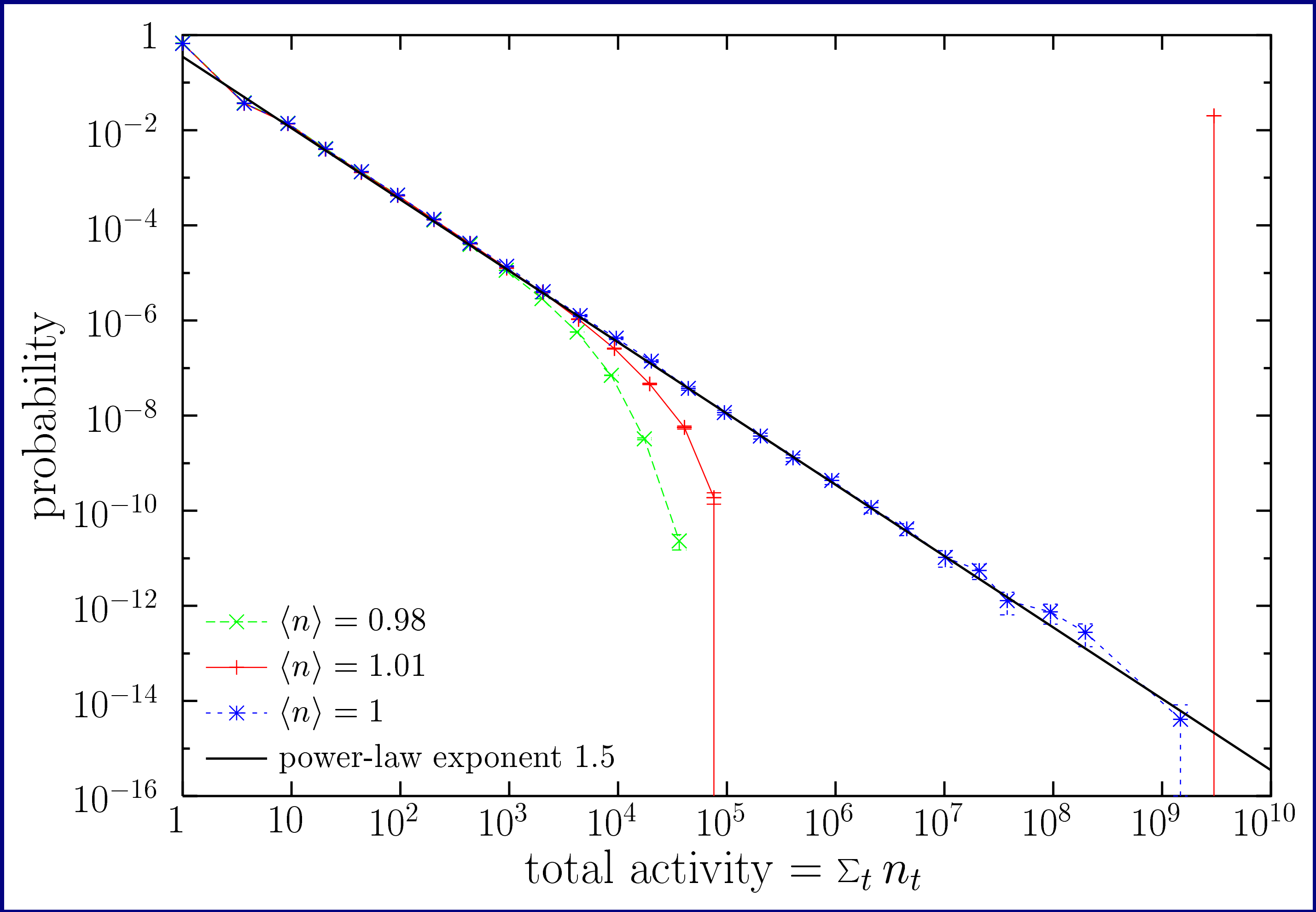
At the end, the activity dies

- Probability distribution of total activity, also for $\langle n \rangle > 1$



Finite probability of infinite activity

- Probability distribution of total activity, including $\langle n \rangle = 1$



All sizes are possible, power-law distribution

⇒ Power law distributions are very difficult to achieve ($\langle n \rangle = 1$)

- After any toppling, we don't know what will happen next

- The perturbation propagates or not to give rise a catastrophic event depending on a huge number of microscopic details which are intrinsically out of control

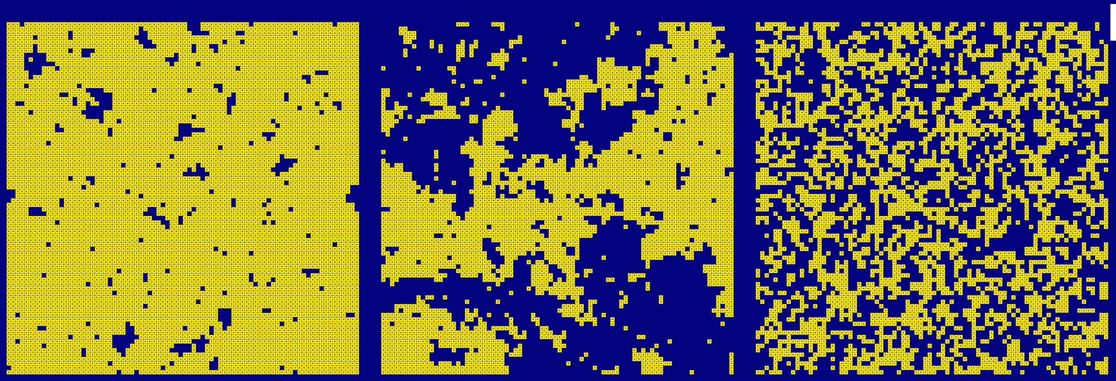
- Consequences for predictability?

- **Another Example: Critical Points of Thermodynamic Phase Transitions**

  Magnetic material: atom $=$ spin with 2 states

  There exists a critical temperature T_c

  ⋆ Above T_c: no magnetization, small clusters
  ⋆ Below T_c: magnetization, one very large cluster
  ⋆ At the precise value $T = T_c$ $\Rightarrow$ clusters of all sizes $\Rightarrow$ power law!



$T < T_c$ $\qquad$ $T = T_c$ $\qquad$ $T > T_c$

Christensen & Moloney, Complexity and Criticality

Self-organized criticality

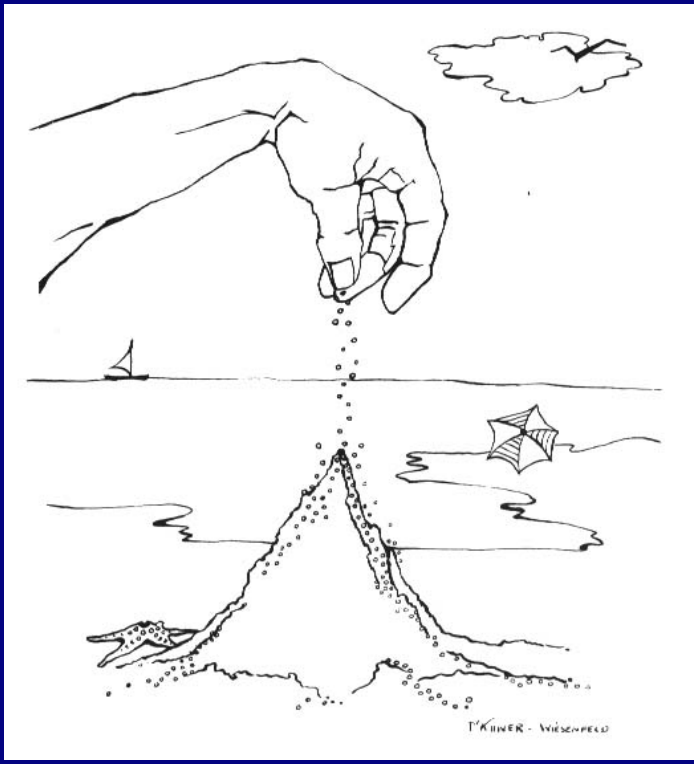
- How is the required fine tuning achieved?

- The power-law response emerges as a consequence of the attraction of the dynamics towards a critical point $\Rightarrow$ sandpile paradigm Bak *et al.* PRL 1987

- **Sandpile metaphor**

  ⋆ If there are few grains (flat pile)
  $\Rightarrow$ small avalanches, pile grows
  ⋆ If there are many grains (steep pile)
  $\Rightarrow$ large avalanches, pile decreases

  This mechanism makes the slope of the pile fluctuate around the critical state

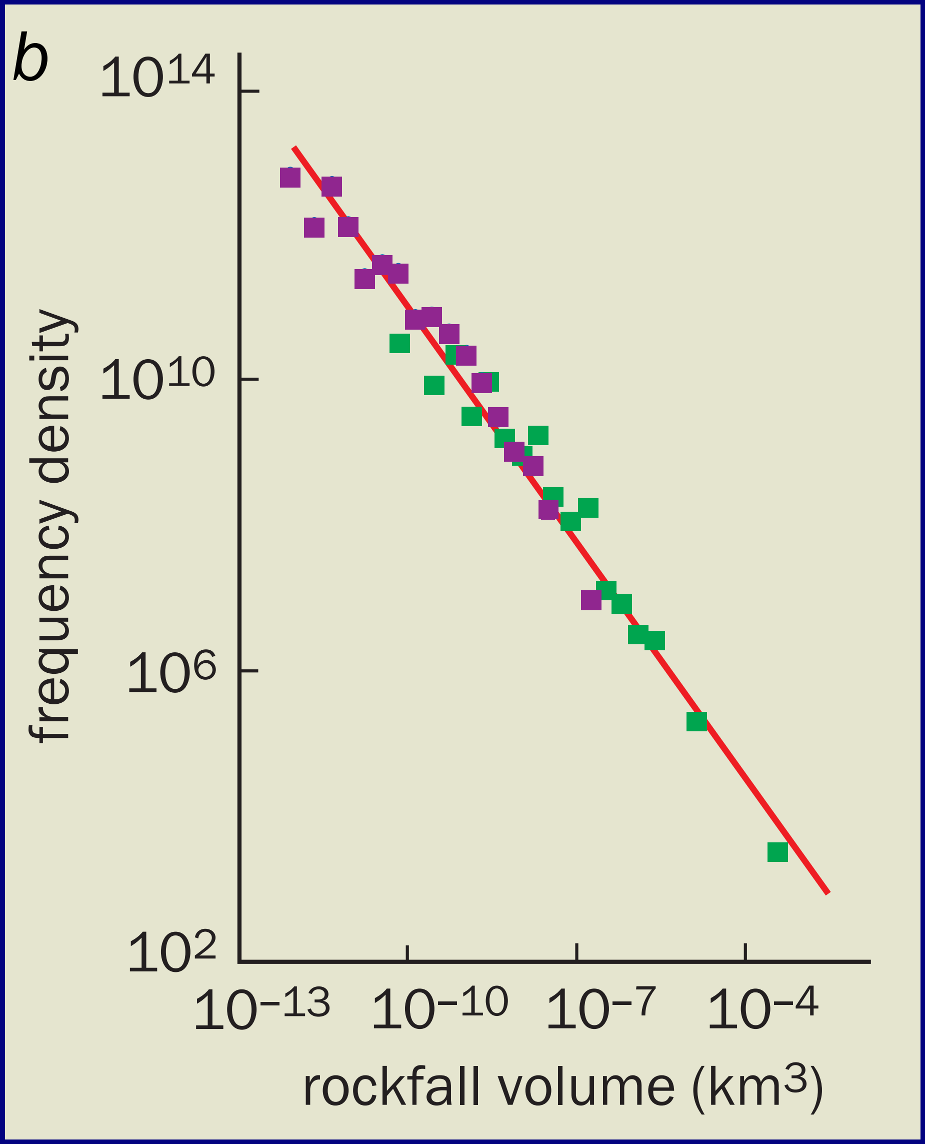


Rockfalls

- Size measured in volume of rocks
  - ⋆ Purple color: earthquake-triggered rockslide event in Umbria (Italy) in 1997
  - ⋆ Green color: rockfalling at Yosemite (USA) from 1980 to 2002

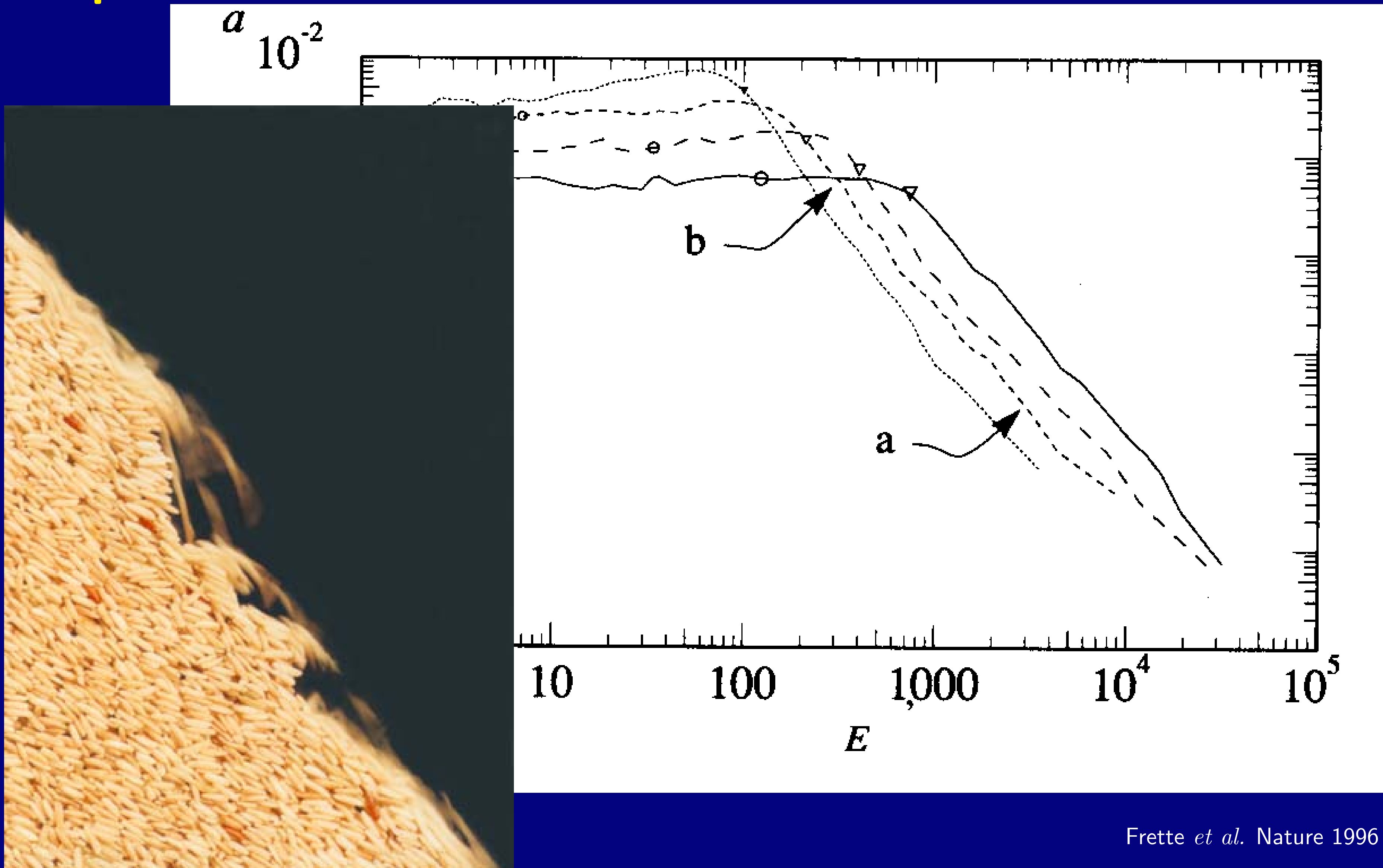
  Exponent 1.1

- Other similar phenomena:
  - ⋆ Landslides
  - ⋆ Snow avalanches
  - ⋆ Sediment gravity flows in the oceans



Malamud, Phys. World 2004

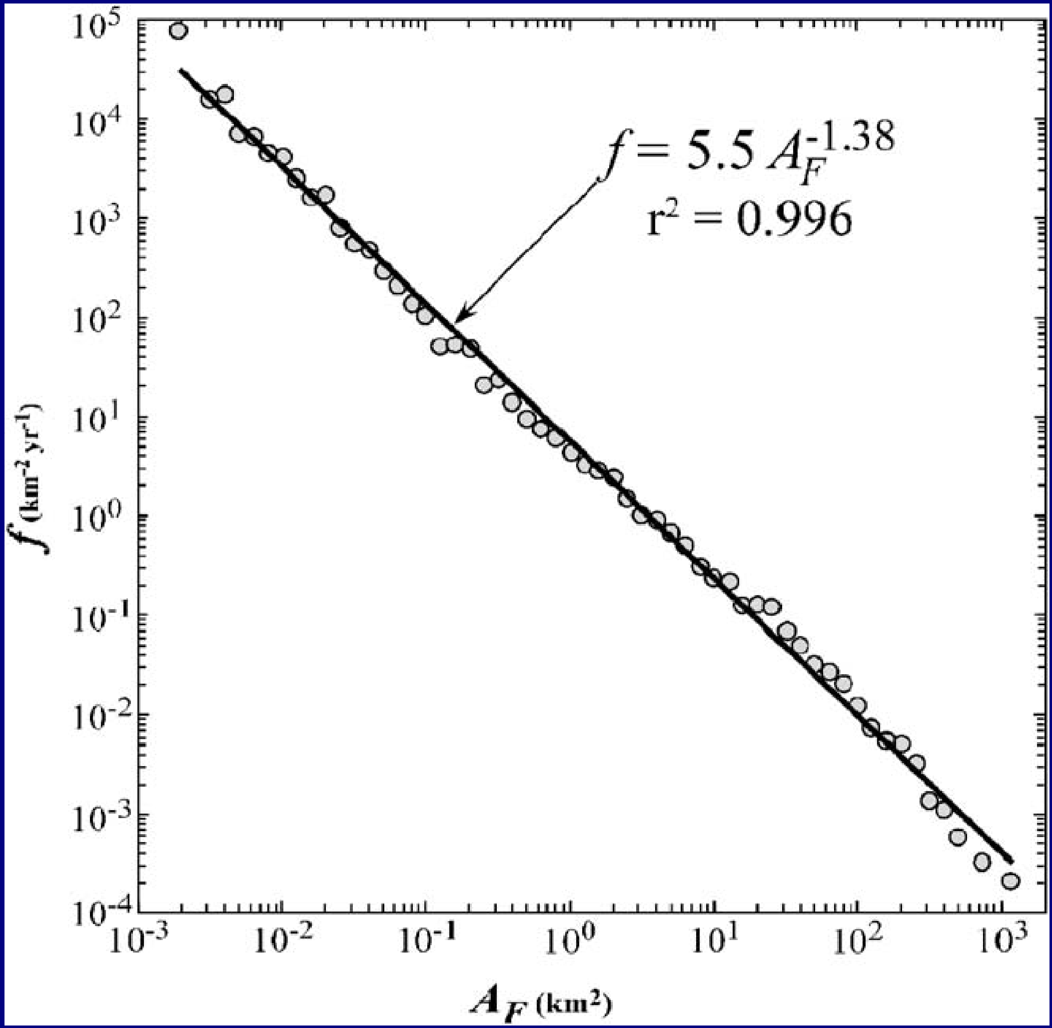
Rice-pile avalanches



Frette *et al.* Nature 1996

Forest fires: Fires at Ontario (Canada), 1976–1996 (15308 fires)

- Size measured as burned area
- Trees store energy which is rapidly released by fire
- Conclusion:
  Forests have the largest number of trees allowed by fires

