A dynamical scaling approach to earthquake occurrence

Lucilla de Arcangelis

Second University of Naples

IfB, ETH, Zurich

Outline

- Phenomenological laws for seismic occurrence
- Critical behaviour and scale invariance
- Scaling hypothesis
- Correlations in seismicity
- Dynamical scaling branching model
- Risk maps
- Generalized Omori law
- Outlook

How big can an earthquake be?

Gutenberg-Richter Law (1954)

$$P(>M) \sim 10^{-b\,M} \quad (b \sim 1)$$

Seismic moment $M_0 = mA\Delta u$

$M = (2/3)\log(M_0) - 6$

(Kanamori,Anderson 1975)

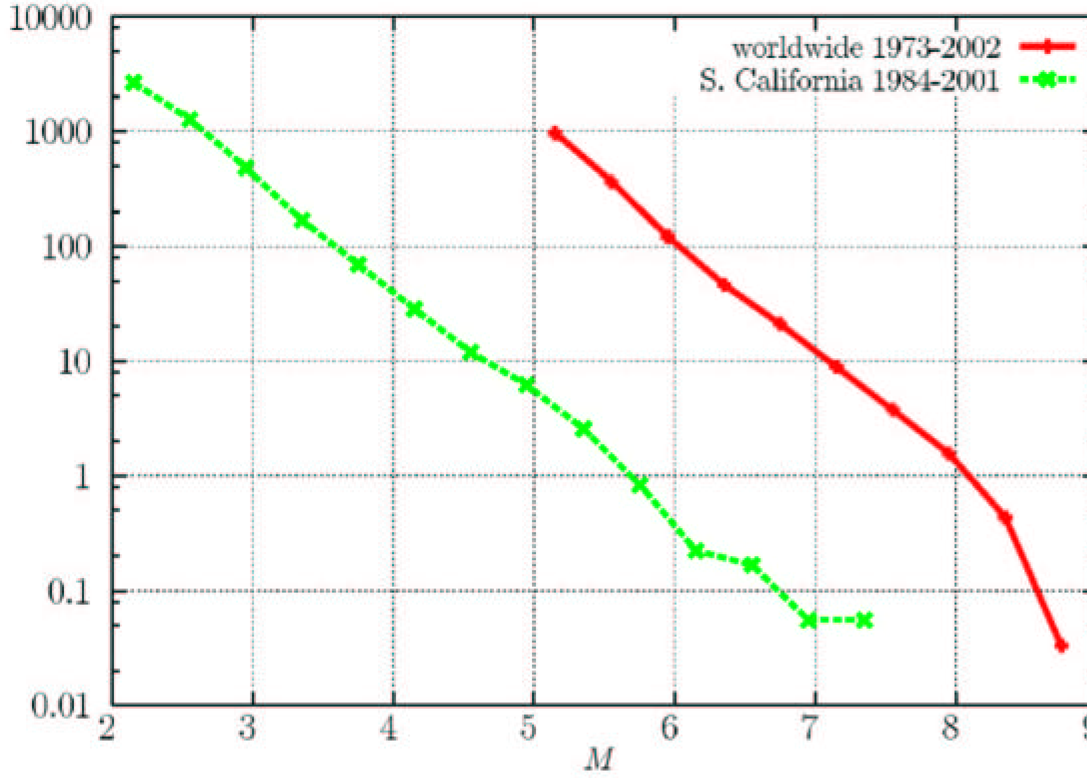
$$P(>M_0) \sim M_0^{-\alpha}$$

Energy

$M = (2/3)\log(E) + cost$

$$P(>E) \sim E^{-\alpha}$$

Universality of $\alpha \sim 0.7$



Sequences of aftershocks

Omori law (JCSIUT,1894)

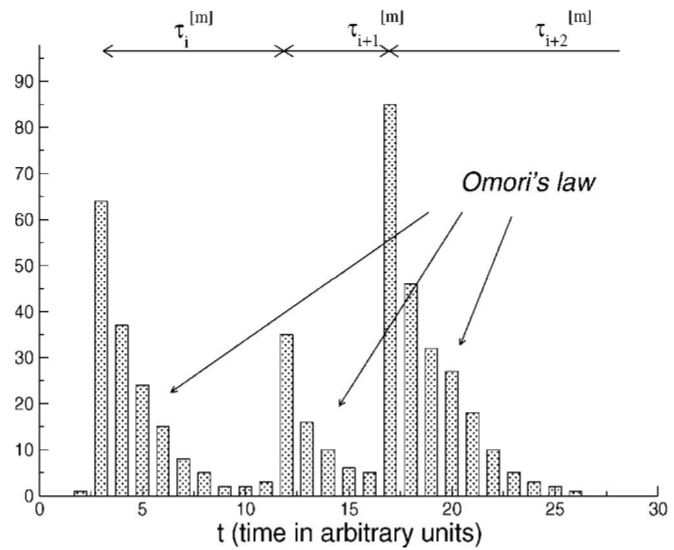
$$n_{AS}(t) \sim (c + t)^{-p}$$

$p \sim 1$

c depends on M main shock and M lower cutoff

(Kagan 2004, Shcherbakov et al 2004, Lise et al 2004)