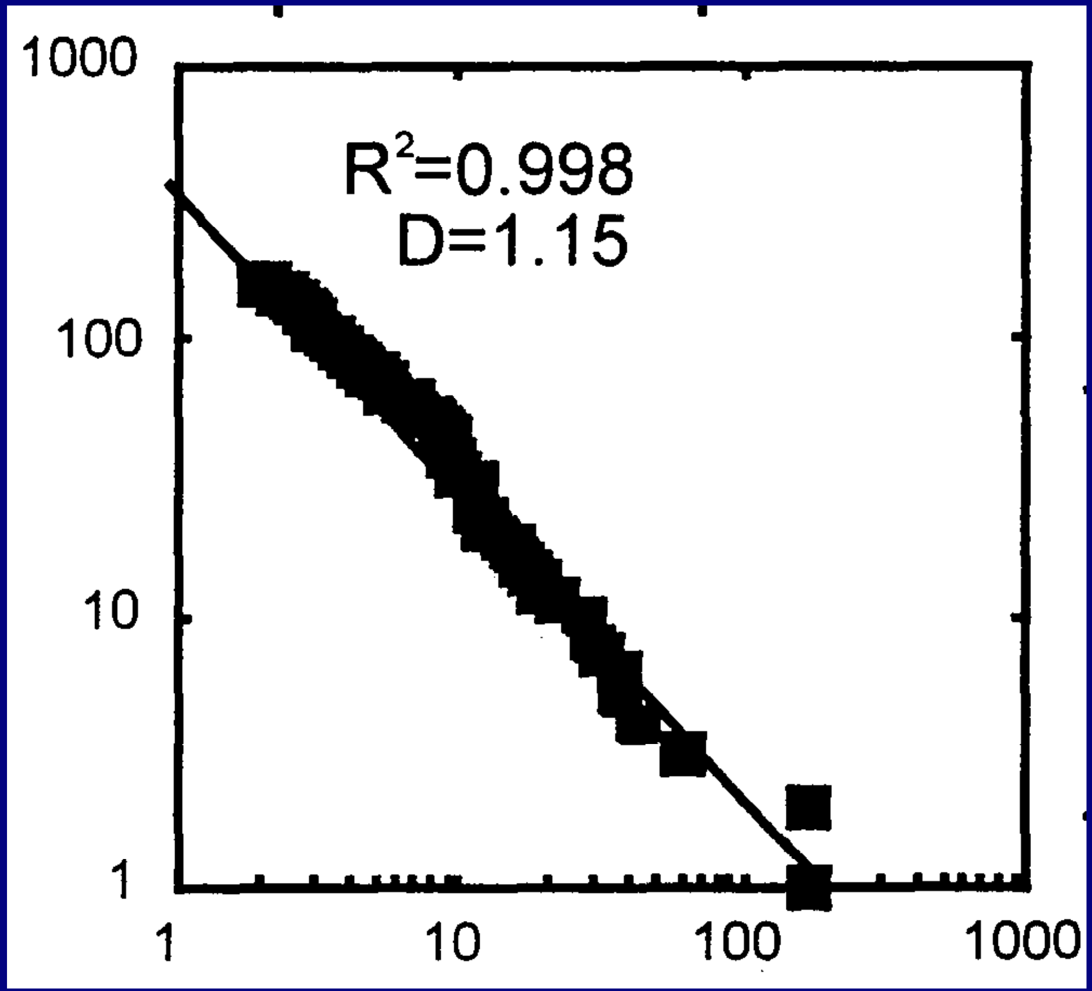


Turcotte & Malamud, Phys. A 2004, also Malamud & Turcotte, Science 1998

Volcanic eruptions

- Area covered by lava flows
  in the Springerville volcanic field, Arizona (USA) between 2.1 Myear and 0.3 Myear ago

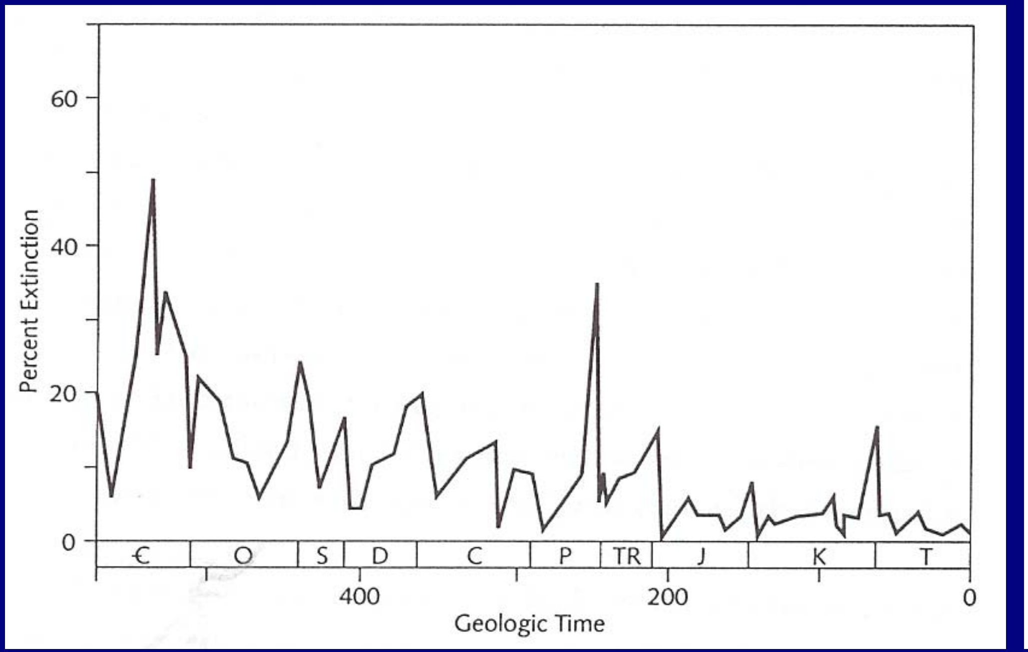
  Cumulative number of eruptions versus area in km^2

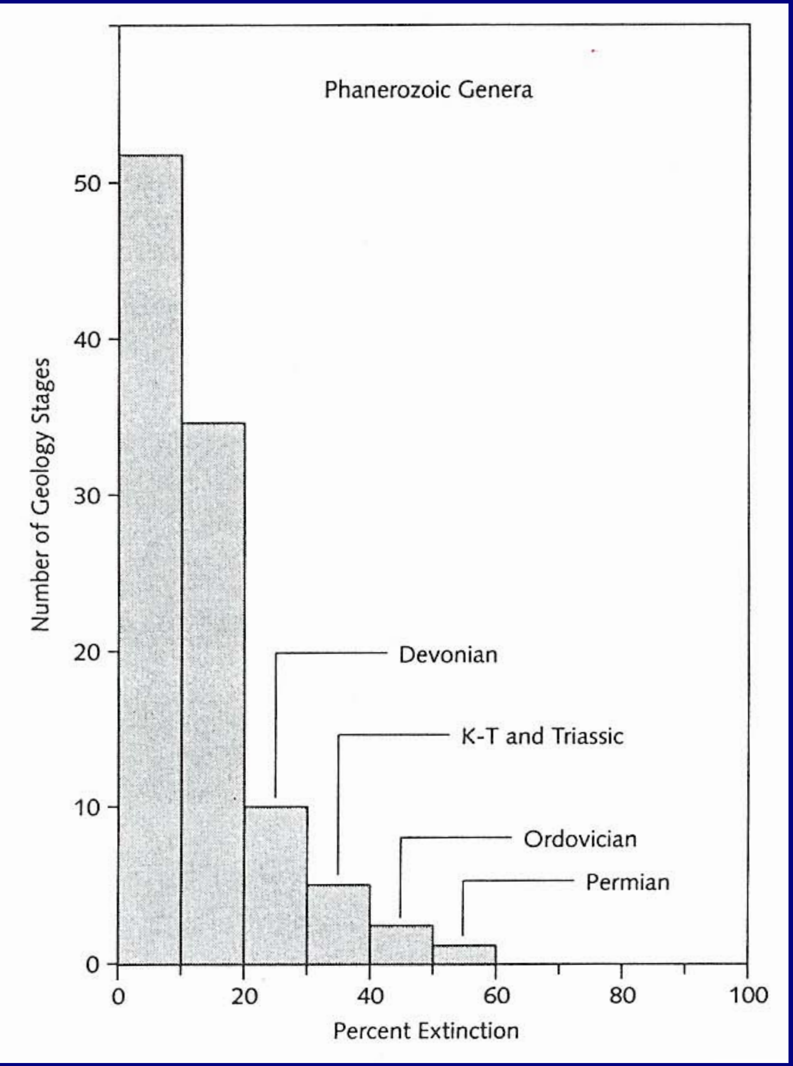


Lahaie and Grasso, JGR 1998

Biological extinctions (?)

- Extinction measured as the percentage of extinct families in periods of 4 million years

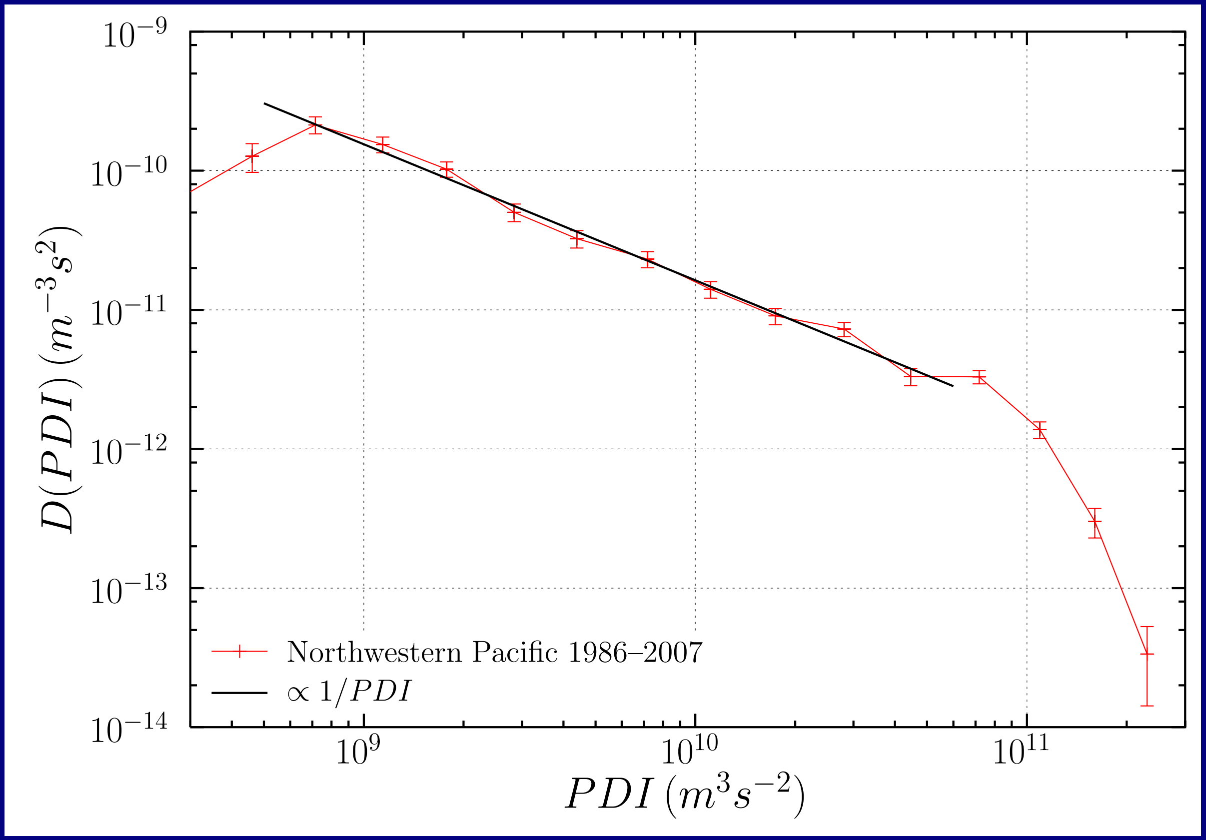




Sepkoski, Paleobio. 1993; Raup, Bad Genes... 1991, shown in Bak 1996

Tropical cyclones (hurricanes):

- Dissipated Energy (PDI) of North Western Pacific typhoons, 1986–2007



Power-law distribution: $D(PDI) \propto 1/PDI$

A. Ossó *et al.* preprint 2009

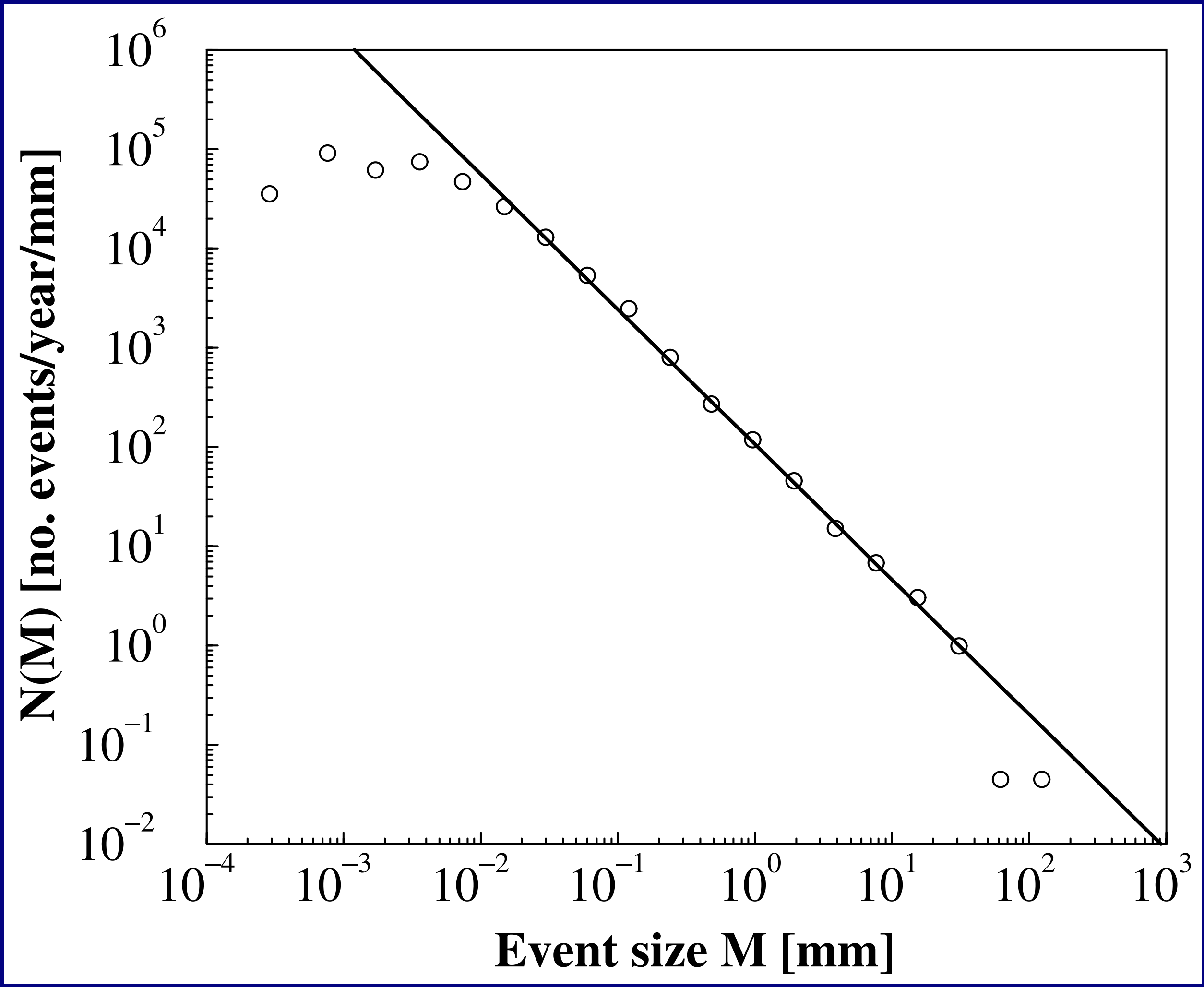
Rainfall: measured at one point of the Baltic coast, Jan-Jul 1999

- A rain event is defined as the continuous occurrence of rain between drought periods of minimum 1 minute

- Dynamics:

  - Solar radiation provides energy
  - Evaporated water stores it
  - If a saturation threshold is reached $\Rightarrow$ rain

Peters et al. PRL 2002



Summarizing:

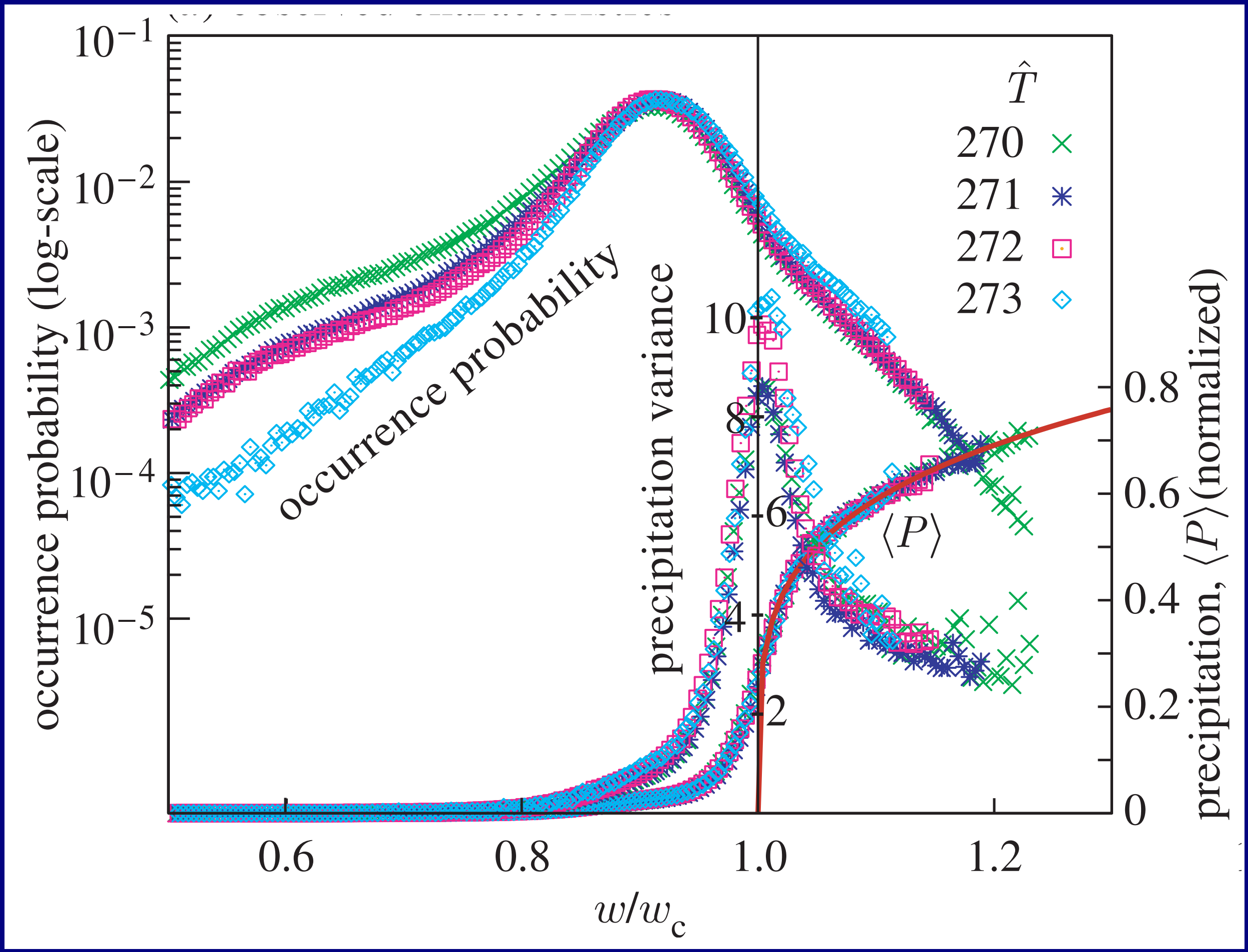
- SOC $\Rightarrow$ sandpile dynamics and power-law distributions
- Many natural disasters $\Rightarrow$ sandpile-like dynamics and power-law distributions

Does this mean that the previous natural disasters are SOC?

Or something else is necessary (?)

First Confirmation of Self-Organized Criticality?

- Rain: there exist a critical point and the system is attracted close to it!



Peters & Neelin, Nature Phys. 2006

- For earthquakes (and others): are there other indications of criticality?

- Do power-law size distributions reflect some degree of self-similarity in time?

  1 year of earthquakes with $M \geq 5 \Leftrightarrow$
  10 years of earthquakes with $M \geq 6$ etc.?

- Complex-System philosophy:

  ⋆ Difficulties studying faults
    ∗ Interaction between faults, no isolated faults exists
    ∗ Problems assigning earthquakes to faults
    ∗ Ambiguity to identify and even define faults
    $\Rightarrow$ Study spatially extended areas

  ⋆ All earthquakes constitute a unique process
    $\Rightarrow$ Do not distinguish between mainshocks, aftershocks, etc.