At time t after a main shock at t=0

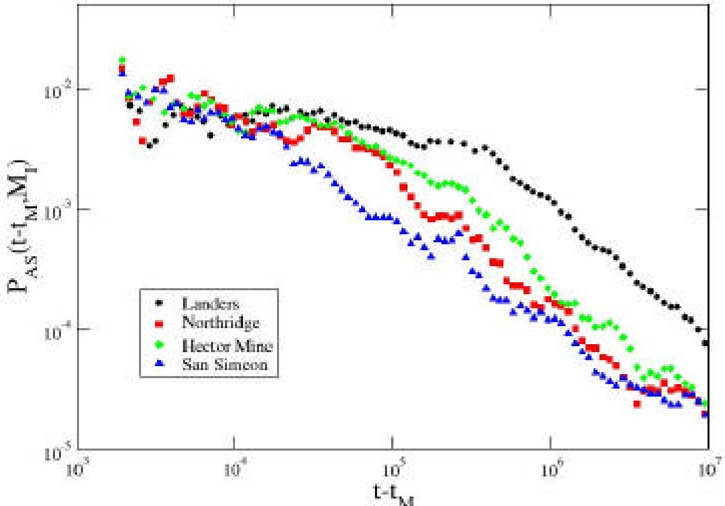


Productivity law

$$N_{AS}(M) \sim 10^{\alpha M}$$

$\alpha \sim b$

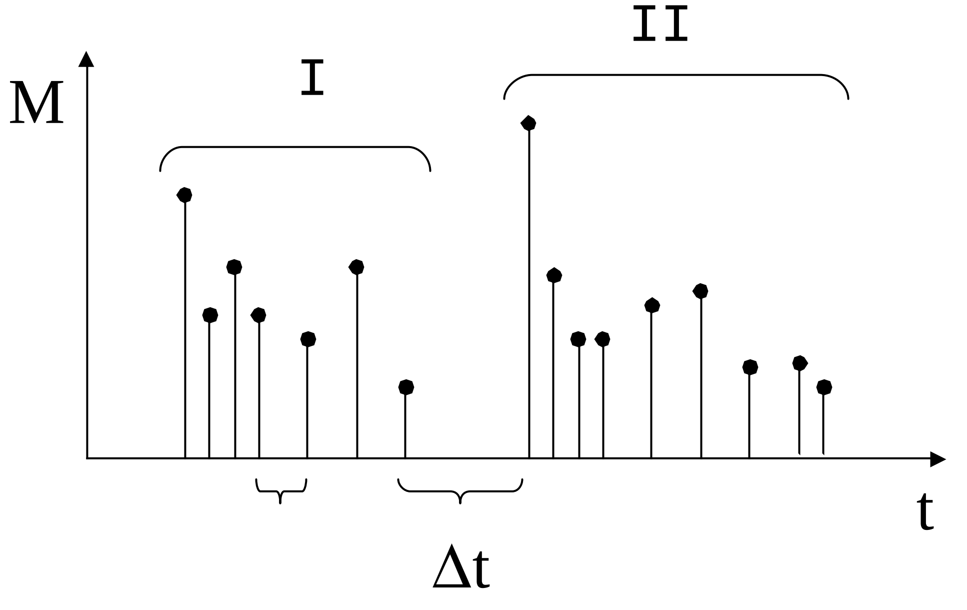
(Helmstetter 2003, Felzer et al 2004, Helmstetter et al 2005, 2006)



Waiting time distribution

Δt time interval between successive events $M > M_c$ lower cutoff magnitude

All events (foreshocks, mainshocks, aftershocks) are considered on the same footing



➡ Bak et al (PRL 2002) divided California in sub-regions of size *l* and computed the distribution $N(\Delta t, M_c)$ in each sub-region. They find the unified scaling law

$$\Delta t^a N_{l,M_c}(\Delta t) = f(10^{-bM_c} l^{d_f} \Delta t)$$

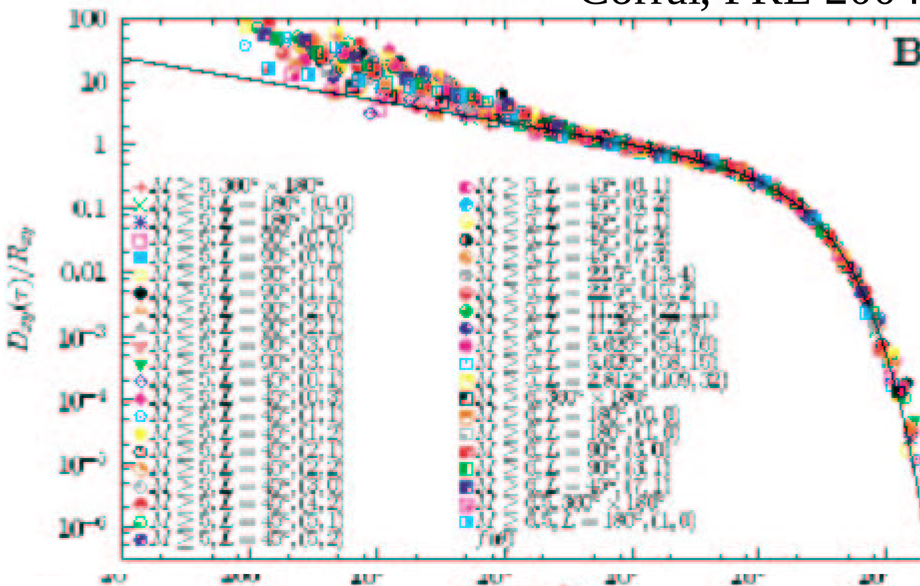
Average number of events with $M>M_c$ occurring in a region of size *l* in the time range Δt

➡ Corral (PRL, 2004) rescaling Δt by the average rate in the area ⟶ universal scaling law for the probability density

$$D(\Delta t, M_c) = R(M_c) f(R(M_c)\Delta t)$$

holds also for Japan, Spain, New Zeland… scaling function not universal (different areas are characterized by different rates)

Corral, PRL 2004



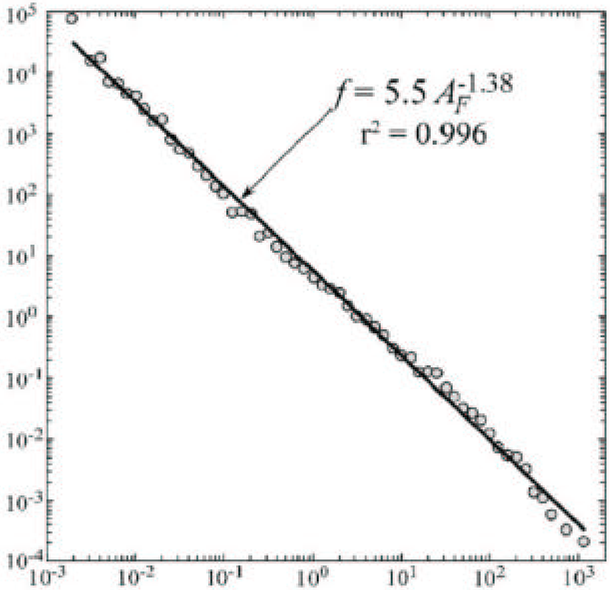
Clustering

Omori law → temporal clustering

Spatial clustering →

- Earthquakes are clustered along hierarchical fault structures
- Ipocenter distribution has fractal dimension $d_f \approx 2.2$
  (Kagan and Knopoff, GJRAS 1980)
- Correlation dimension for epicenters $D_2 \approx 1.5$
  (Helmstetter and Sornette, PRE 2002)
- Generalized dimensions for the epicenter distribution of California and Southern Italy (Davidsen and Goltz, GRL 2004; Godano et al, GJI 1996)
- Scale free networks of aftershocks, degree distribution has exponent $? \approx 2.$ (Baiesi and Paczuski, PRE 2004)