$\Rightarrow$ The robustness of the results will corroborate the coherence of the approach

Bak *et al.* PRL 2002

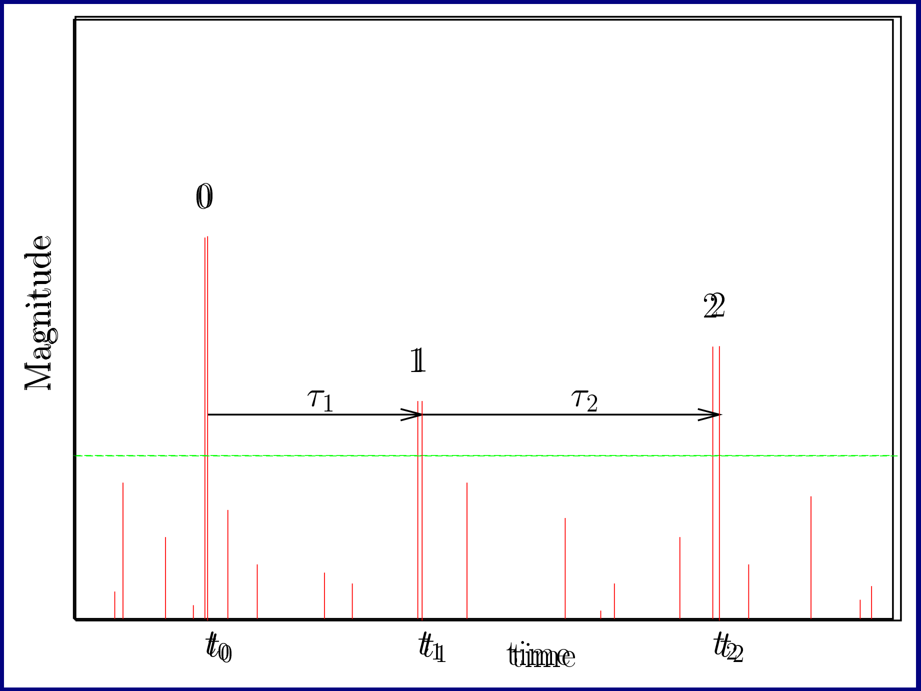
Waiting times

- Consider a fixed spatial region
- Consider earthquakes with magnitude larger than a threshold, $M \geq M_c$
- Compute waiting time as the

  time between consecutive earthquakes

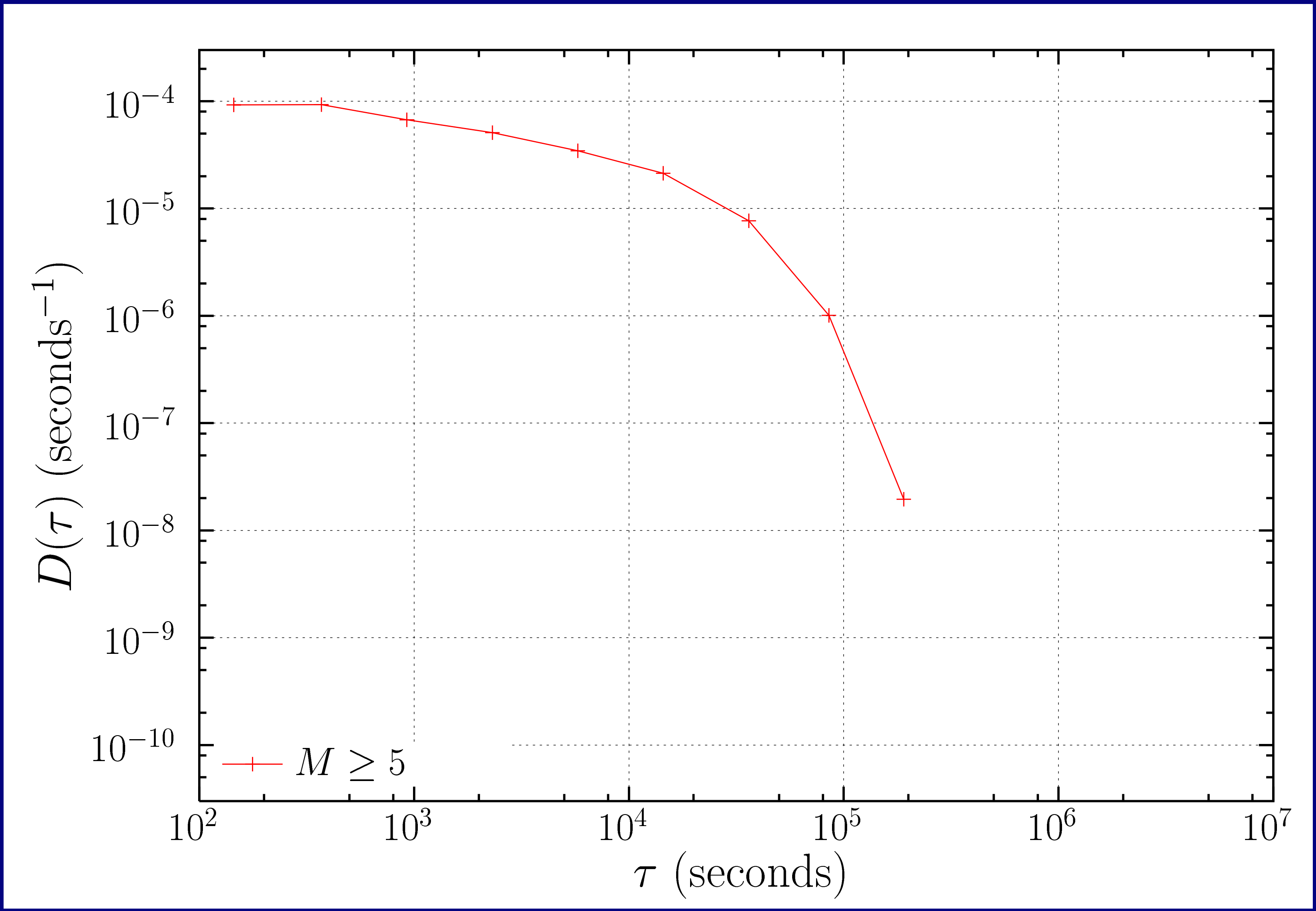
$$\tau_i \equiv t_i - t_{i-1}$$

$$i = 1, 2, 3 \ldots$$

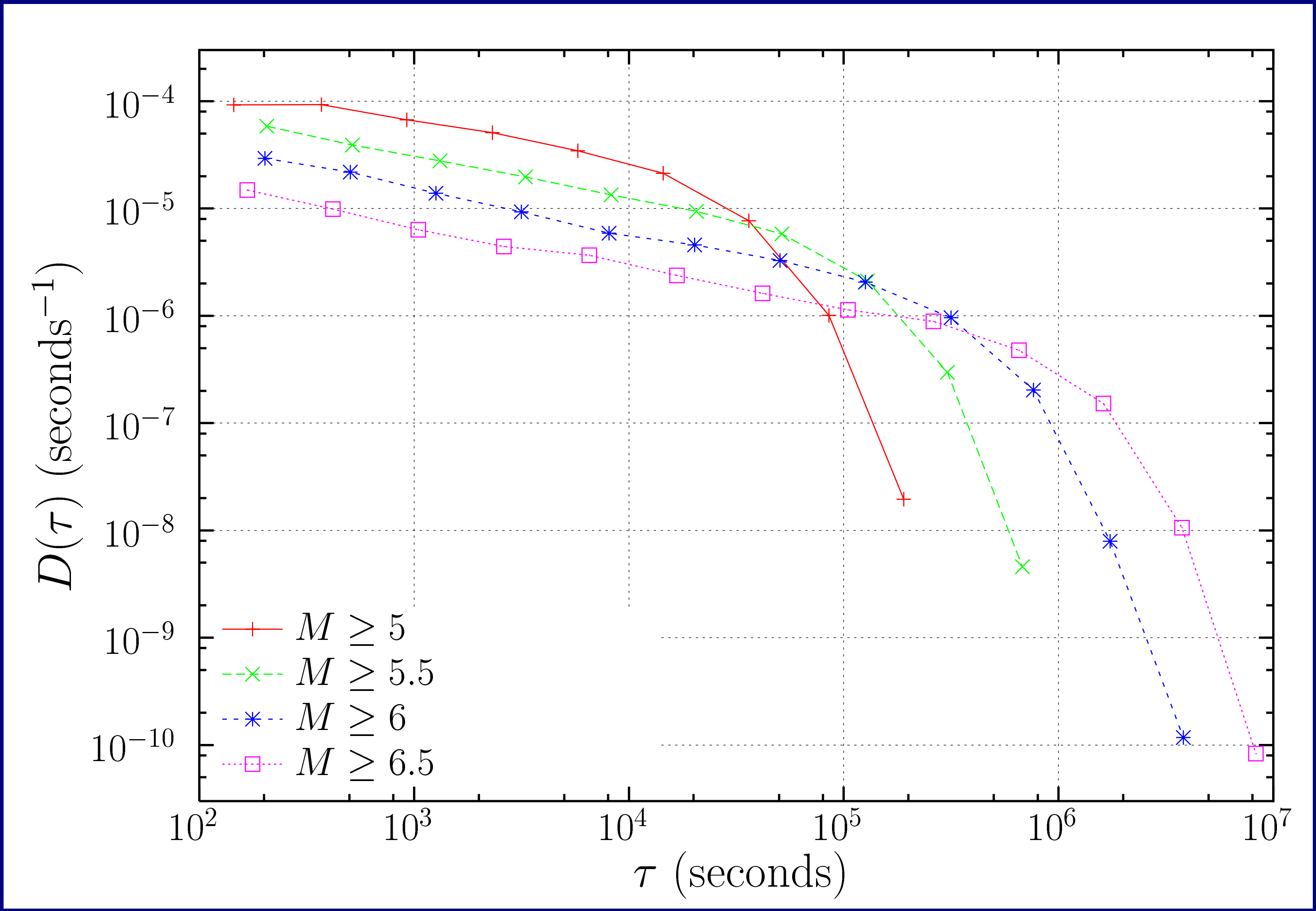


$\Longrightarrow$ Broad scale of times $\Rightarrow$ Gutenberg-Richter gives a poor description!

Worldwide seismicity for $M \geq 5$, from 1973 to 2002



Worldwide seismicity for $M \geq M_c$, with M_c variable, from 1973 to 2002



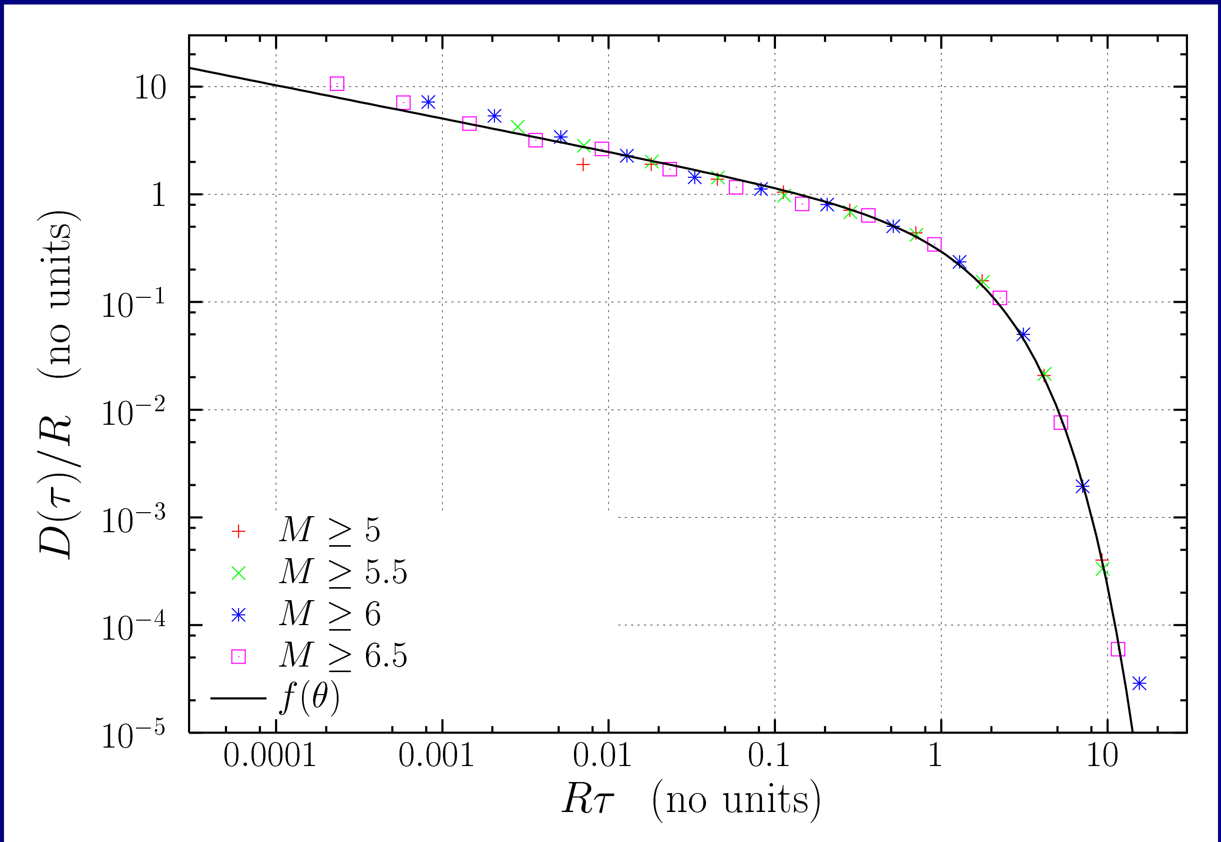
Scale transformation of the axes

$$\tau \longrightarrow R_c\,\tau$$
$$D(\tau, M_c) \longrightarrow D(\tau, M_c)/R_c$$

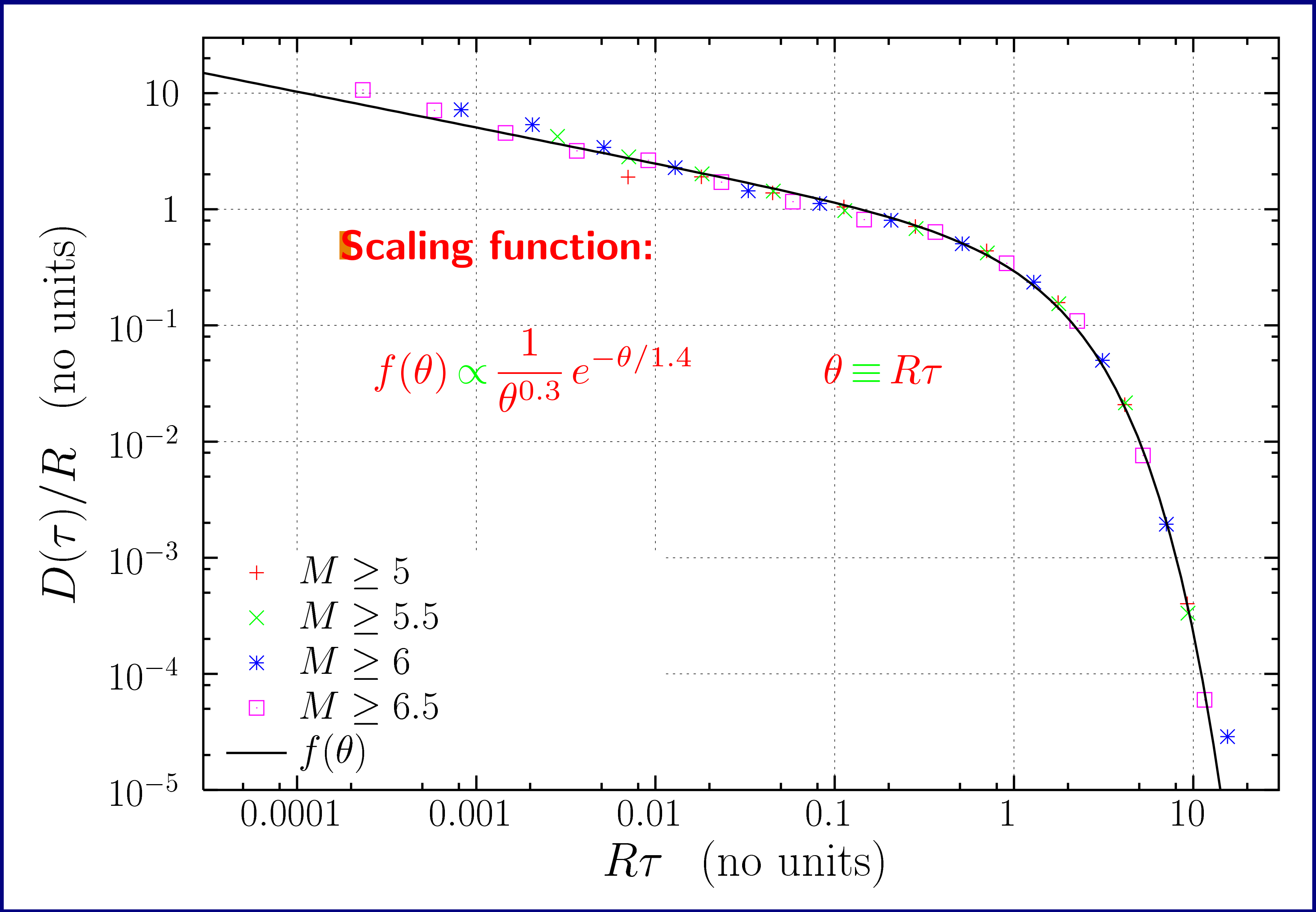
with $R_c(M_c)$ the rate of seismic activity: number of earthquakes per unit time

Scaling law:

$$D(\tau, M_c) = R_c f(R_c \tau)$$



A.C. PRL 2004


Scaling function:
$f(\theta) \propto \frac{1}{\theta^{0.3}} e^{-\theta/1.4}$
$\theta \equiv R\tau$
$M \geq 5$
$M \geq 5.5$
$M \geq 6$
$M \geq 6.5$
$f(\theta)$
$D(\tau)/R$ (no units)
$R\tau$ (no units)
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0.0001
0.001
0.01
0.1
1
10

Two main properties:

- Clustering

$$f(\theta) \propto \frac{1}{\theta^{0.3}}\, e^{-\theta/1.4}$$

It is valid independently of the fit of $f(\theta)$

- Scaling

$$D(\tau, M_c) = R_c f(R_c \tau)$$

In fact, the existence of clustering is clear before rescaling

Poisson process

- A dice decides if an earthquake happens or not
  - ⋆ The dice has many faces (probability of occurrence very small, $p \longrightarrow 0$)
  - ⋆ The dice is thrown continuously in time, $N \longrightarrow \infty$

$$\text{Prob}[\ n \text{ events in } N \text{ throwns}] = \begin{pmatrix} N \\ n \end{pmatrix} p^n (1-p)^{N-n} \longrightarrow$$

$$\longrightarrow e^{-\lambda} \frac{\lambda^n}{n!} = \text{Prob}[n \text{ events in time } T]$$

with $pN \equiv \lambda = RT$. The waiting-time cumulative distribution function is

$$S(\tau) \equiv \text{Prob}[\text{waiting time} \geq \tau] = \text{Prob}[0 \text{ events in time } \tau] = e^{-R\tau}$$

$$\Rightarrow D(\tau) = -\frac{dS(\tau)}{d\tau} = Re^{-R\tau}$$

Clustering

- Fit of the scaling function: gamma distribution

$$f(\theta) \propto \frac{1}{\theta^{0.3}} e^{-\theta/1.4} \qquad \theta \equiv R\tau$$

Note that rescaling imposes $\bar{\theta} = 1$
$\Rightarrow$ Only one parameter is independent

- The gamma distribution gives an increased probability for short waiting times (in comparison with a Poisson process, $f(\theta) = e^{-\theta} \simeq 1$ for $\theta < 1$) $=$ clustering

$\Rightarrow$ Earthquakes tend to attract each other

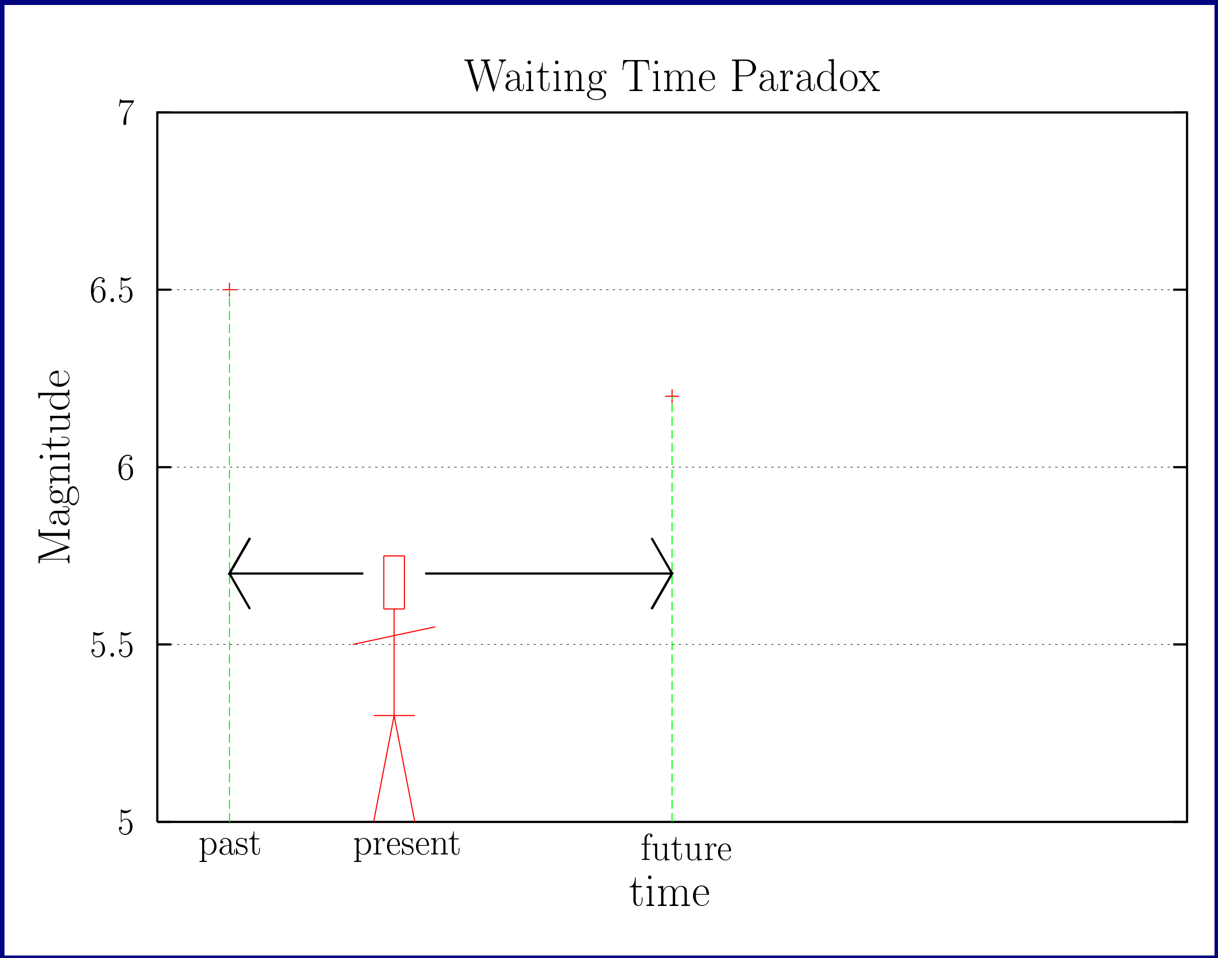
$\Rightarrow$ Counterintuitive consequences:

The longer you have been expecting for an earthquake
the longer you will still have to wait

A.C. PRE 2005

Consequence of clustering: waiting-time paradox

- *The longer you have been expecting for an earthquake the longer you will still have to wait*



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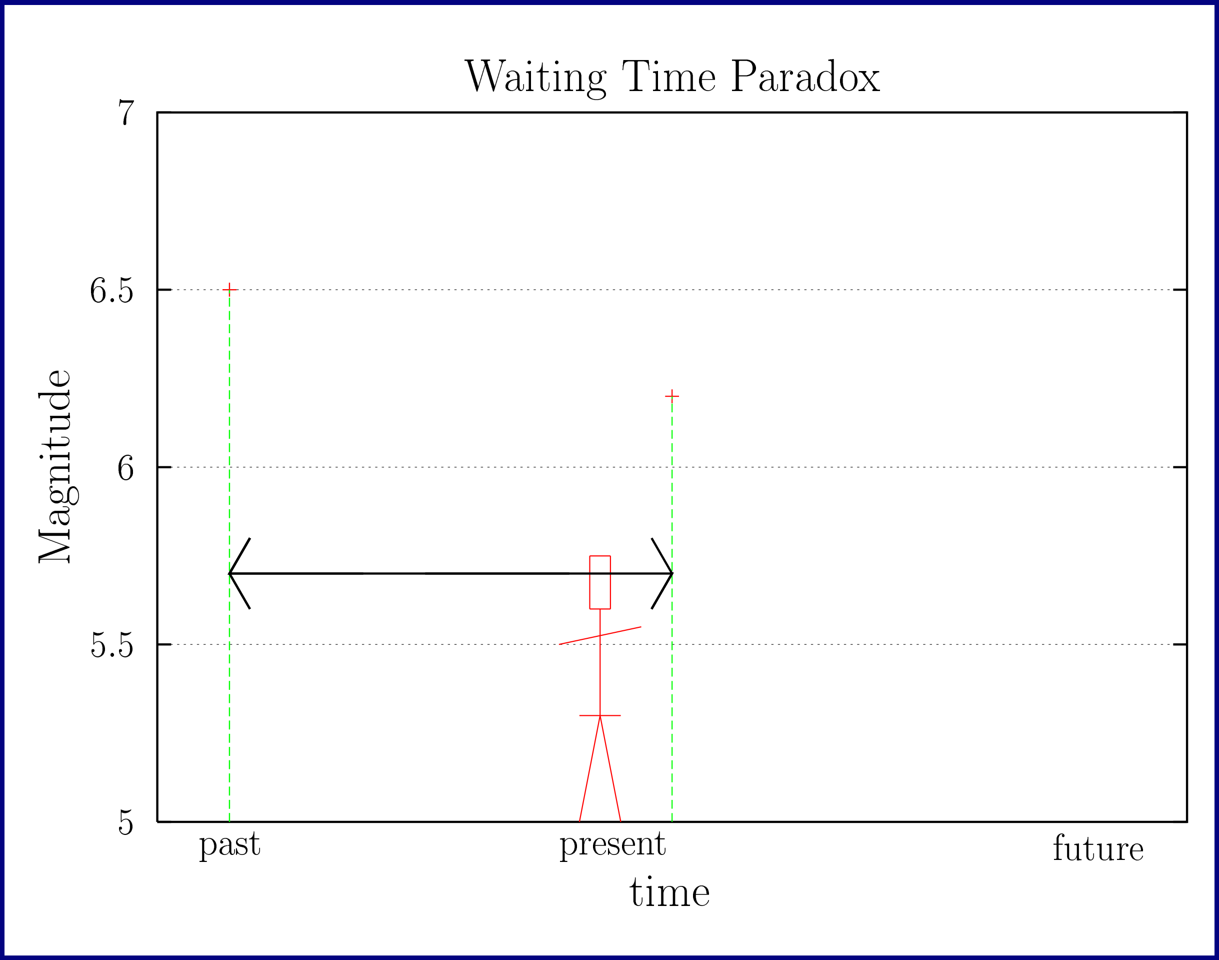

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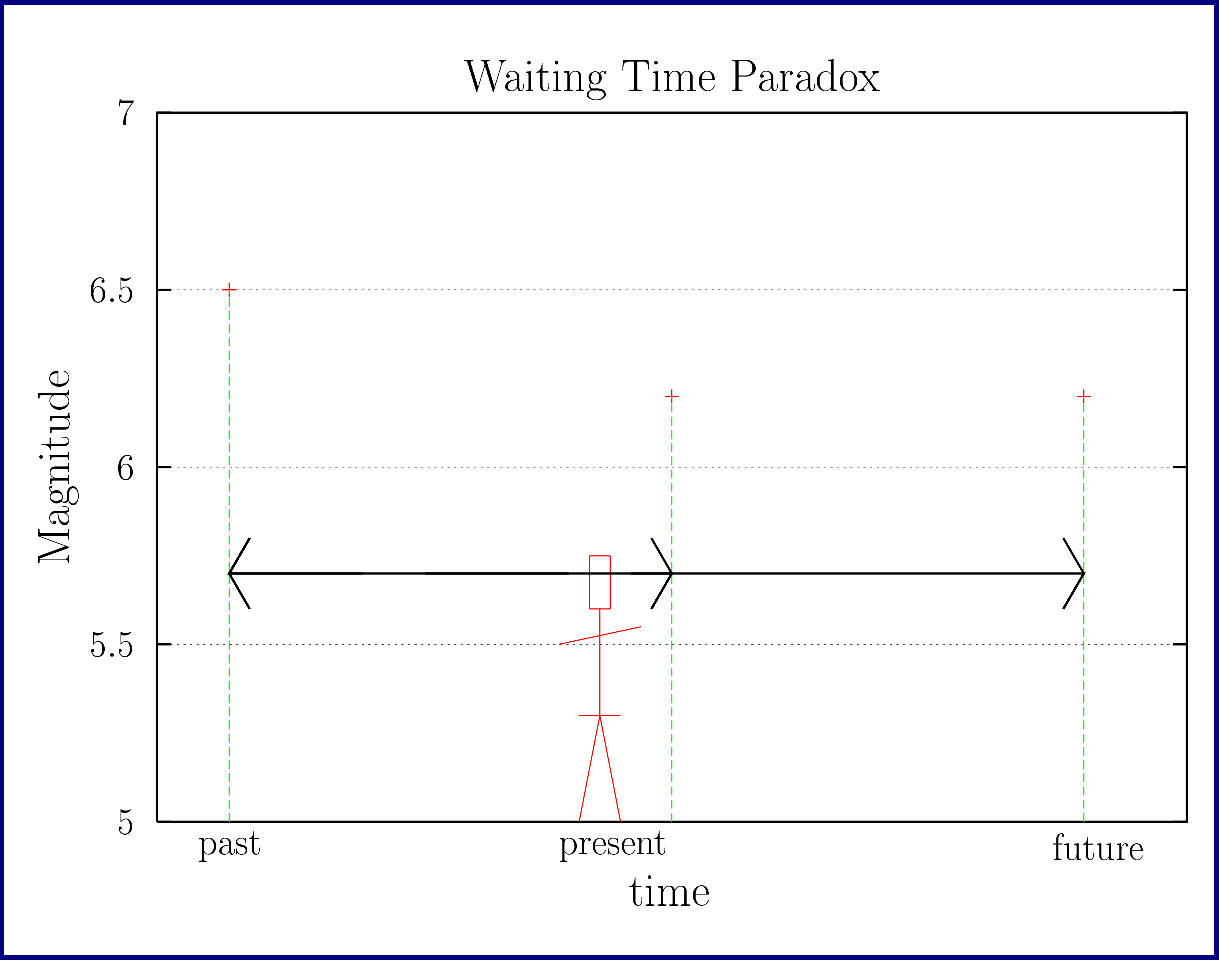

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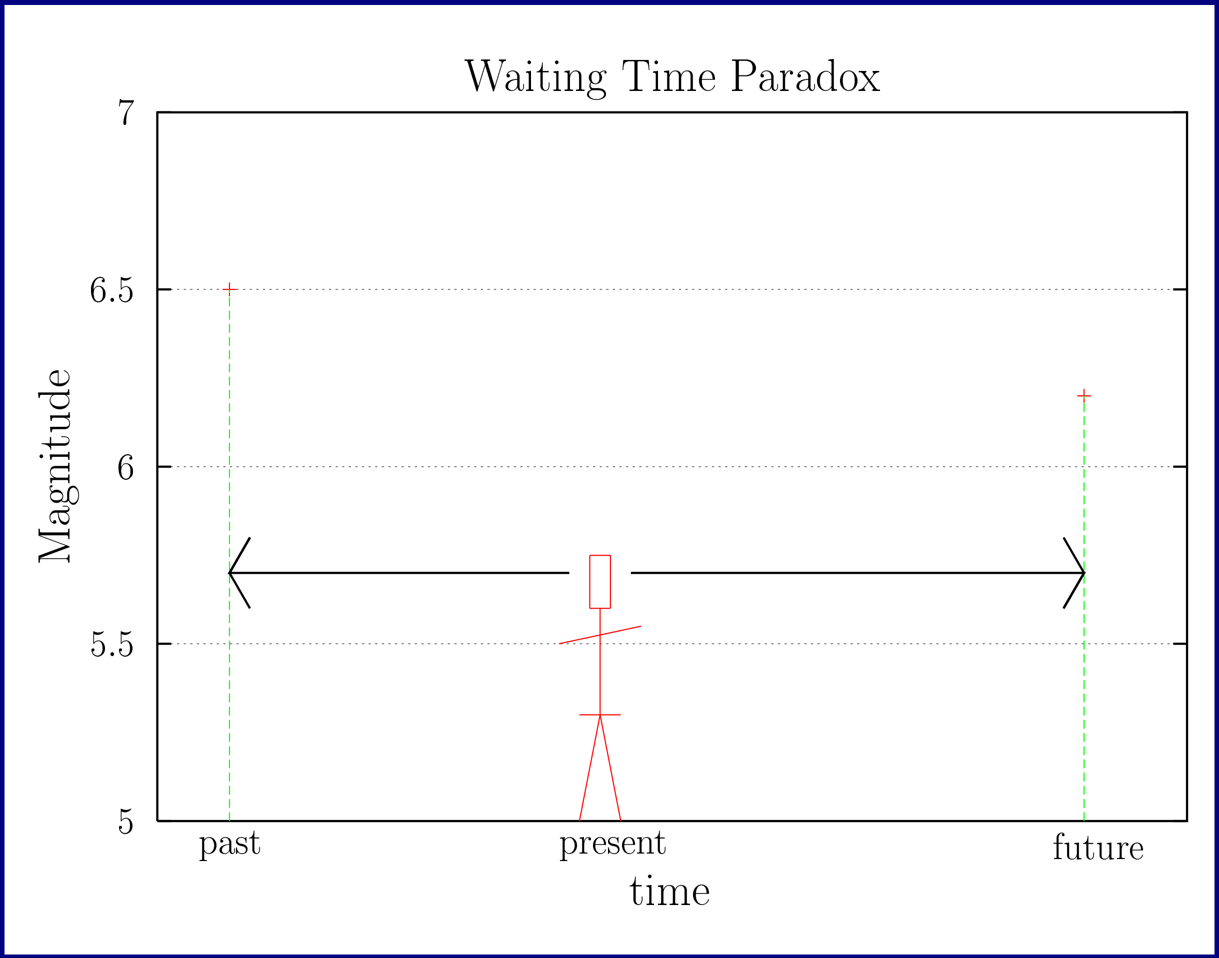
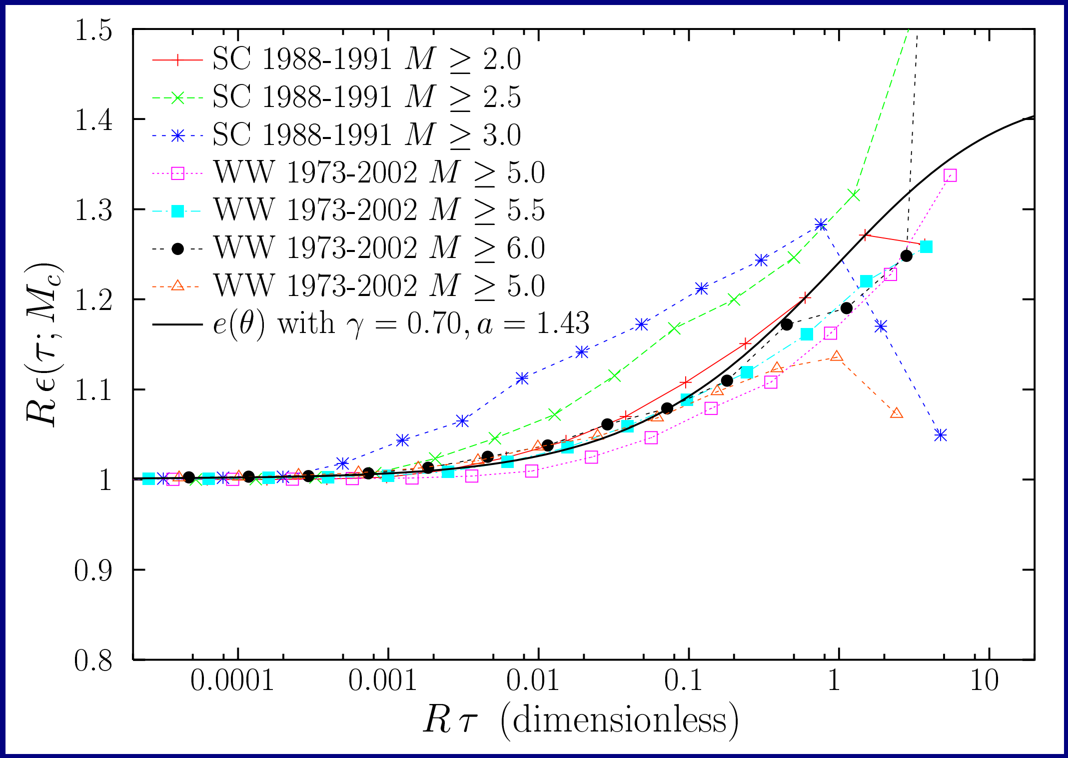


How is this paradoxical effect measured?

Expected Residual Recurrence Time

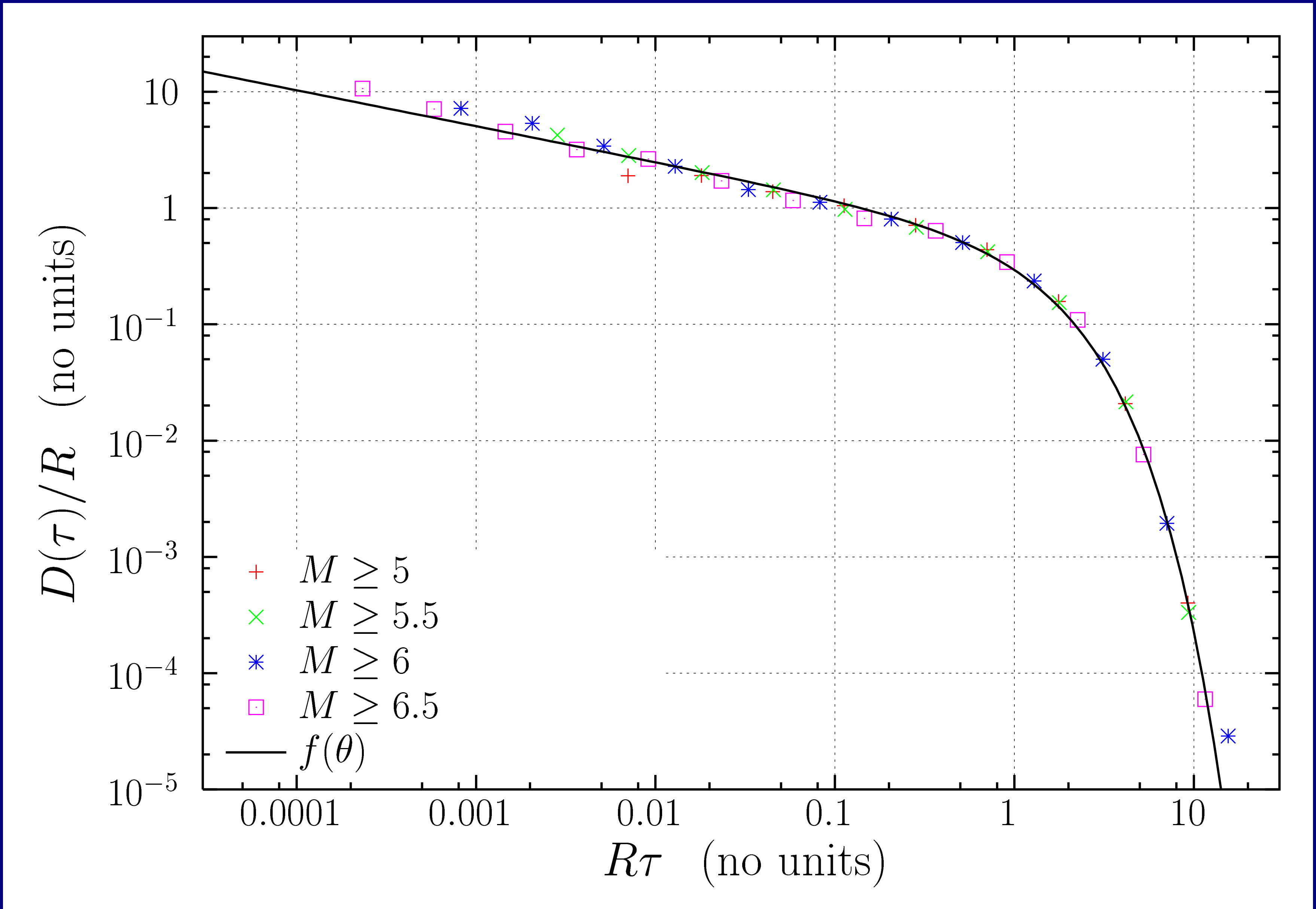
$$\epsilon(\tau_0) \equiv \langle \tau - \tau_0 \,|\, \tau > \tau_0 \rangle = \frac{\int_{\tau_0}^{\infty} (\tau - \tau_0) D_w(\tau) d\tau}{\int_{\tau_0}^{\infty} D_w(\tau) d\tau} \simeq \frac{\sum_{\forall i \text{ s.t. } \tau_i > \tau_0} (\tau_i - \tau_0)}{\text{num of equakes s.t. } \tau_i > \tau_0}$$



- This result seems certainly paradoxical, as for example:
  - ⋆ If you are waiting for the metro, you expect the next train is approaching
  - ⋆ When you celebrate your birthday, you may feel you are consuming your life

- From a statistical point of view, this is only counterintuitive, as there are counterexamples
  - ⋆ Newborns become “healthier” as time passes
  - ⋆ Companies become more solid with time (it is not preferable to invest your money in a very new company!)

- For earthquakes, this seems even more counterintuitive:
  - ⋆ The increase of time implies the increase on stress on the faults
  - ⋆ The occurrence of earthquakes decreases the stress in some areas, but we have no occurrence since the last one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$D(\tau)/R$ (no units)
$R\tau$ (no units)
$M \geq 5$
$M \geq 5.5$
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$f(\theta)$

- **Scaling law**

$$D(\tau, M_c) = R_c f(R_c \tau)$$

Gutenberg-Richter law: $R_c \propto 10^{-bM_c}$

In terms of energy: $R_c \propto 1/E_c^{\beta}$, so:

$$D(\tau, E_c) = E_c^{-\beta}\, \hat{f}(E_c^{-\beta}\tau)$$

This is the condition of scale invariance for 2d functions:

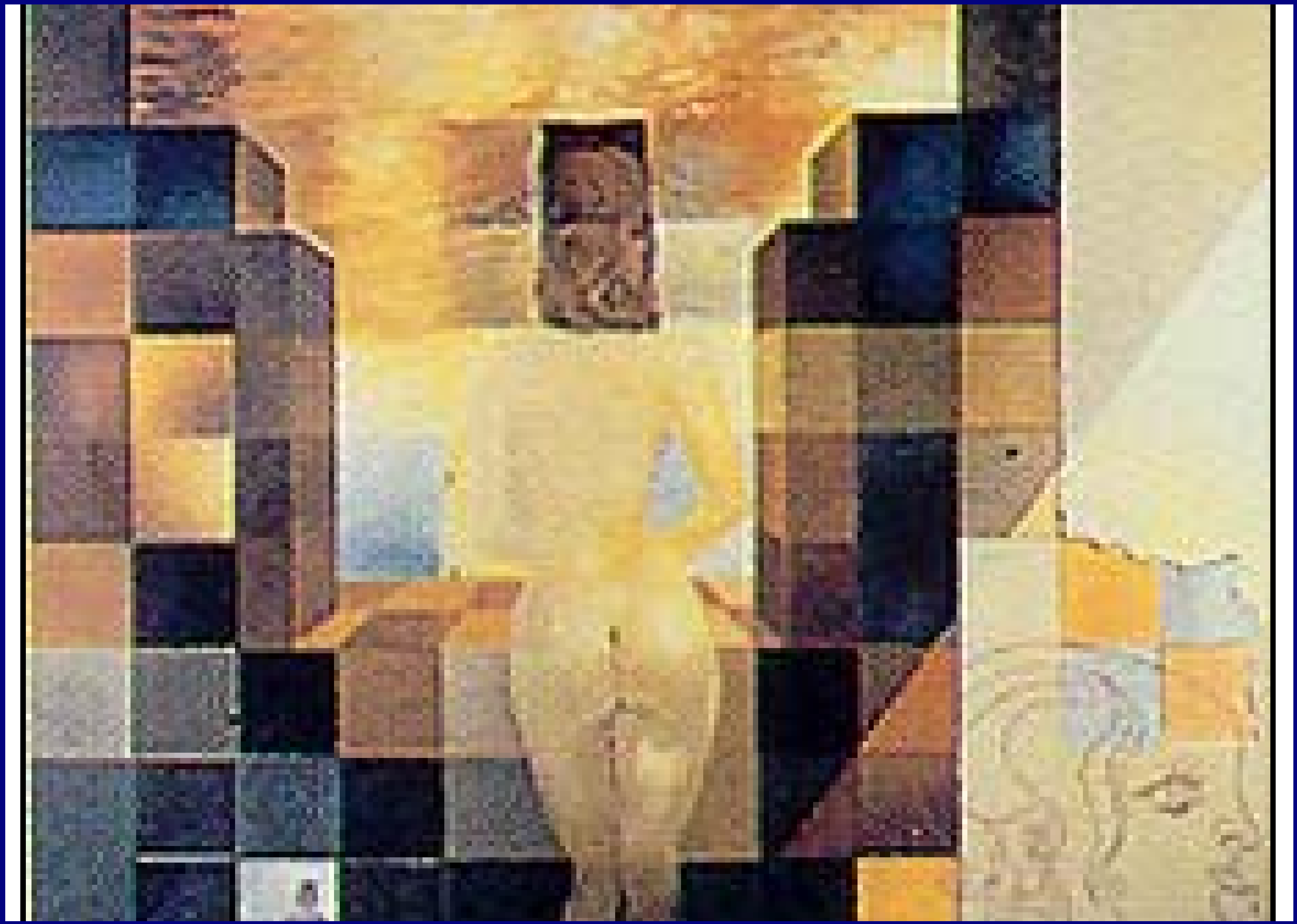
$$F(x, y) = cF(x/a_1, y/a_2) \Rightarrow F(x, y) = x^{\alpha} f(y/x^{\beta})$$

with f arbitrary, $\alpha = \ln c/\ln a_1$ and $\beta = \ln a_2/\ln a_1$

Why is this remarkable?

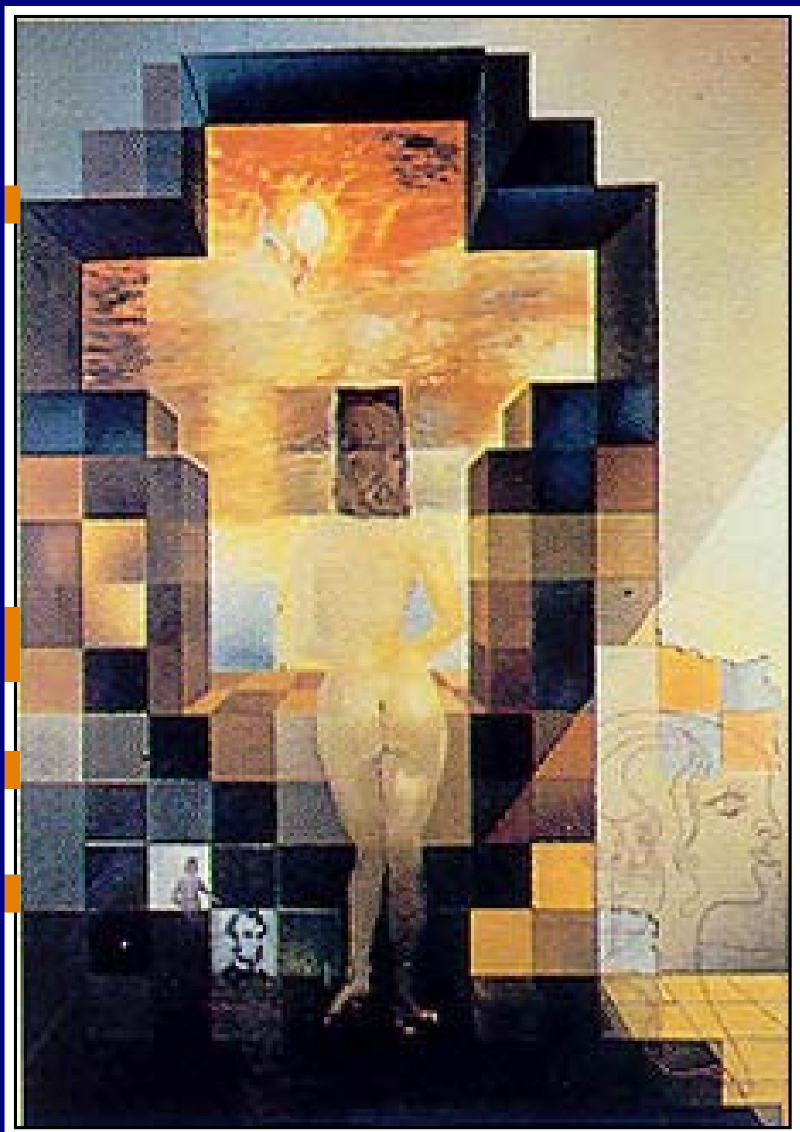
Relation with renormalization-group (RG) transformations

S. Dalí



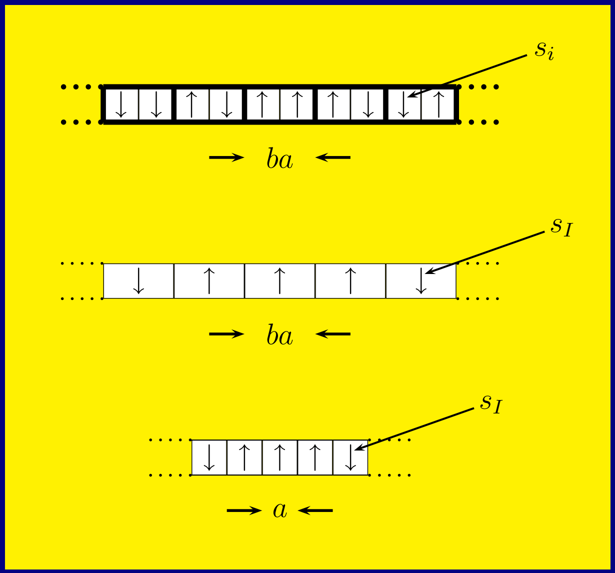
Relation with renormalization-group transformations

S. Dalí



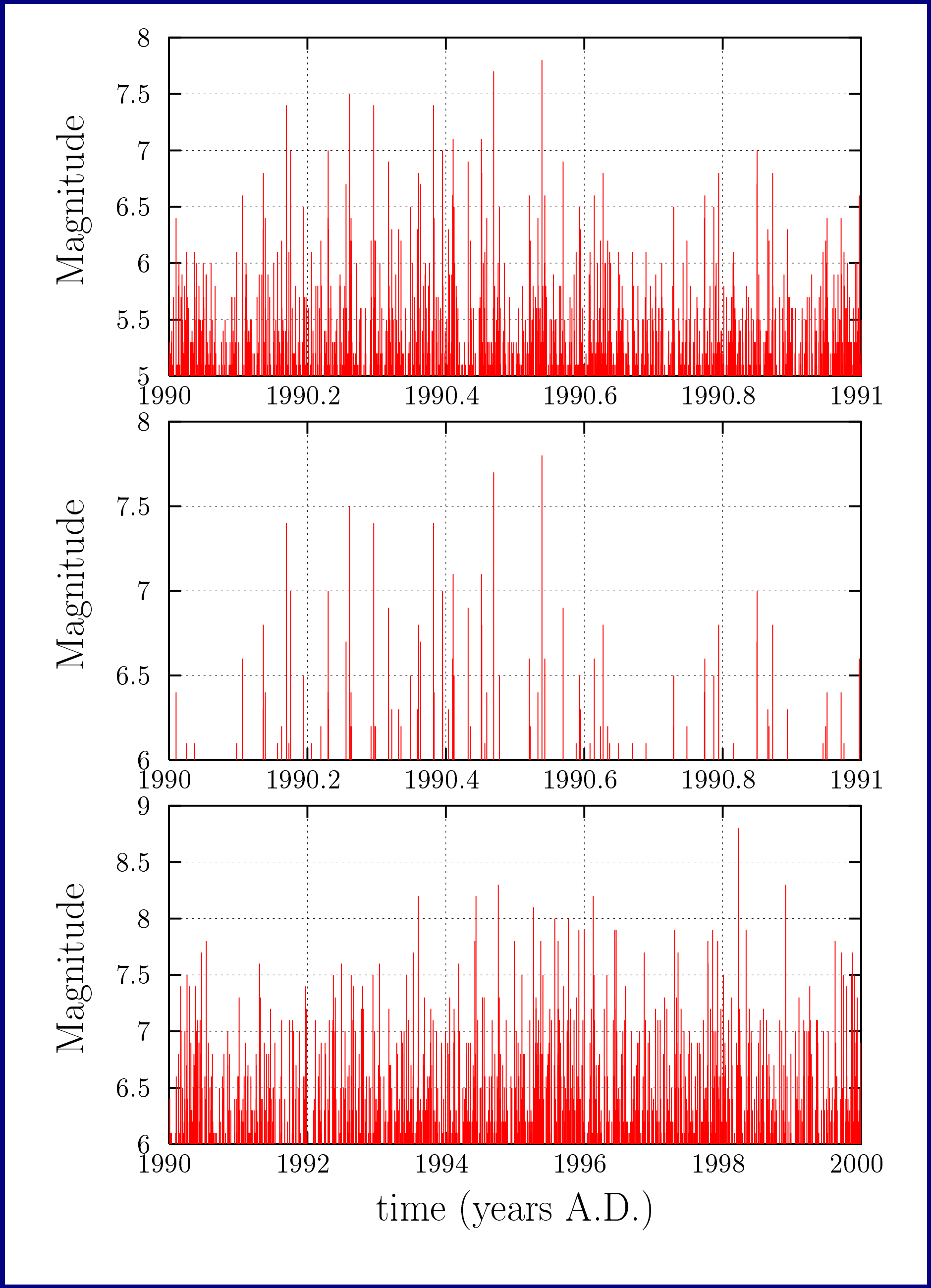
$\Rightarrow$ **Gala renormalizes into Lincoln!**

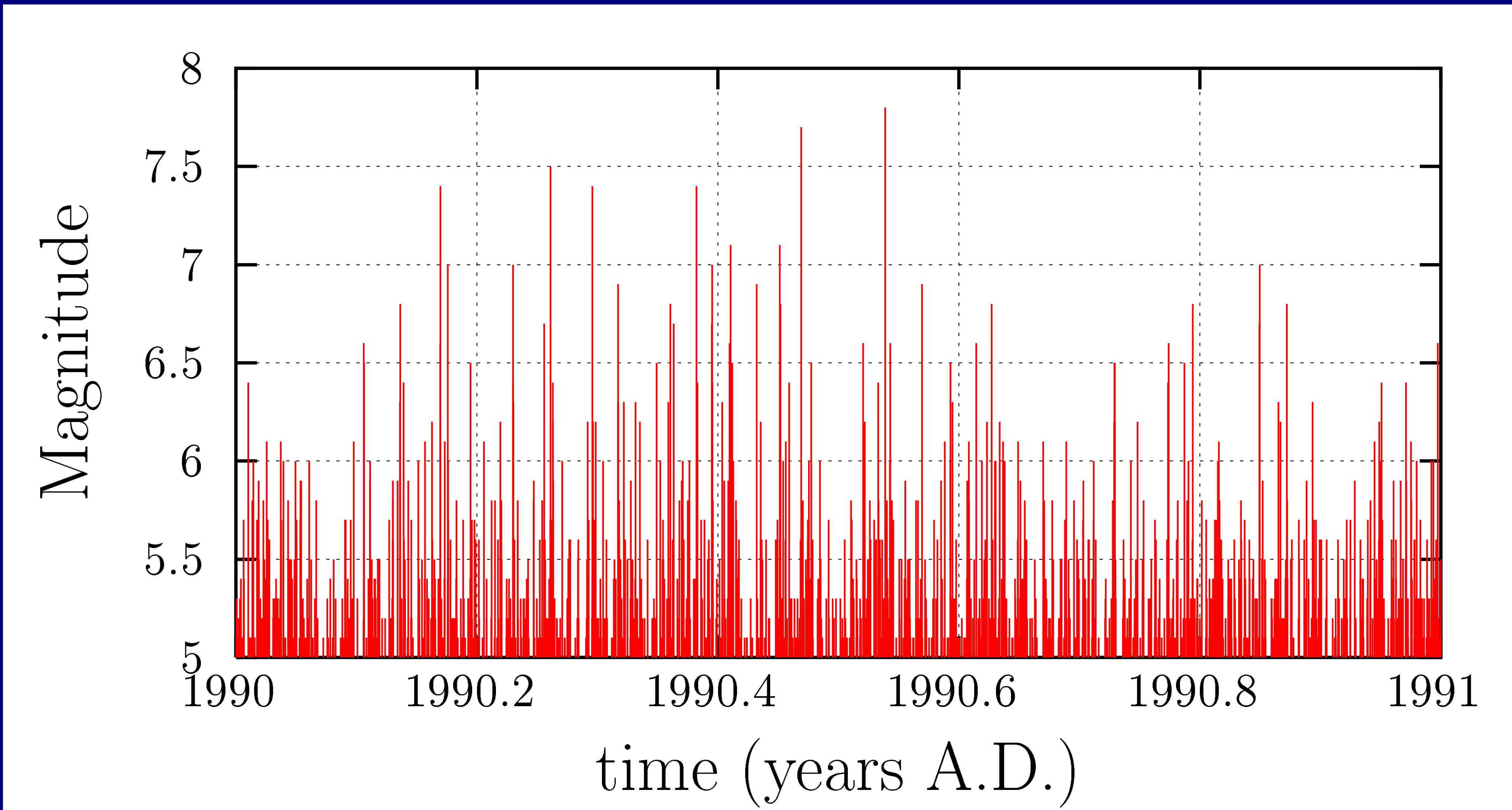
- **RG transformation:**
  The change of M_c (decimation)
  plus the re-scaling with R_c
  is analogous to a
  renormalization-group transformation

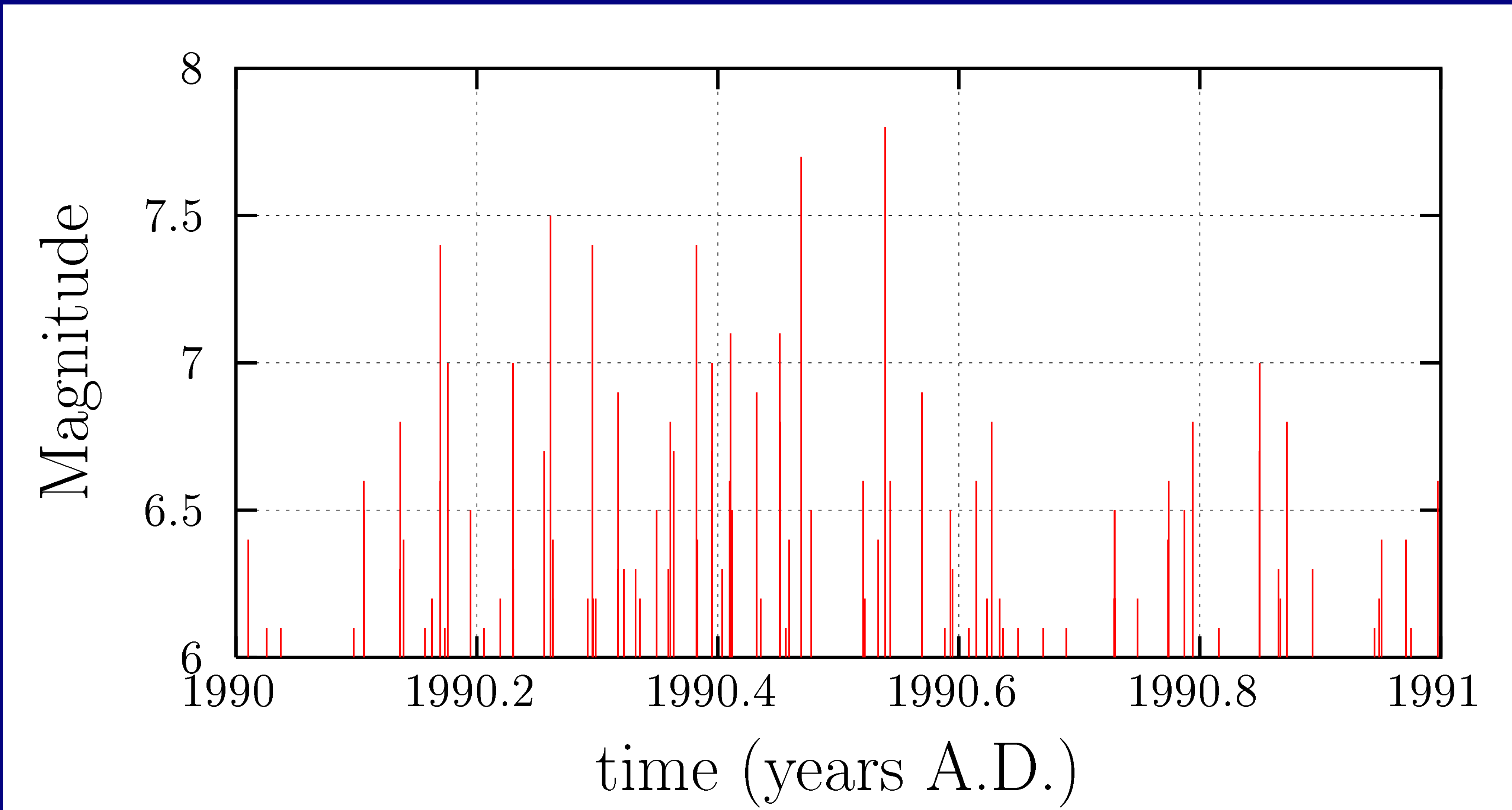


Christensen & Moloney

Complexity and Criticality




Magnitude
8
7.5
7
6.5
6
5.5
5
1990
1990.2
1990.4
1990.6
1990.8
1991
time (years A.D.)