Power laws in natural hazards

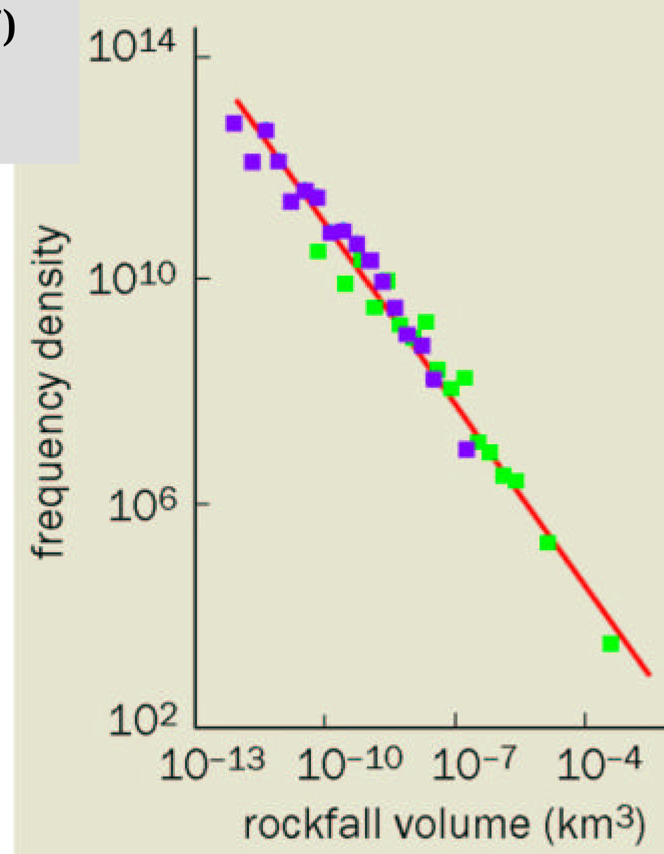


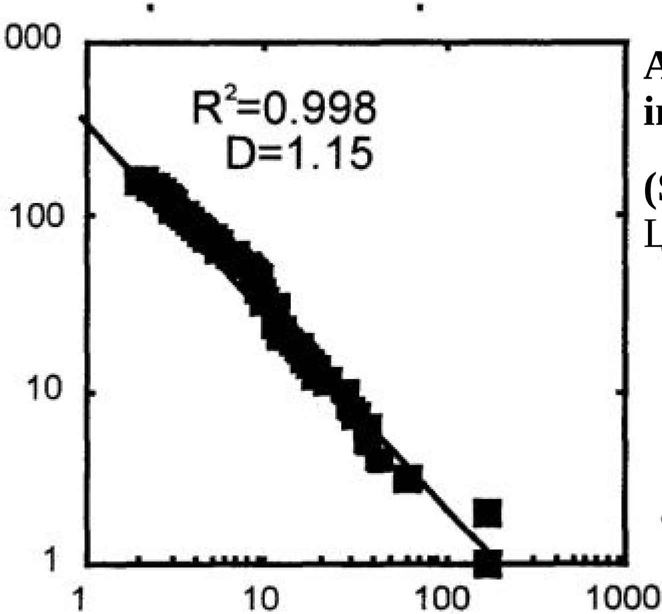
Forest fires in Ontario (Canada) 1976-1996