8
7.5
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Magnitude
1990
1990.2
1990.4
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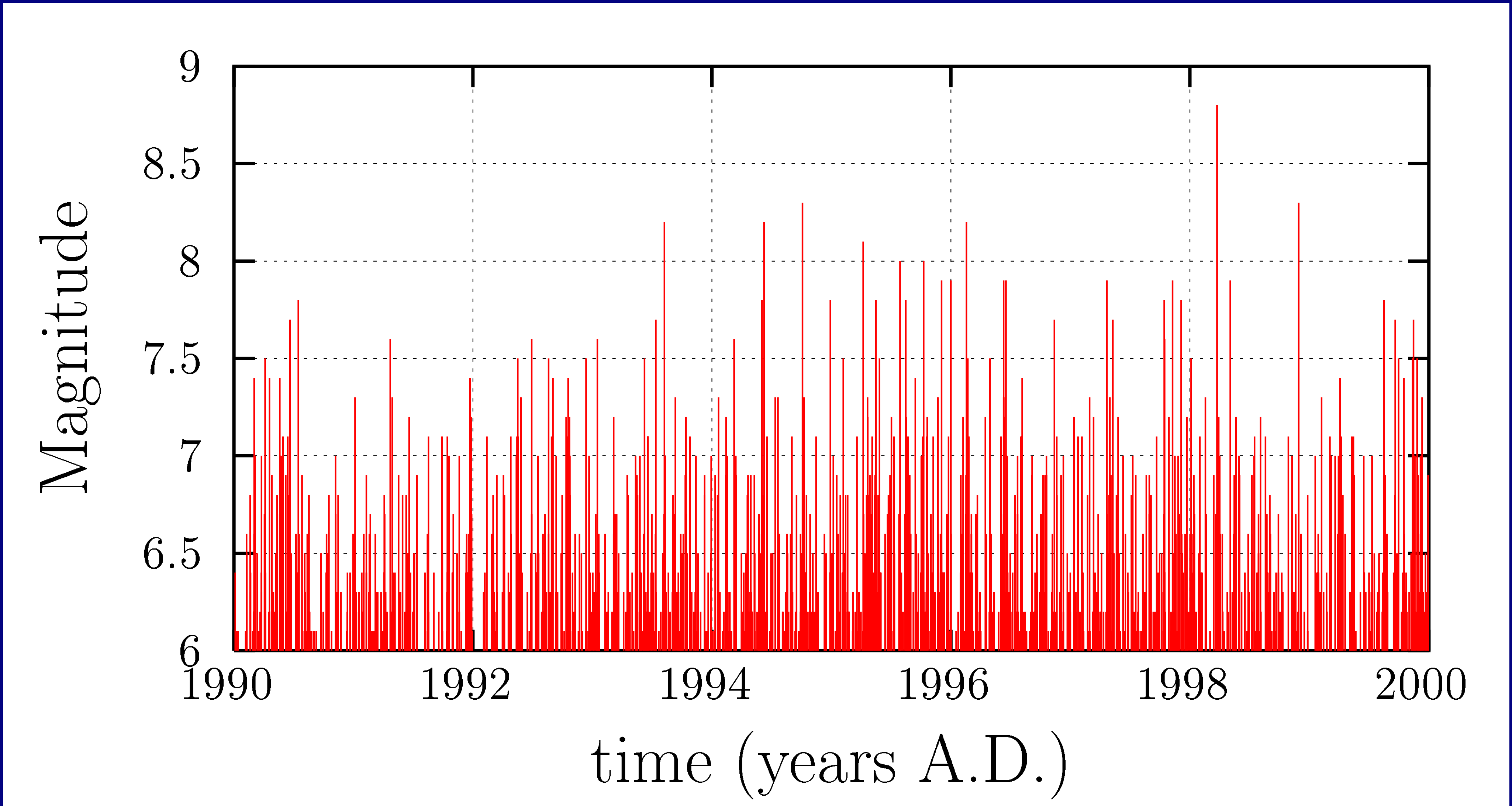
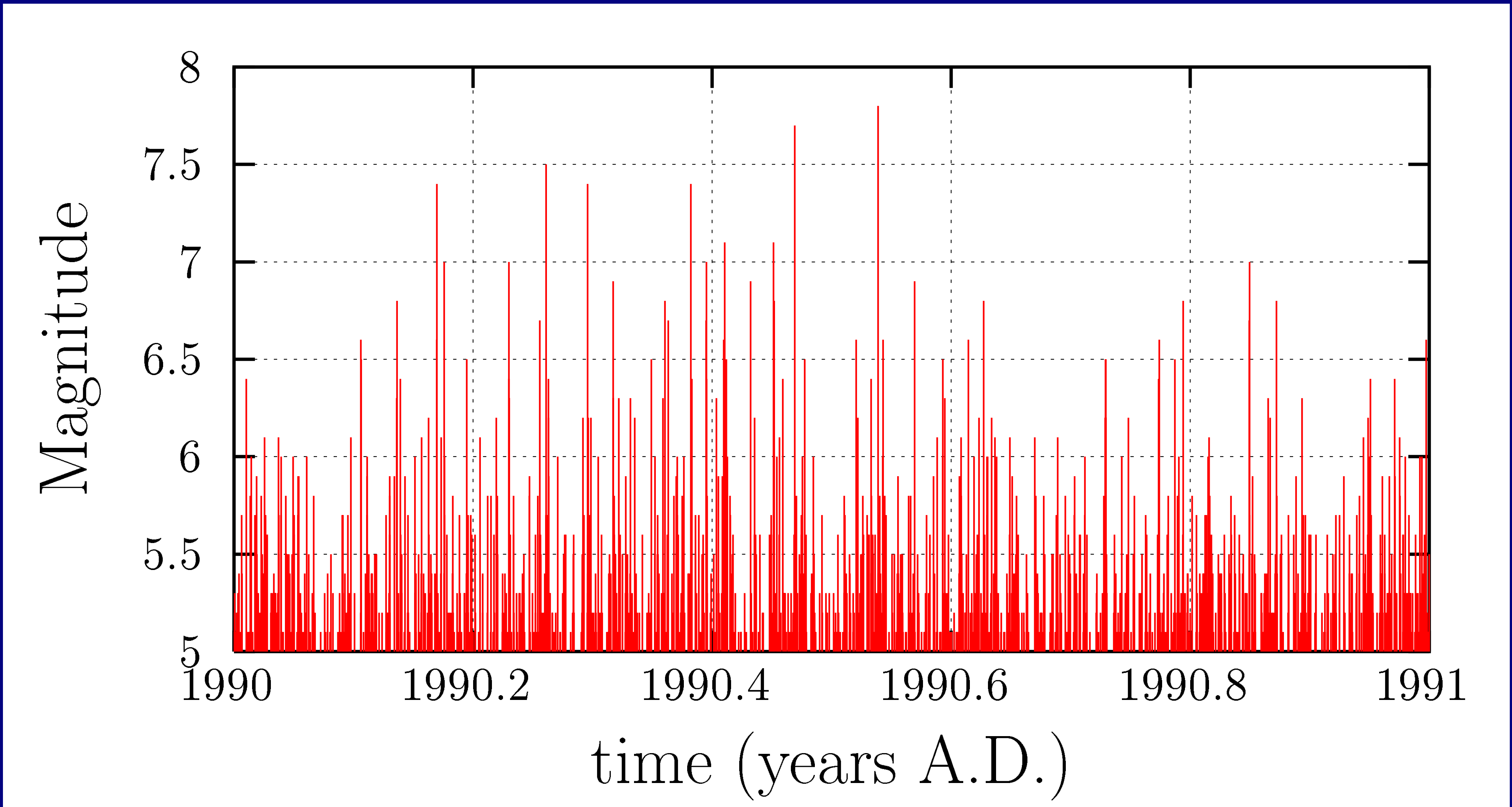
1991
time (years A.D.)


9
8.5
8
7.5
7
6.5
6
1990
1992
1994
1996
1998
2000
Magnitude
time (years A.D.)


Magnitude
8
7.5
7
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1990
1990.2
1990.4
1990.6
1990.8
1991
time (years A.D.)

Mathematical results for renewal processes A.C. JSTAT 2009
(no correlations, magnitudes and times independent)

- Scaling $\Rightarrow$ Invariance of seismicity under RG transformations

- RG Transformation $=$ *random thinning* (decimation) $+$ rescaling:

$$\top D(s) = \frac{pD(ps)}{1 - qD(ps)}$$

with $D(s)$ the Laplace transform of $D(\tau)$ and $p \equiv R(M_c')/R(M_c)$, $q \equiv 1 - p$

- The only fixed point is Poisson $\top D(s) = D(s) \Rightarrow D(\tau) = Re^{-R\tau}$ ∎

- Moreover, the Poisson process is the attractor for all waiting time distributions with finite mean

- Summary: for processes with no correlations and a finite mean
  $\Rightarrow$ the Poisson process is a trivial fixed point

- But the observed $D(\tau)$ is not exponential

  $\Rightarrow$ There must be correlations

- Correlations are fundamental to determine the form of $D(\tau)$

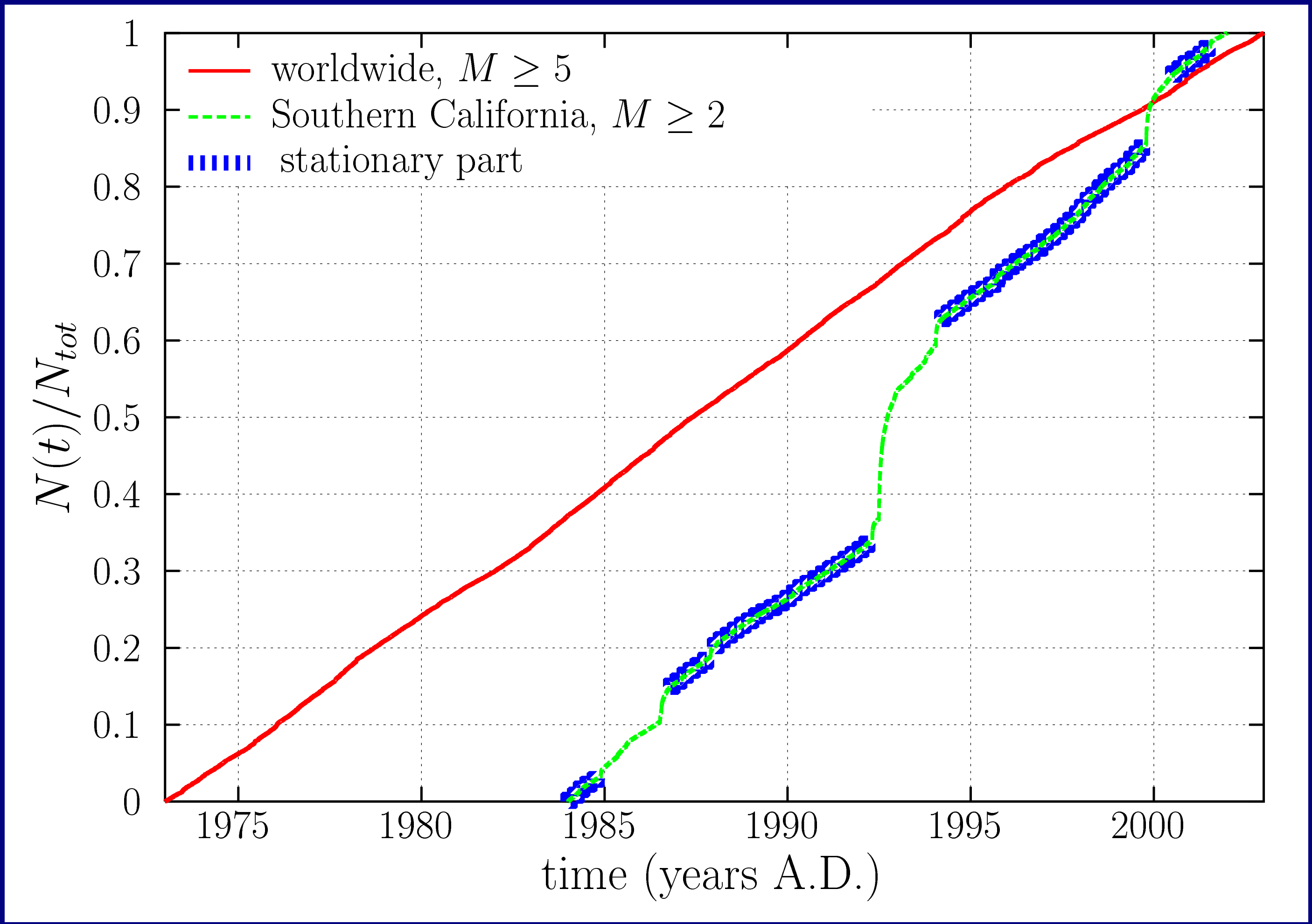
- Short-range correlations do not seem enough to escape from Poisson

- Correlations should be long ranged (Lennartz *et al.*, EPL 2008)

- A RG approach could provide scaling relations between the exponent of $D(\tau)$, the exponent of correlations, and the Gutenberg-Richter exponent

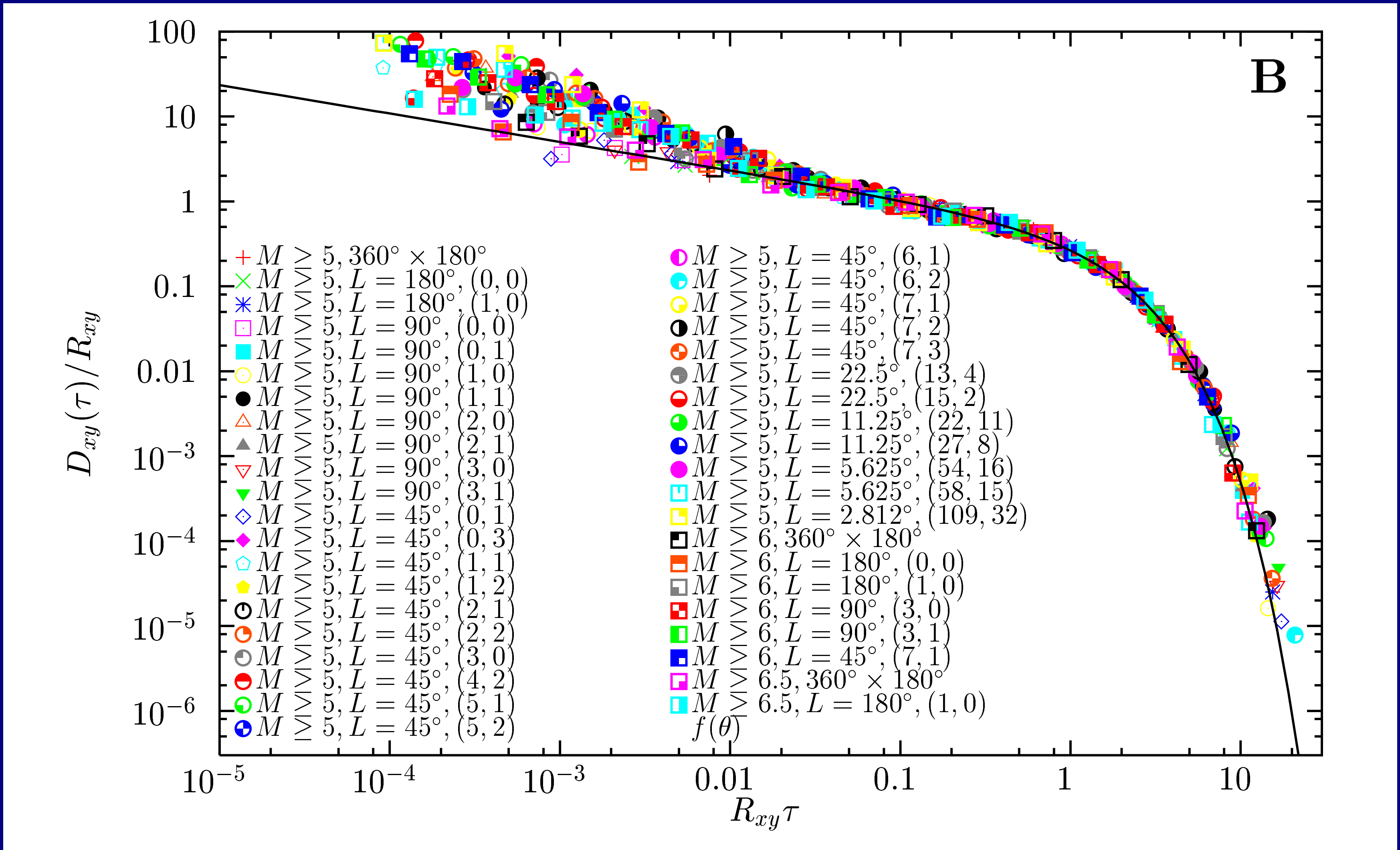
- Connection with critical phenomena A.C. PRL 2005

Accumulated number of earthquakes (normalized) versus time
with $N_{tot} = 84771$ for Southern California and $N_{tot} = 46054$ for worldwide

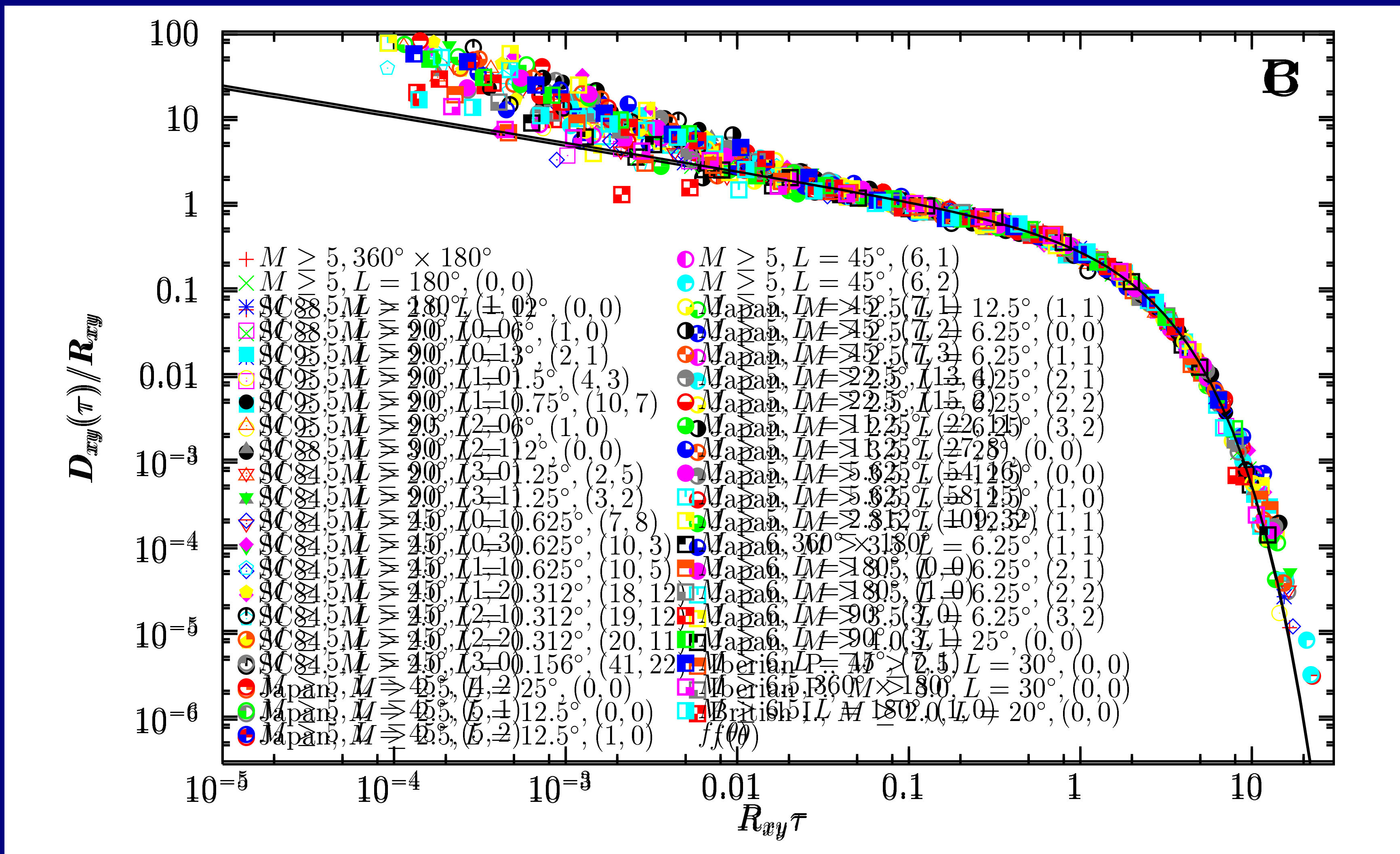


$\Rightarrow$ worldwide seismicity is stationary, Southern California is not stationary

Changing the spatial region: WW stationary seismicity up to $2.8°$ (300 km)



Universal scaling law: also California, Japan, and Spain, for stationary periods



ETAS model (epidemic-type aftershock sequence)

- Each earthquake (i) triggers other earthquakes with
  - ⋆ Probability proportional to the Omori law, $1/(t-t_i)^p$
  - ⋆ and proportional to the productivity law, 10^{aM_i} (power law)
  - ⋆ Magnitude given by the Gutenberg-Richter law, 10^{-bM} (power law)

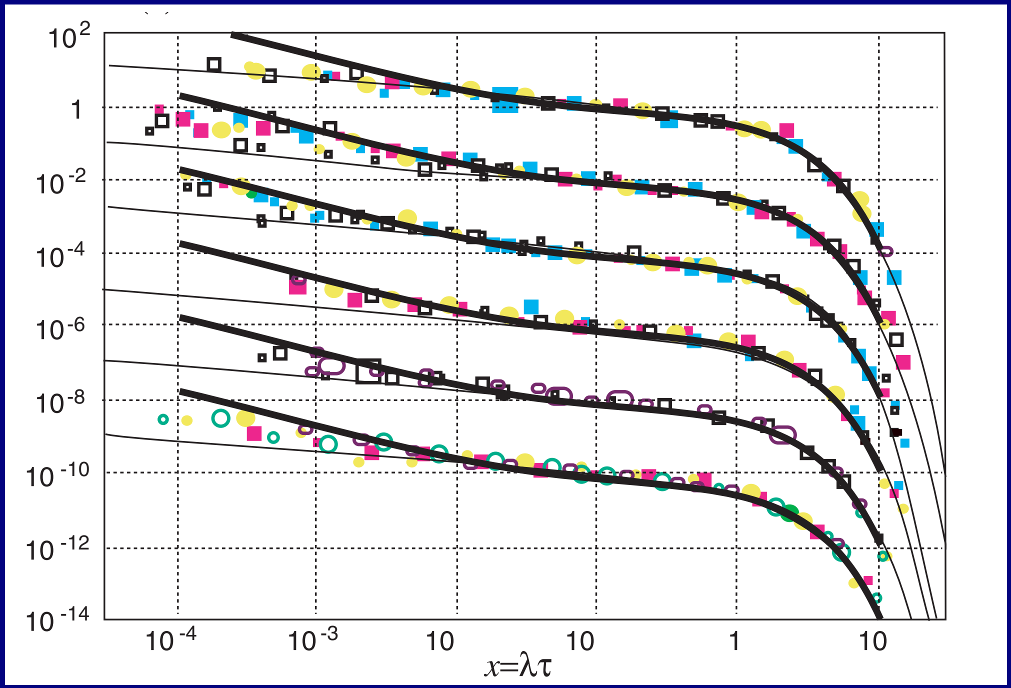
- Hard mathematics show that
  - ⋆ The scaling function is different
  - ⋆ Even more, a scaling law cannot hold exactly!

  $\Rightarrow$ there must be (very) slow variations with magnitude

  $\Rightarrow$ the ETAS distribution renormalizes to an exponential

Saichev & Sornette, PRL 2006

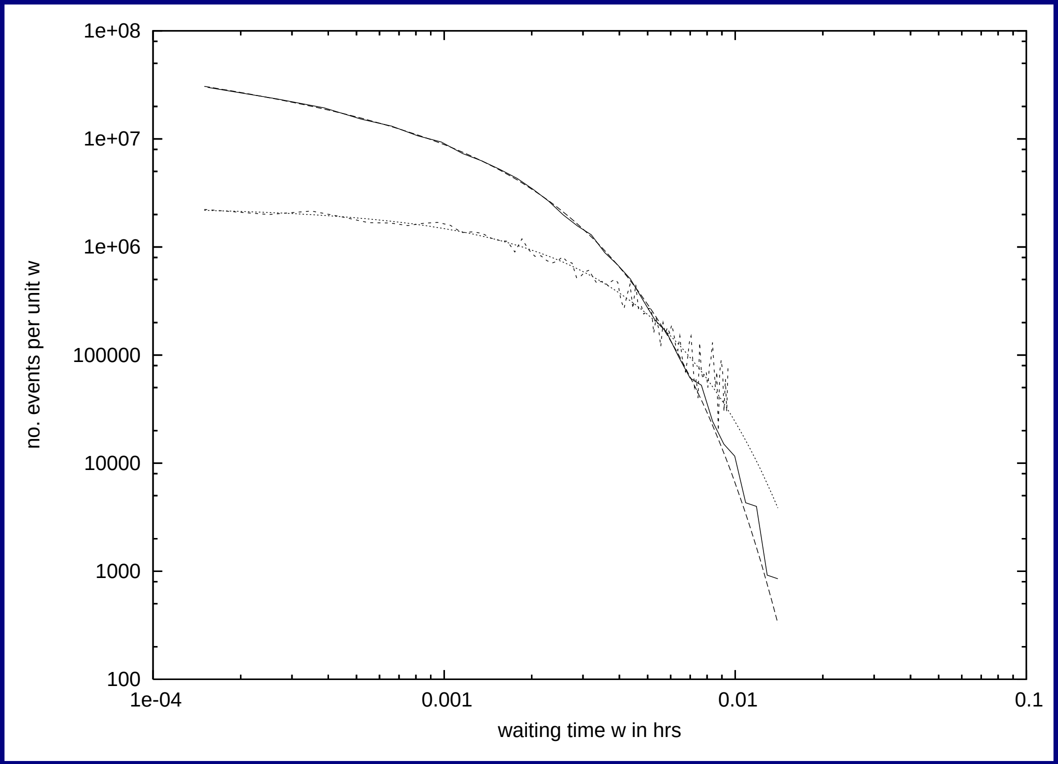
- Saichev & Sornette's fit: 4 parameters:



- But the ETAS model is not scale invariant (from its definition)!

- Try with a scale-invariant model?
  For instance: Vere-Jones model, AAP 2005;
  DS model, Lippiello *et al.* PRL 2007;
  BASS model, Turcotte *et al.* GRL 2007

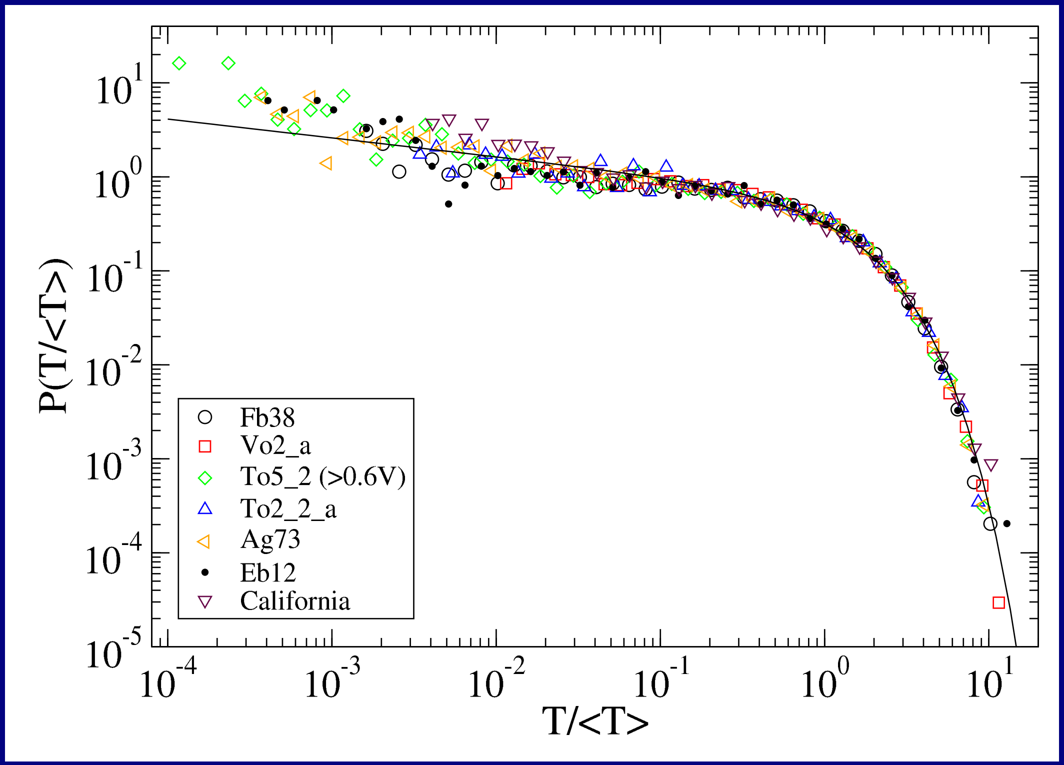
- Unexpected pulses detected in the CRESST project for dark matter search at the Gran Sasso Laboratory Åstrom *et al.* PLA 2006
  Cryogenic detector (at milliKelvin) made by a sapphire monocrystal



⋆ Radioactive contamination? $\Longmapsto$ No Poisson distribution!
⋆ Origin: nanofractures in the crystal due to the tight clamping of the detector

- Acoustic emission from laboratory rock fractures

Davidsen *et al.* PRL 2007



⋆ Materials: sandstones (wet conditions), granite (dry), Etna basalt (dry)
⋆ Loading conditions: constant displacement rate, AE activity feedback control of loading, punch-through loading
⋆ Confined pressures: from 5 to 100 MPa

- Enormous range of validity of the scaling law:

  ⋆ From nanofractures involving the breaking of only several hundreds of covalent bonds (5 keV $\simeq 8 \cdot 10^{-16}$ J)

  to very large earthquakes ($M \geq 7$ or radiated energy $\geq 2 \cdot 10^{15}$ J)

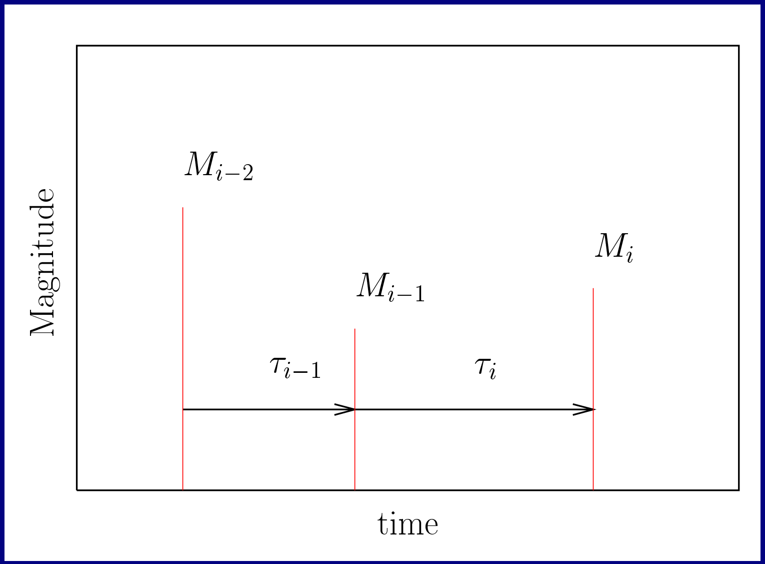
  $\Rightarrow$ More than 30 orders of magnitude of validity!

  ⋆ Profound differences between the homogeneity and regularity of a monocrystal at milli-Kelvin temperatures and the heterogeneity of fault gouge producing (and produced by) earthquakes

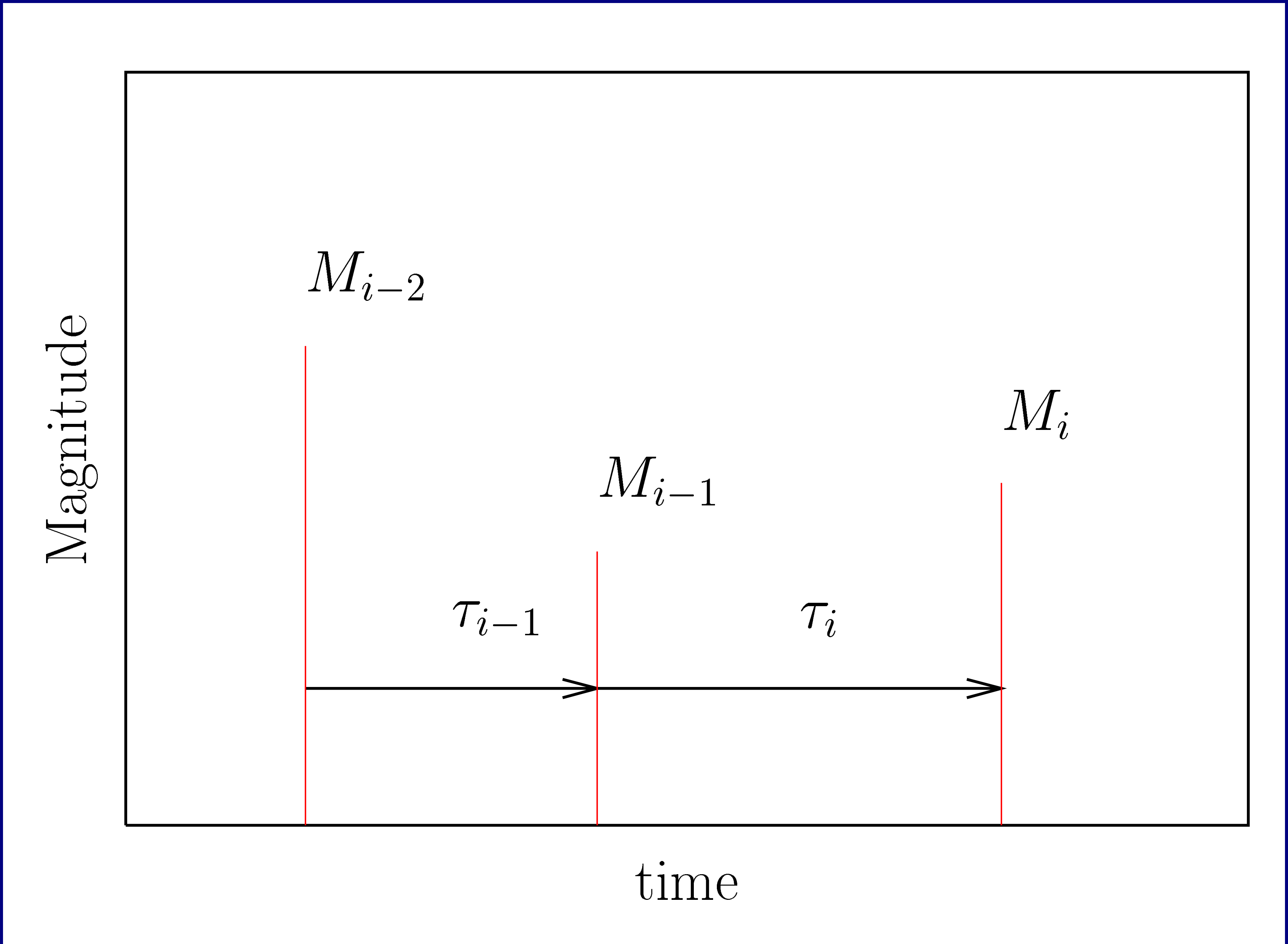
$\Rightarrow$ Universality

Conditional probability density (for recurrence times)

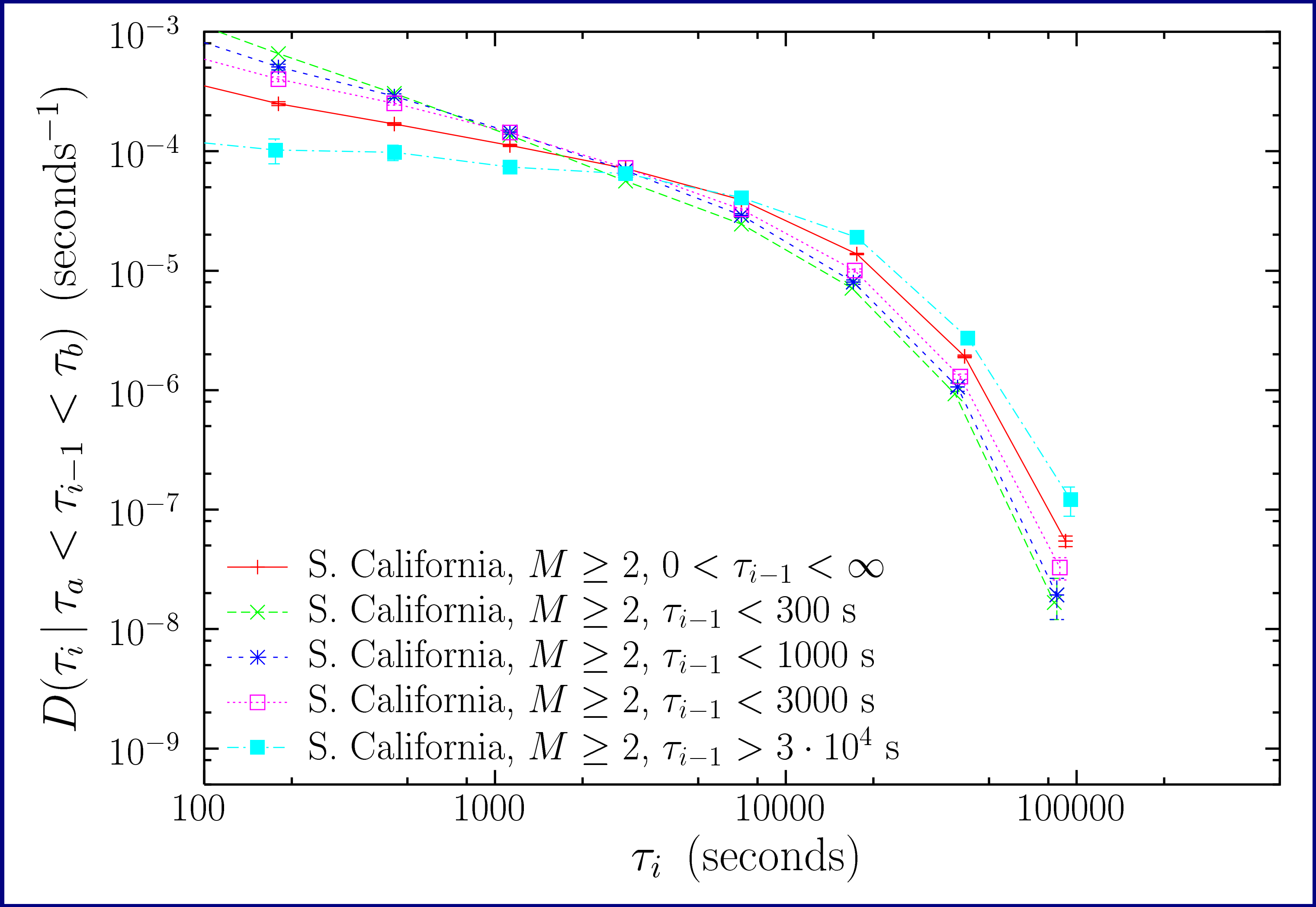
$$D_w(\tau|X) \equiv \frac{\text{Prob}\left[\tau \leq \text{recurrence time} < \tau + d\tau \text{ conditioned to } X\right]}{d\tau}$$



- For each τ_i, X will be different sets of values (large, small, etc.) of
  - ⋆ M_i (current magnitude)
  - ⋆ M_{i-1} (previous magnitude)
  - ⋆ τ_{i-1} (previous recurrence time)
- if $D_w(\tau|X) = D_w(\tau) \Rightarrow \tau$ and X independent
- if $D_w(\tau|X) \neq D_w(\tau) \Rightarrow \tau$ and X correlated (linearly or nonlinearly)

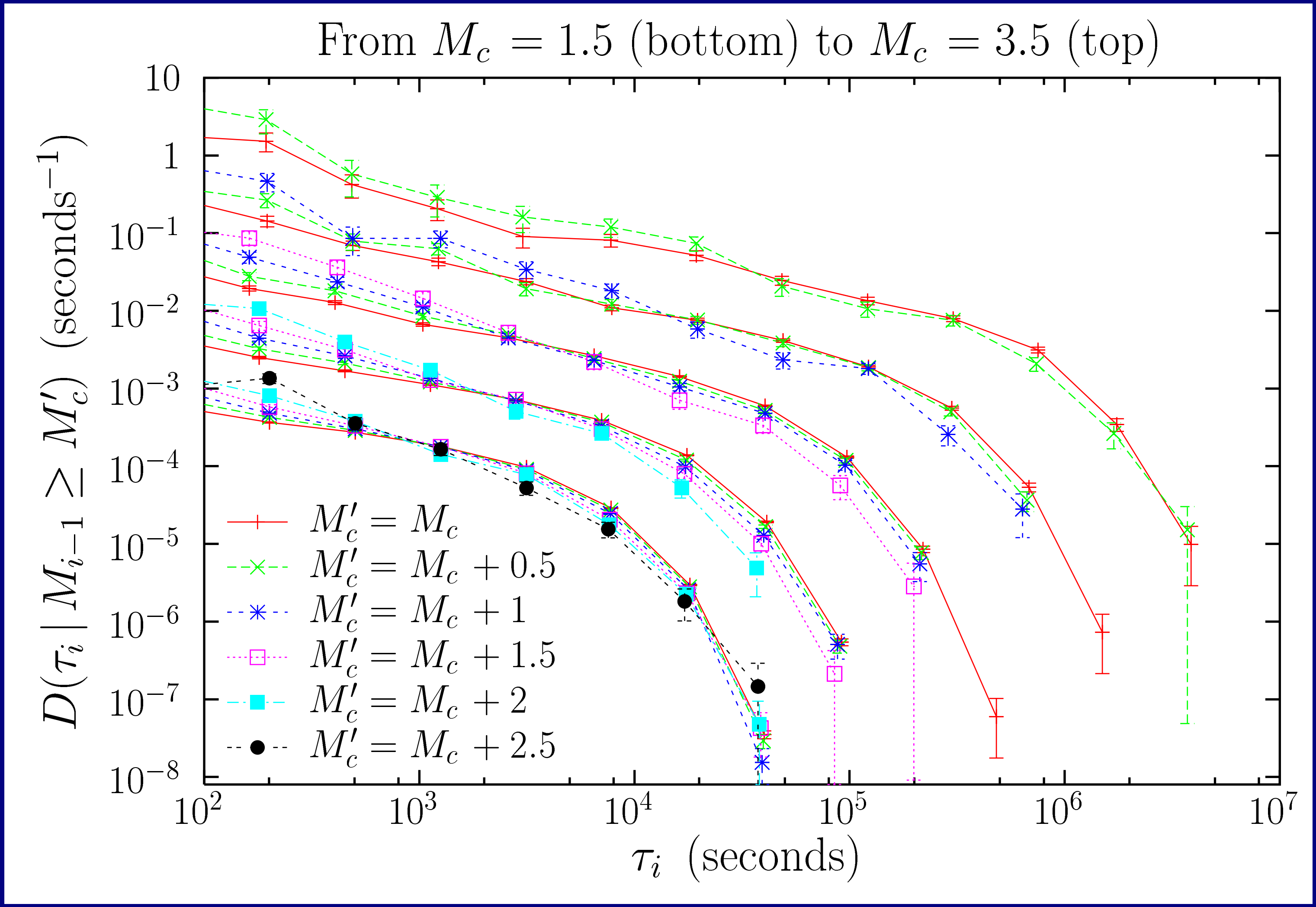

M_{i-2}
M_i
M_{i-1}
τ_{i-1}
τ_i
Magnitude
time

Relation of τ_{i-1} with τ_i for Southern-California stationary seismicity



$D_w(\tau_i|\tau_a \leq \tau_{i-1} < \tau_b) \neq D_w(\tau_i)$ $\Rightarrow$ τ_i does depend on τ_{i-1} (positive correlation)