Turcotte & Malamud 2004

Exponent 1.1

Rockfall in Umbria (1997) & Yosemite (1980-2002)
Malamud 2004

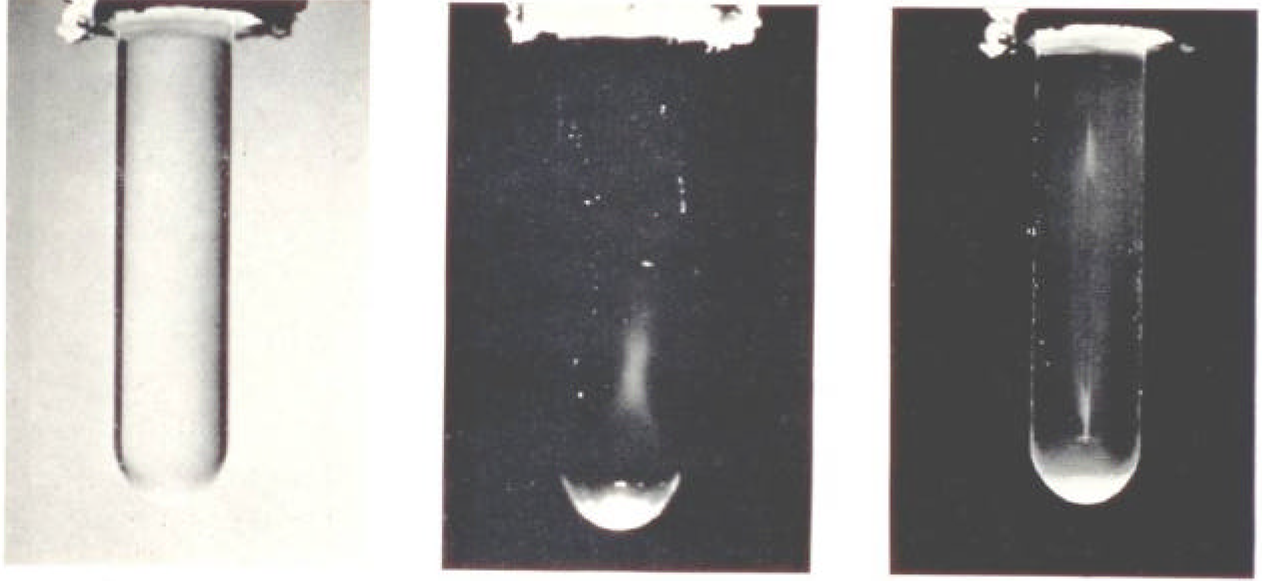




Areas covered by lava in volcanic eruptions

(Springerville, Arizona)
Lahaie & Grasso 1998

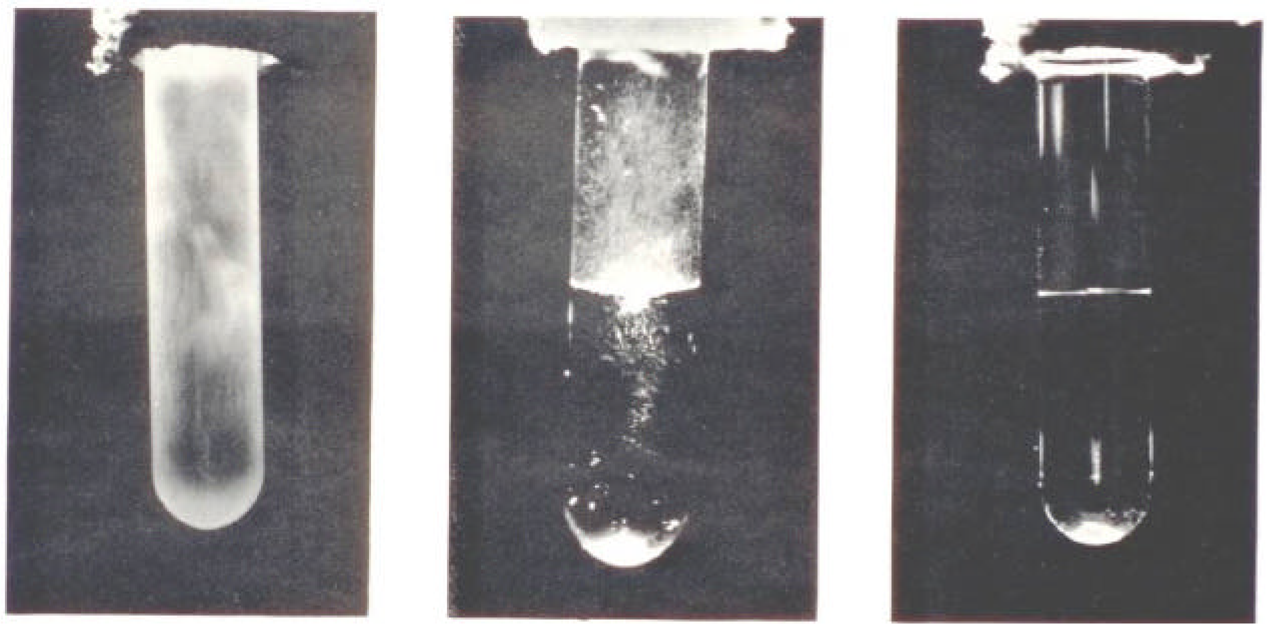
$T \gg T_c$



a *b* *c*

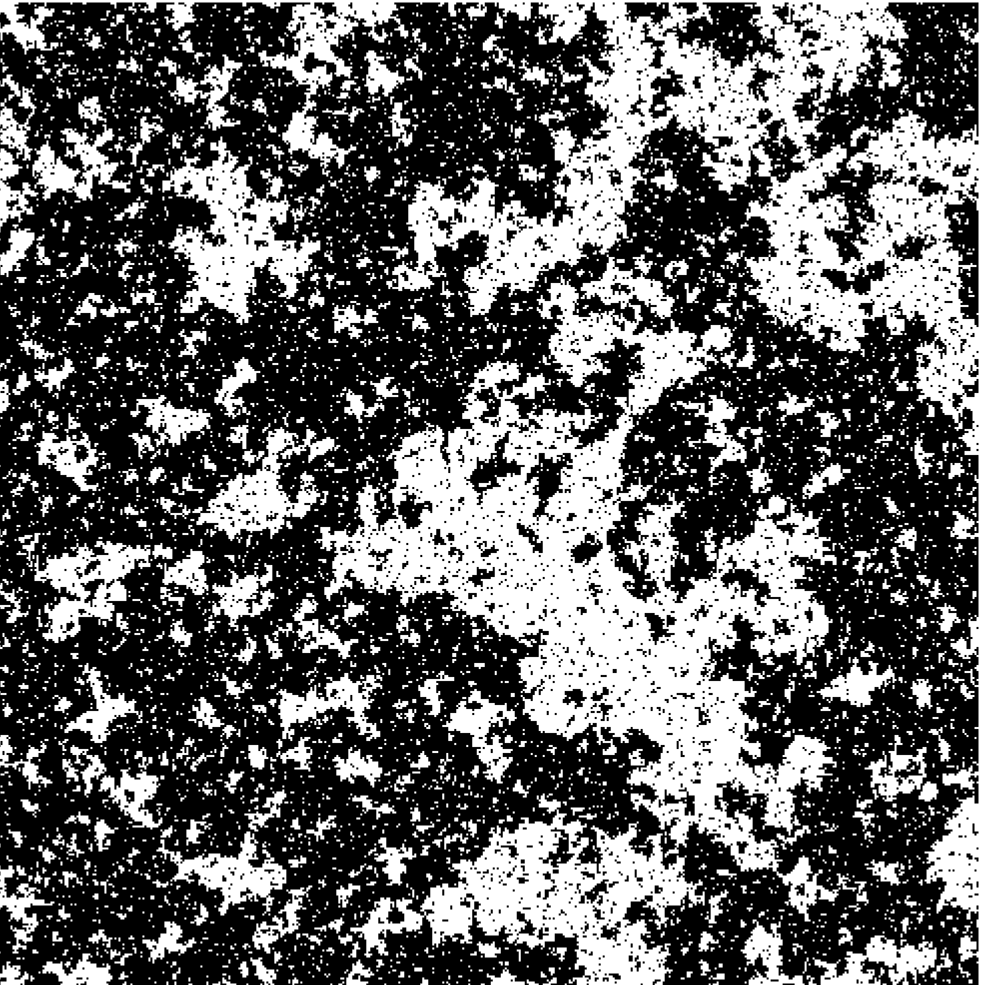
critical opalescence

$T \approx T_c$



$T \ll T_c$

- clusters of all possible sizes are present
- divergence of correlation range
- divergence of fluctuations
- Self-similarity ⟶ the largest cluster is fractal



At the critical point physical properties behave as power laws

Self-similarity

Diffusion Limited Aggregation

Power laws and scaling

Near the critical point the main physical properties exibit power law behaviour

Nice properties of power laws ⟶ invariant under rescaling!

uppose $$y = f(x) = x^a$$

Iake the scale transformation $x \to x' = bx$ $y \to y' = cy$

Jnder rescaling $$f(x) \to f(x') = cf(x'/b)$$

f $c = b^a$ the function is invariant

⟶ **Scale invariance** $f(lx) = g(l)f(x)$

homogeneous functions

For functions of more than one variable

$$f(l^a x, l^b y) = l f(x, y) \qquad \forall l$$

Choose $\quad l = y^{-1/b}$

Obtain

$$f(x, y) = y^{1/b} f(y^{-a/b} x, 1) = y^{1/b} g(y^{-a/b} x)$$

Are there correlations in seismicity?

The conditional probability $D_M(\Delta t \mid \Delta t_0)$ **of a waiting time** $?t$ **followin**
waiting time $?t_0$ **does depend on** $?t_0$ (Livina et al, PRL 200

ystematic analysis of <u>stationary seismicity</u> for world wide and California catalog
n terms of conditional probability distribution indicates: Corral, Tectonophys. 200

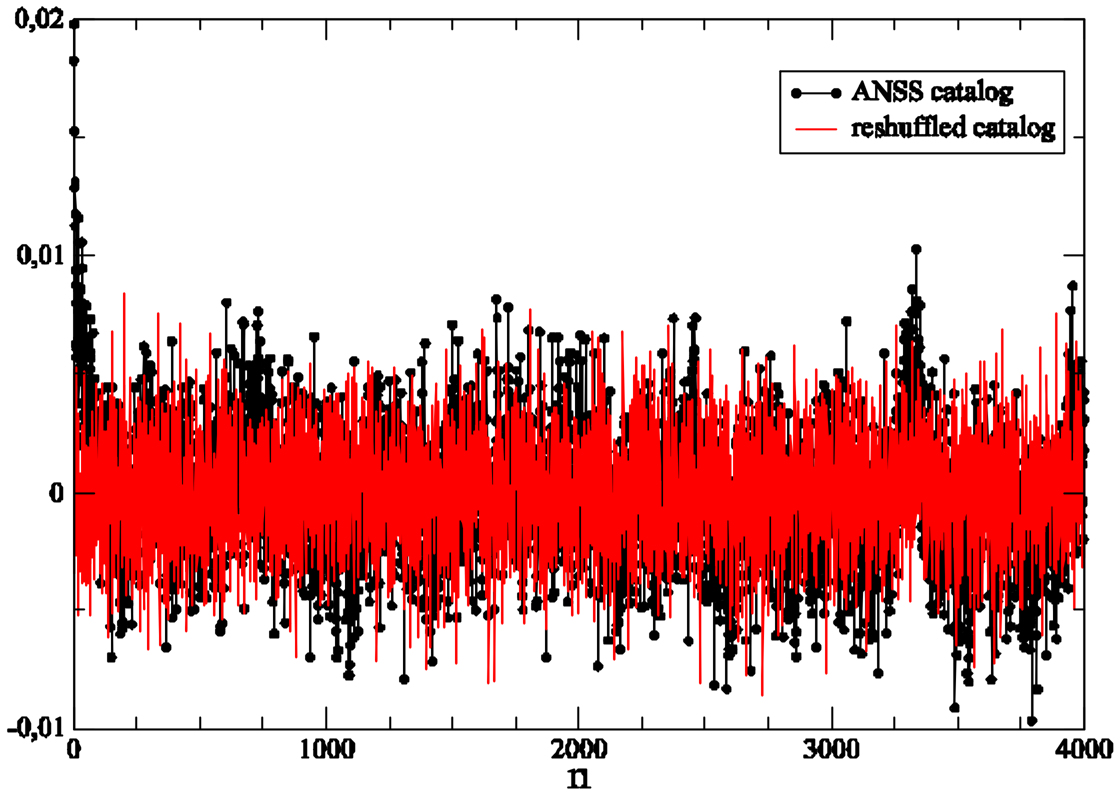
- **Positive correlations between waiting times:** **short** $?t$ **close to each other**
  the shorter the time to get an earthquake, the shorter till the next
- **Anticorrelation between waiting times and magnitudes:**
  large *M* tend to increase number of short $?t$ **and decrease large** $?t$
  the bigger the size of an earthquake, the shorter the time till the next
- **<u>No significant correlations</u> between earthquake magnitudes:**
  values comparable with statistical fluctuations
  <u>an earthquake does not know how big it will become</u>

avidsen & Paczuski (PRL 2005) → **waiting times and distances between epicenter**
of successive earthquakes are independent

Magnitude correlations

valuating the $<M_i M_j> - <M_i>^2$ **gives values comparable with statistical noise**

ed data represent the correlations evaluated in a catalog where magnitudes are eshuffled with respect to occurrence time ⟶ uncorrelated magnitudes



Spatio-temporal correlations

Lippiello, LdA, Godano, PRL 2008

We define for any couple of successive events of the NCEDC catalog:

$\Delta r_i = |\vec{r}_{i+1} - \vec{r}_i|$ epicenter distance, $\Delta t_i = t_{i+1} - t_i$ time distance,

for the magnitude $\Delta m_i = m_{i+1} - m_i$ and $\Delta m_i^* = m_{i+1} - m_{i^*}$

where we reshuffle the previous magnitude, with $i^* \neq i$ a random index

We neglect events in a temporal window $\propto 10^{m-m_c}$

after each earthquake of magnitude m (Helmstetter, Kagan, Jackson JGR 2005)

We evaluate the conditional probability

$$P(\Delta m_i < m_0 \,|\, \Delta t_i < t_0) = \frac{N(m_0, t_0)}{N(t_0)}$$

couples of subsequent events with both $\Delta m_i < m_0, \Delta t_i < t_0$

couples of subsequent events with $\Delta t_i < t_0$

We calculate the conditional probabilities $P(\Delta m_i < m_0 \mid \Delta r_i < r_0)$ and $P(\Delta m_i < m_0 \mid \Delta t_i < t_0)$ in the California catalog

and for 10^4 realizations of the reshuffled catalog (Gaussian distributed)