Relation of M_{i-1} with τ_i for Southern-California stationary seismicity



$D_w(\tau_i|M_{i-1} \geq M_c') \neq D_w(\tau_i)$ $\Rightarrow$ τ_i does depend on M_{i-1} (anticorrelation)

Conclusion

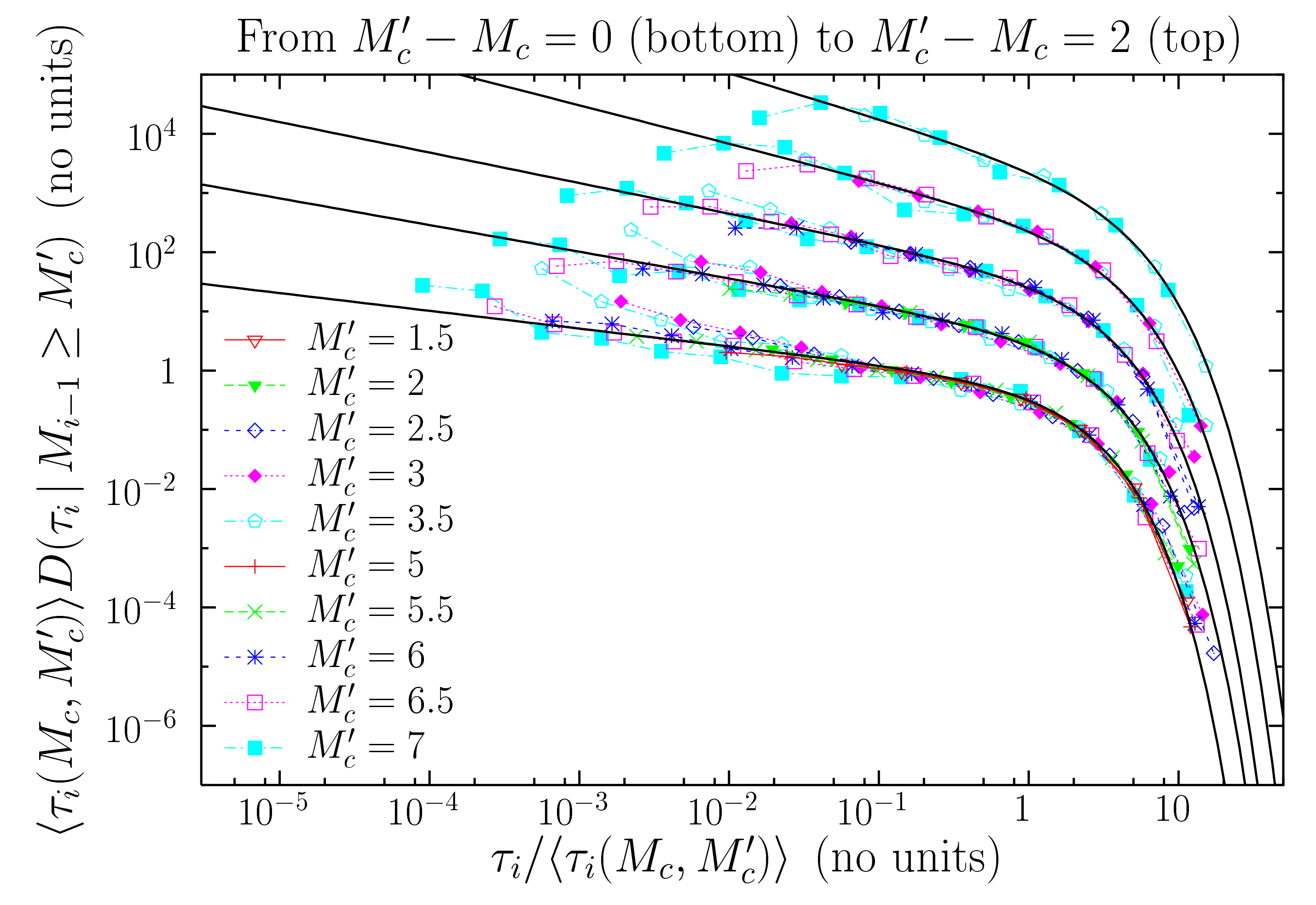
- *"The shorter the time between 2 earthquakes,*
  the shorter the time to the next"

- *"The larger the magnitude, the shorter the time to the next earthquake"*

$\Rightarrow$ Recurrence times depend on history

- Possible existence of long-range correlations [Lennartz et al. 2007]

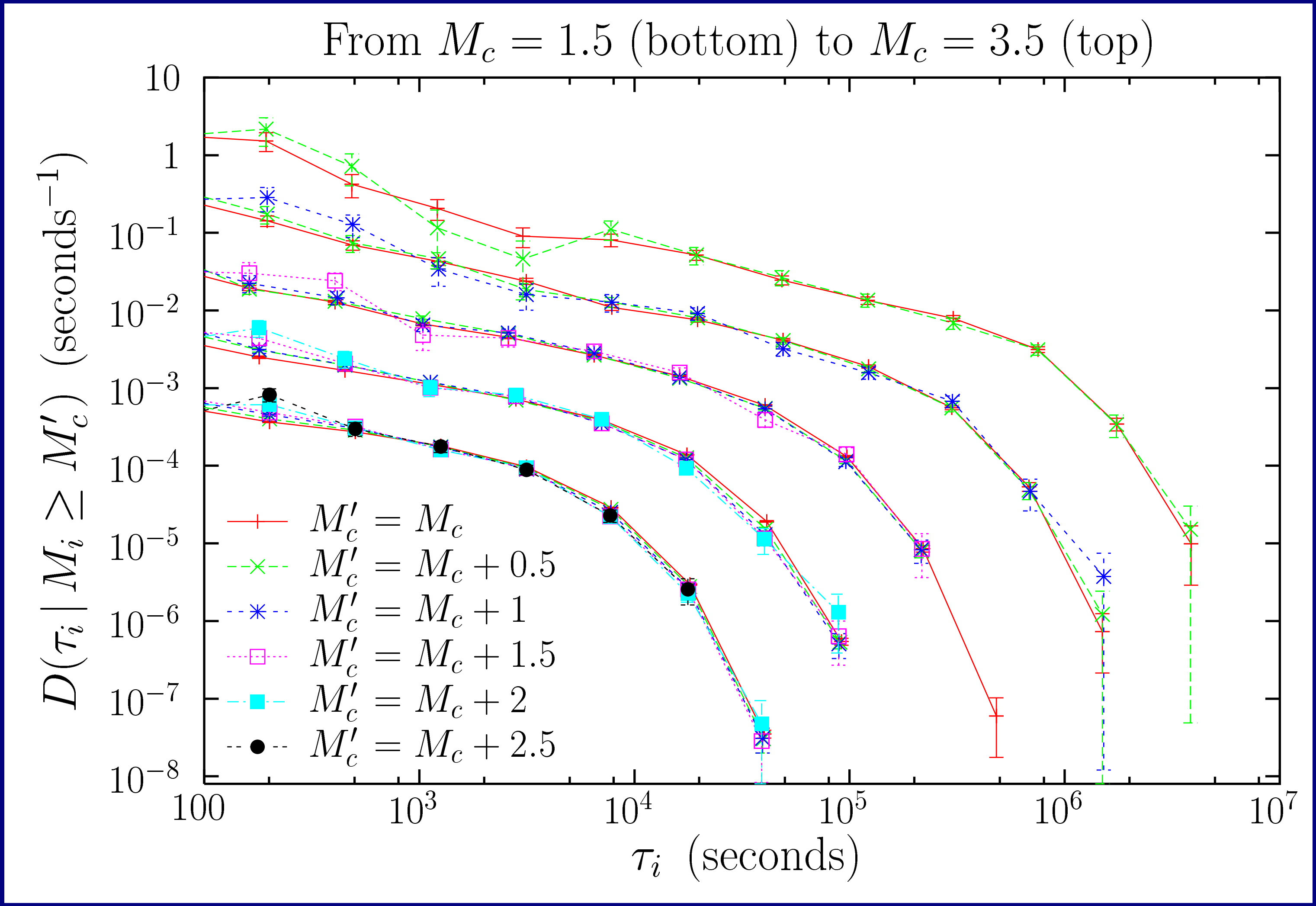
$D_w(\tau_i|M_{i-1} \geq M'_c; M_c)$ **only depends on** τ **and** $M'_c - M_c$



Moreover, a new scaling law holds, if $R(M_c, M'_c) \equiv 1/\langle \tau(M_c, M'_c)\rangle$,

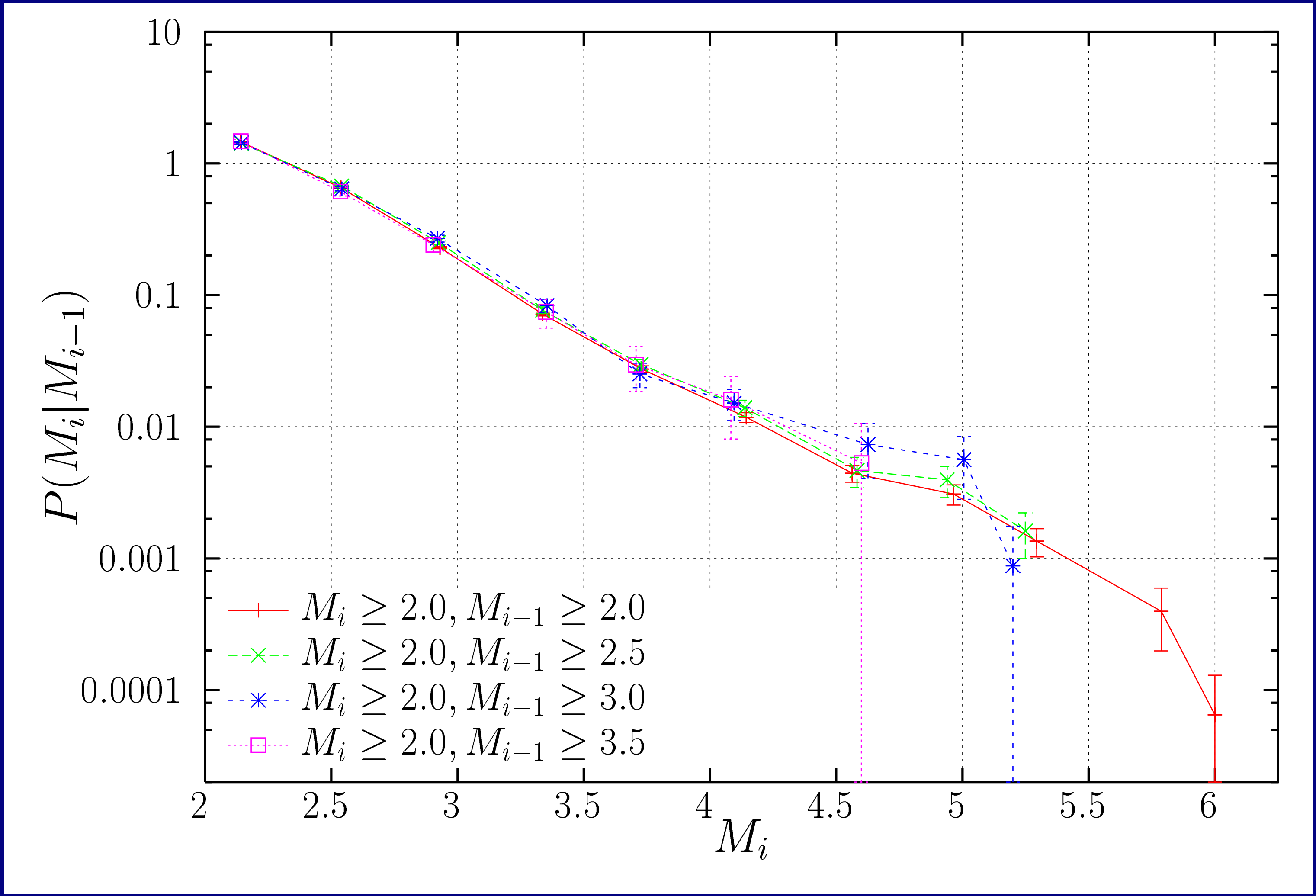
$\Rightarrow D_w(\tau_i|M_{i-1} \geq M'_c; M_c) = R_w f(R_w \tau_i, M'_c - M_c)$

Relation of M_i with τ_i for Southern-California stationary seismicity



$D_w(\tau_i|M_i \geq M'_c) \simeq D_w(\tau_i)$ $\Rightarrow$ M_i is independent on τ_i

Magnitude probability densities conditioned to the preceding magnitude



$D_w(M_i|M_{i-1} \geq M_c') \simeq D_w(M_i)$ $\Rightarrow$ M_i is independent on M_{i-1} (for $\tau_i > 30$ min)

Conclusion

- *"The time you have been waiting for an earthquake does not influence its magnitude"*

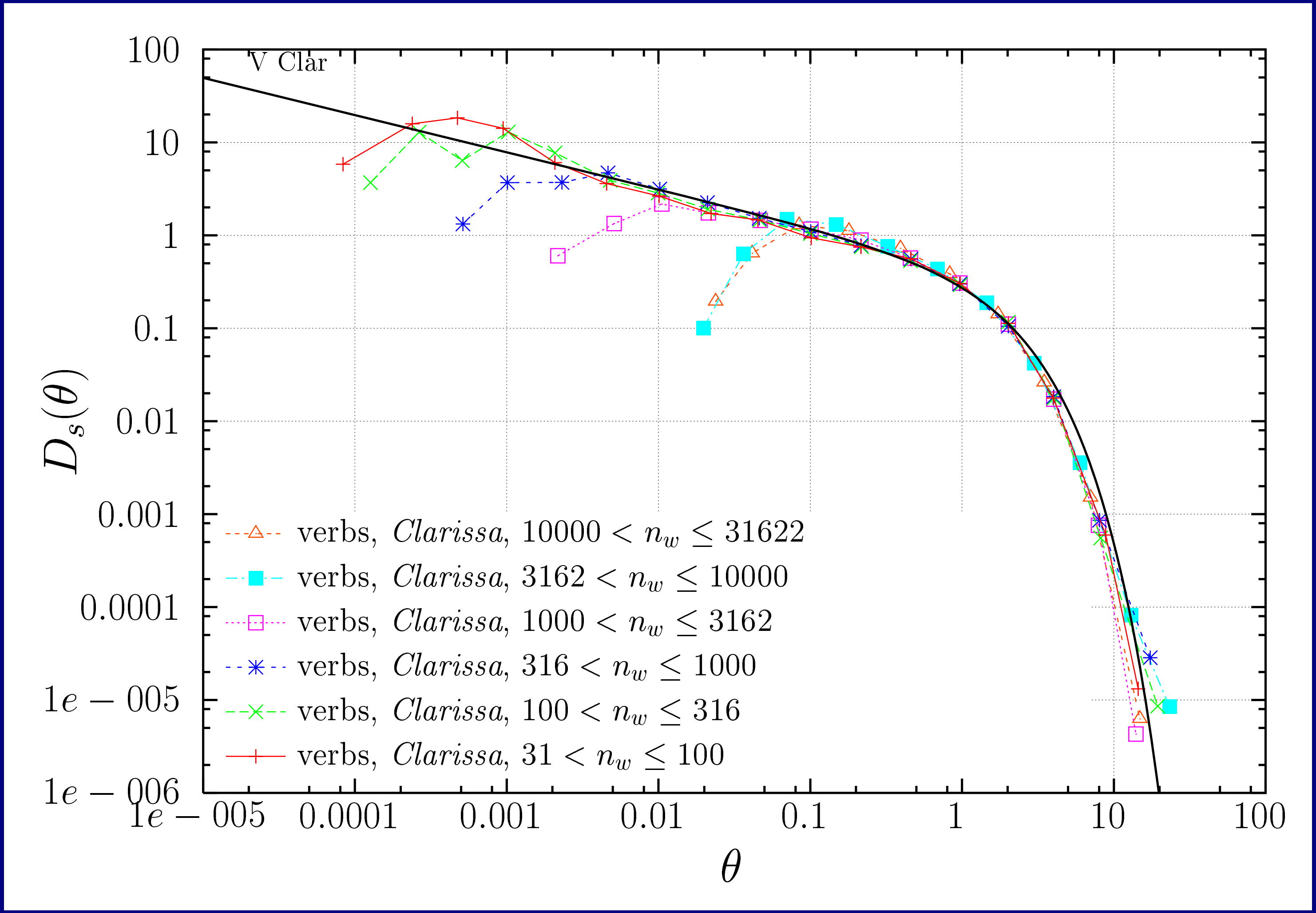
- *"The magnitude of a given event does not influence the magnitude of the next one"*

$\Rightarrow$ Magnitude seems to be independent on history

$$\boxed{\textit{An earthquake does not know how big is going to be}}$$

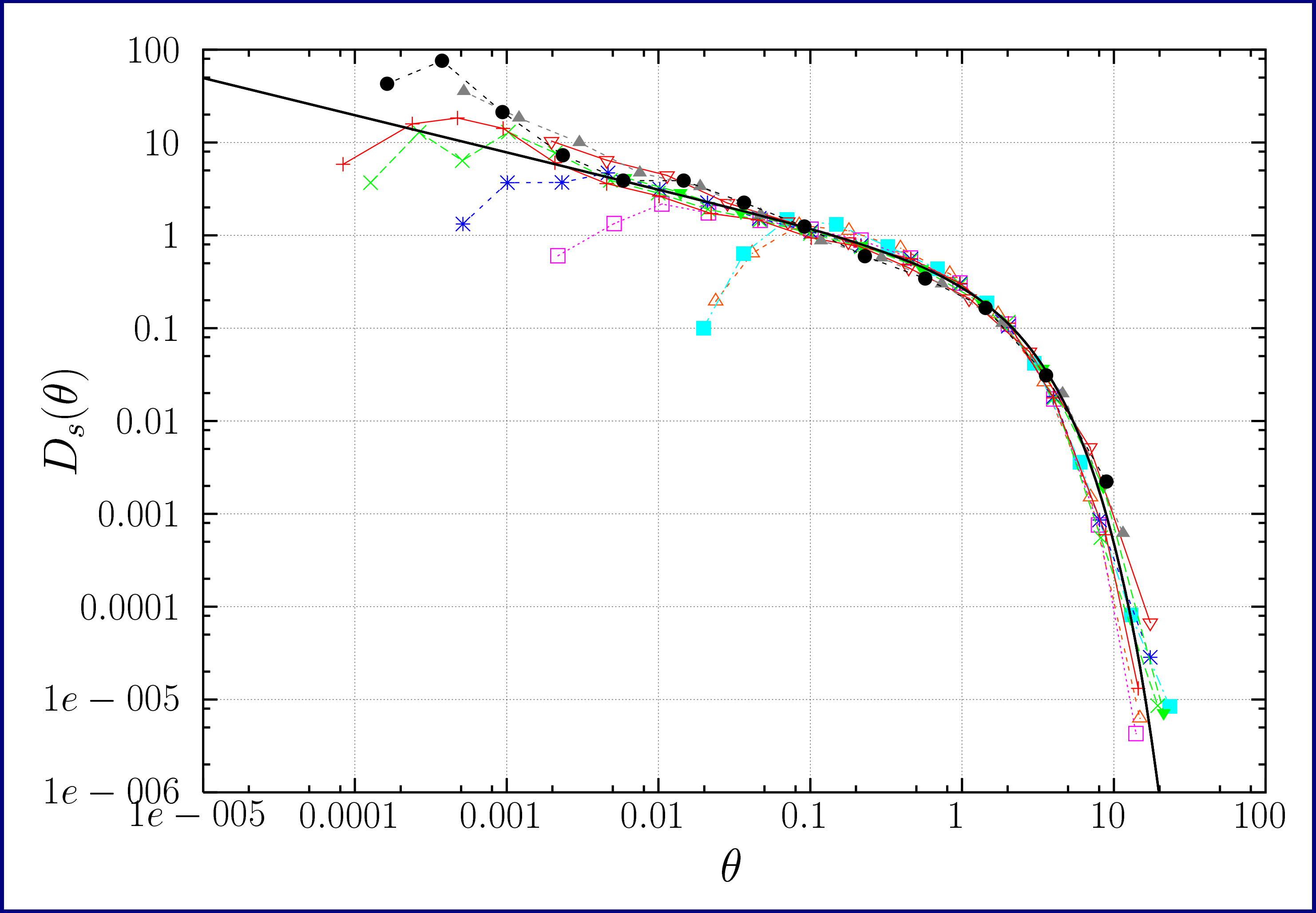
But see Lippiello et al. PRL 2007

Verbs in novel *Clarissa*, by S. Richardson (year 1748, 1 million words)



⇒ Scaling and clustering (attraction)! Fit: gamma distribution with $\gamma = 0.6$

Comparing verbs in *Clarissa* with earthquakes in S. California, 1995-1998



remember, $\theta = \ell/\bar{\ell}_w$ for words, $\theta = R\tau = \tau/\bar{\tau}$ for earthquakes

- The dynamics of earthquake occurrence shows self-similar clustering, described by a universal scaling law
- The same law holds for fractures up to very small scales (Davidsen *et al.*, Åstrom *et al.*)
- The scaling law is equivalent to the invariance of the system under renormalization transformation
- Correlations are essential to the existence of the scaling law
- References at `http://einstein.uab.es/acorralc`