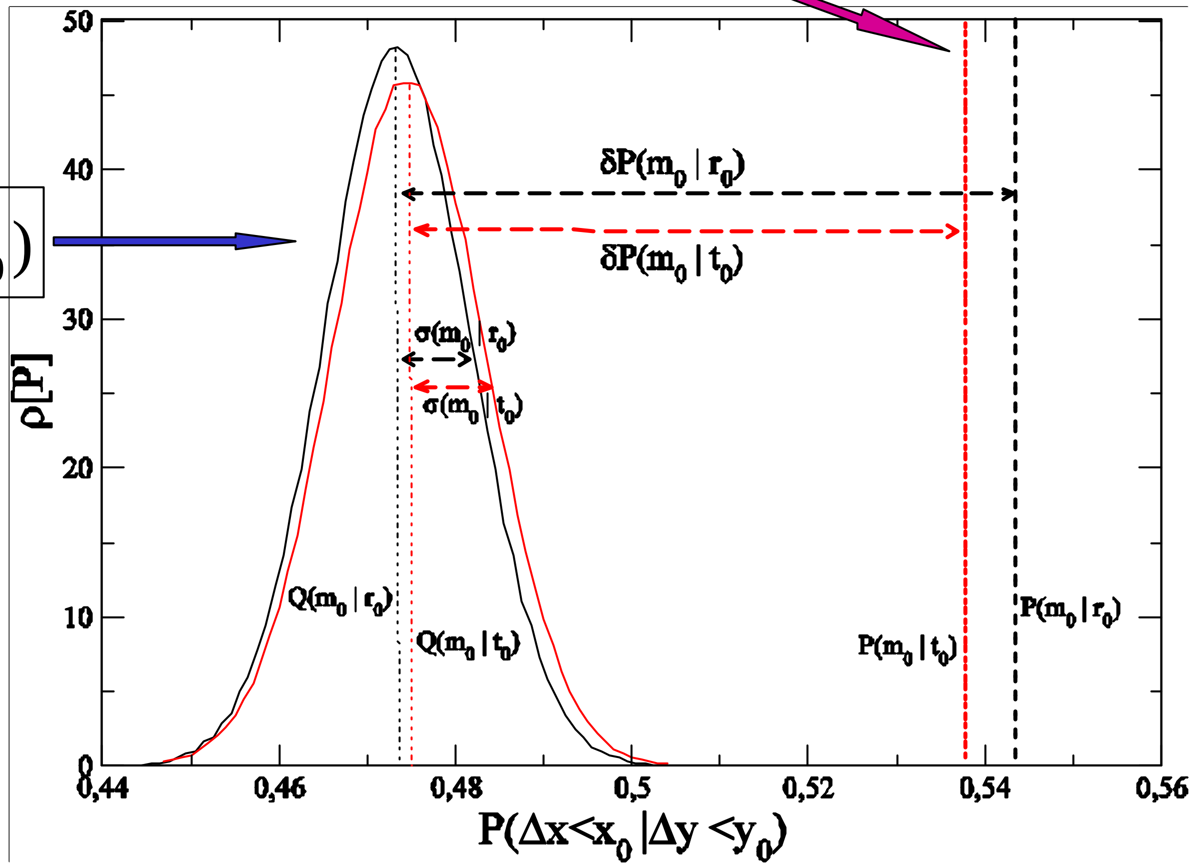
$P(\Delta m_i^* < m_0 \mid \Delta r_i < r_0)$

$P(\Delta m_i^* < m_0 \mid \Delta t_i < t_0)$

r_0=10km

t_0=1h

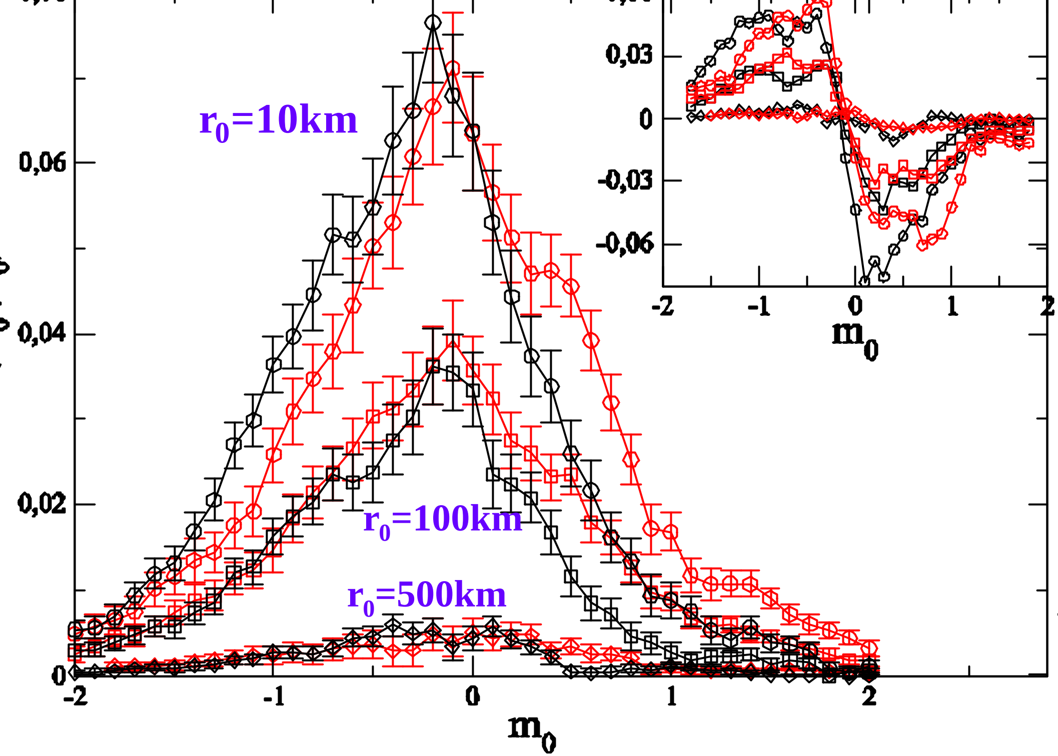
m_0=0



If

$$dP(m_0 \mid t_0) = P(\Delta m_i < m_0 \mid \Delta t_i < t_0) - Q(m_0 \mid t_0) > s(m_0 \mid t_0)$$

Evidence for magnitude correlations

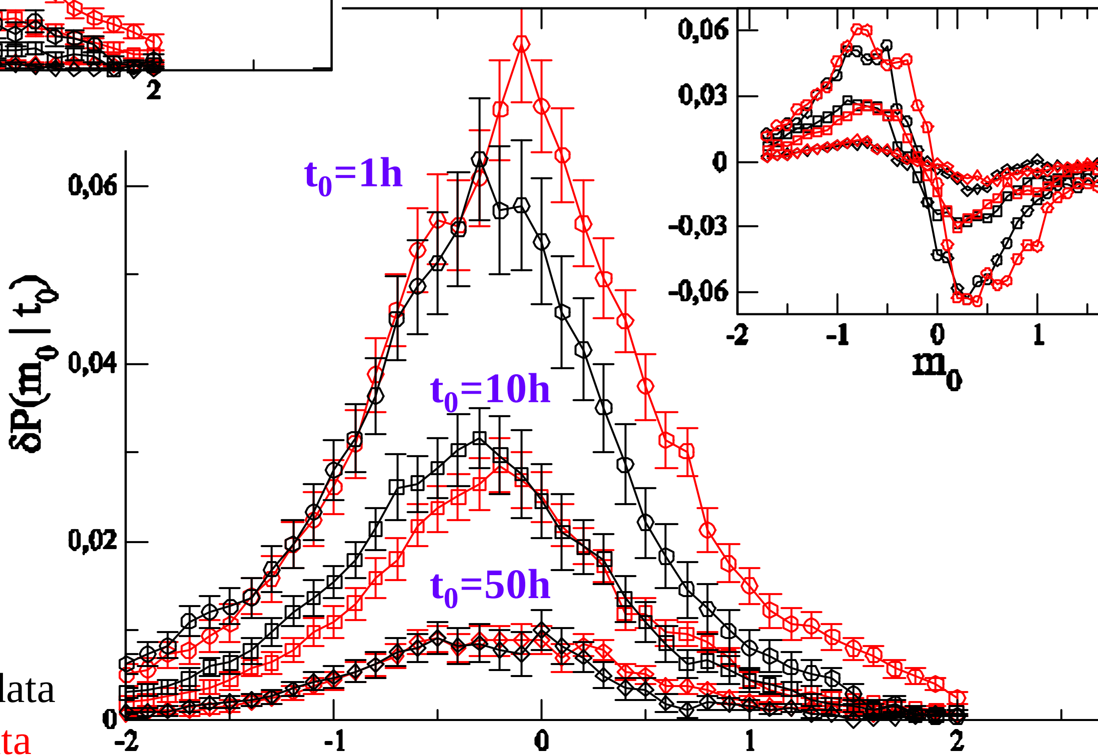


$$\frac{ddP(m_0 \mid r_0)}{dm_0} \rightarrow$$ probability difference

For $m_0 < 0$ the probability is larger in the real than in the reshuffled catalog where magnitudes are uncorrelated

Maximum for m_0 in [-1,-0.5]

Analogously for $dP(m_0 \mid t_0)$



The next earthquake tends to have magnitude close but smaller than the previous one

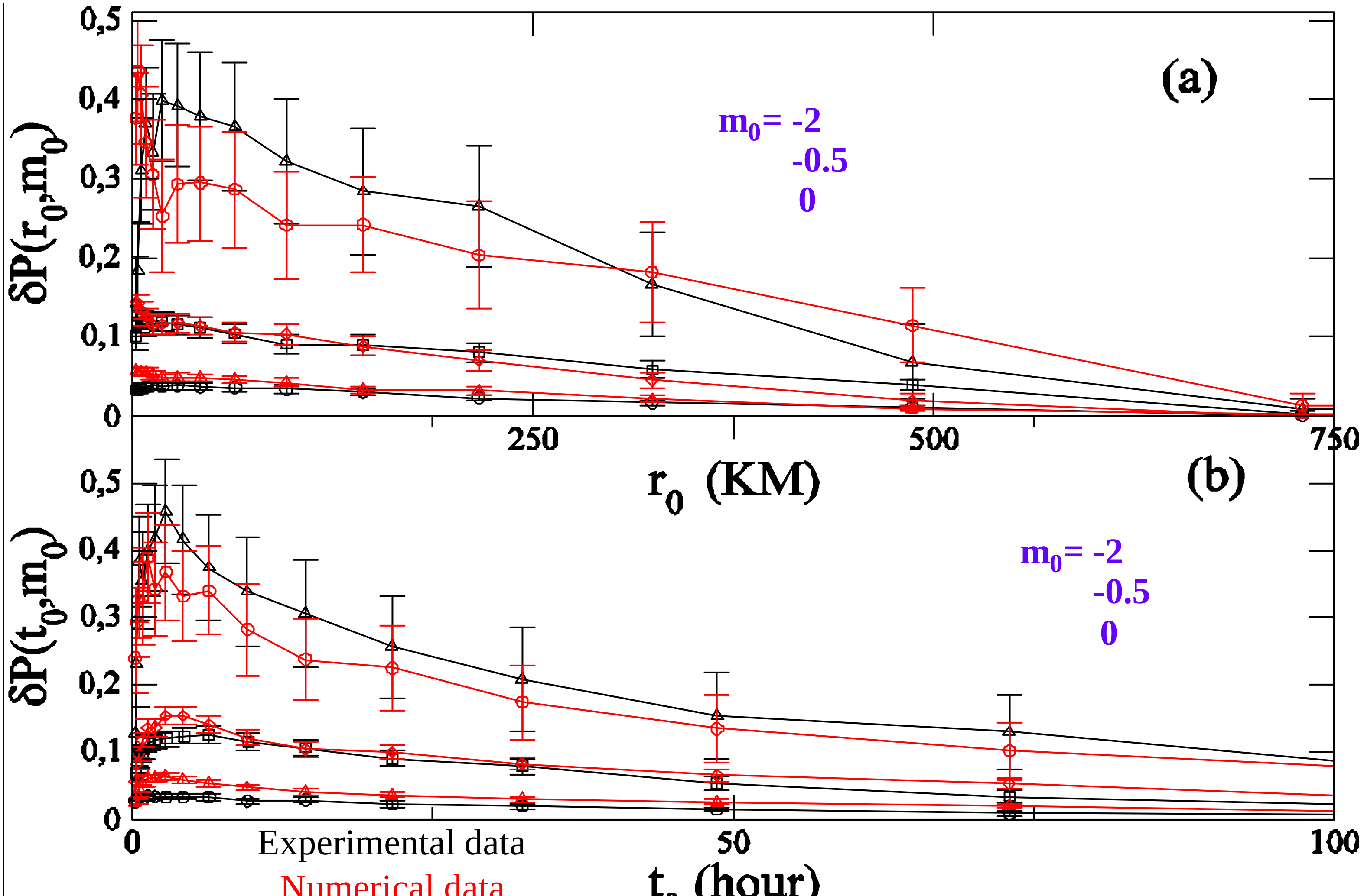
Experimental data

Numerical data

Calculating $P(\Delta r_i < r_0 \mid \Delta m_i < m_0)$ and $P(\Delta t_i < t_0 \mid \Delta m_i < m_0)$

→ **Influence of temporal / spatial clustering on magnitude correlations**

→ **Better description of seismicity if *space – time – magnitude* correlations are taken into account**



Branching model for seismicity

We treat seismicity as a point process in time , where $\{m_i(t_i)\}$ **the history of past events**

Given the history, one assumes that each event can trigger future ones according to a two point conditional rate and therefore the rate of events of magnitude *m* at time *t* is

$$r\left(m(t) \mid \{m_i(t_i)\}\right) = \sum_{i:t_i<t} r\left(m(t) \mid m_i(t_i)\right) + \mu P(m)$$

where μ **is a constant rate of independent sources and *P(m)* their magnitude distribution**

In the ETAS model (Ogata, JASA 1988) **the magnitude *m* is independent of previous events**

$$r\left(M_i(t) \mid M_i(t_i)\right) = P(m_i) g(t_i - t_j; m_j) \propto 10^{-bm_i} 10^{am_j} (t_i - t_j + c)^{-p}$$

Magnitude correlations must be introduced via a multiplicative term

(Vere-Jones, AAP 2005)

$$S(m_i - m_j) = 10^{-d|m_i - m_j|}$$

Dynamical scaling

Lippiello, Godano, LdA, PRL 2007, 20

We assume that the magnitude difference fixes a characteristic time

$$t_{ij} = t_0 10^{b(m_j - m_i)}$$

where t_0 is a constant measured in seconds

and that $r\left(m_i(t_i) \mid m_j(t_j)\right)$ is invariant for $\Delta t \rightarrow l\,\Delta t = \frac{\Delta t}{t}$

This time represents the temporal scale for correlations:

A m=2 earthquake is correlated to a previous m=6 event over a time scale of about 2 years

A m=5 earthquake is correlated to a previous m=6 event over a time scale of few days

Therefore the conditional rate becomes with time rescaled by t_{ij}

$$r\left(m_i(t_i) \mid m_j(t_j)\right) = F\left(\frac{t_i - t_j}{t_{ij}}\right.$$

where $F(x)$ is a normalizable function

⇒ **On the basis of this scaling hypothesis we recover the GR law:**

Total number of daughter earthquakes

$$\int_{t_0}^{\infty} r(m(t) \mid m_0(t_0))dt = t_0 10^{-b(m-m_0)} \int_0^{\infty} F(x)dx$$

and the Omori law: $$r(m, t - t_0) = \int_{-\infty}^{\infty} r(m(t) \mid (m_0(t_0))P(m_0)dm_0$$

Rate of m events at time t

$$\propto \frac{10^{-bm}}{t - t_0} \int_{-\infty}^{\infty} F(z)dz$$

Numerical catalog

By choosing explicitly the function *F* we can generate a catalog of events

$$F(z) = A/(z^{l} + g) \qquad \text{or} \qquad F(z) = A/(e^{z} - 1 + g)$$

- **At t=0 choose a random event with *m* in [m_{inf}, m_{sup}]**
- **t ⟶ t+1 choose a random *m***
- **Evaluate the probability of the event *m(t)* by contribution of all rates due to previous events $m_j(t_j)$ and constant rate of independent sources** μ

$$r\left(m(t) \mid m_j(t_j)\right) = F\left(\frac{t_i - t_j}{t_0 10^{b(m - m_j)}}\right)$$

- **Compare probability with random number to select event**
- **Construct a catalog of 245000 events (30 year California catalog)**

⟹ **Gutenberg Richter law**

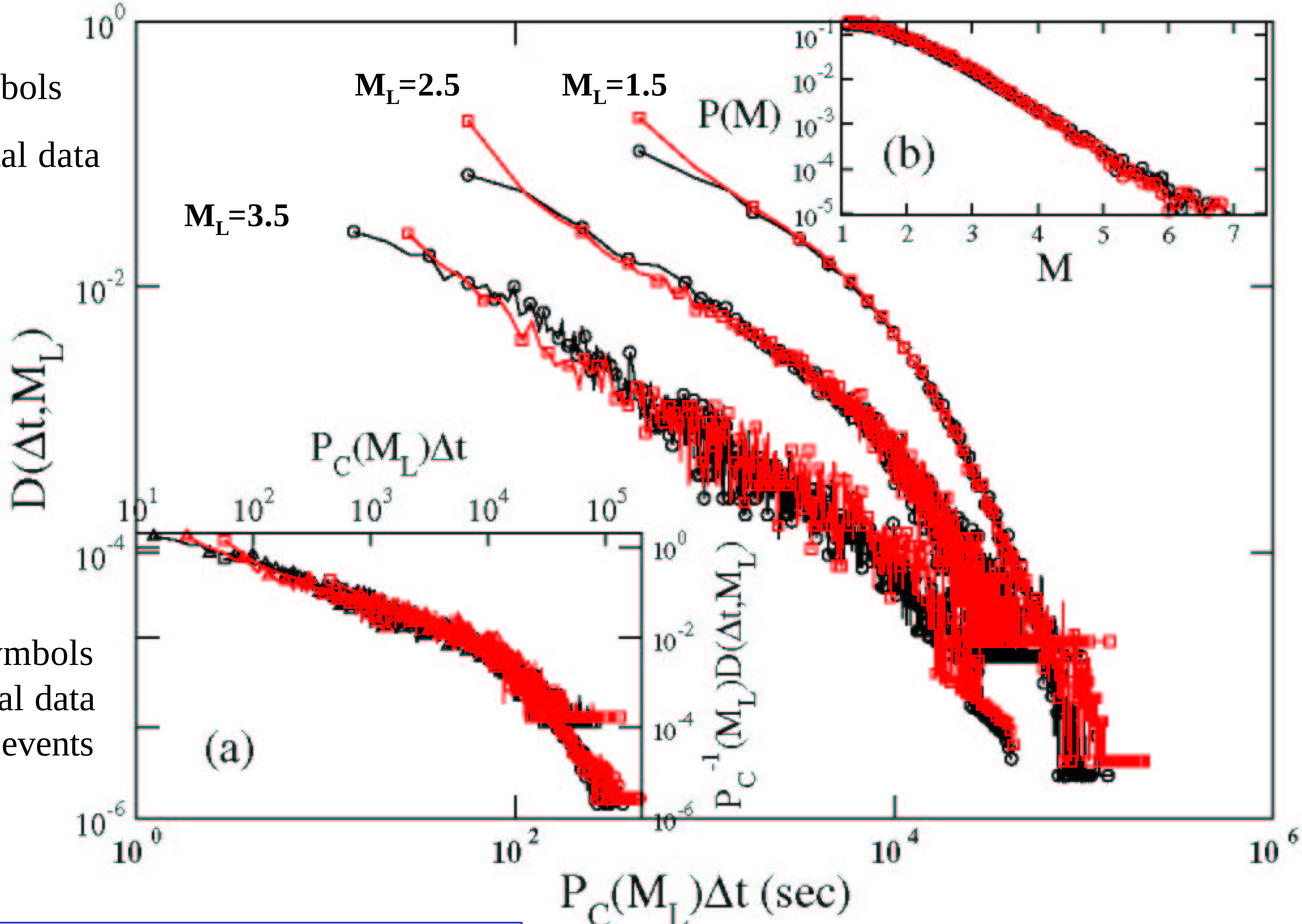
Waiting time distribution

$$F(z) = A/(e^z - 1 + g)$$

Gutenberg Richter law

Red symbols

experimental data



Black symbols
numerical data
245000 events

Waiting time distribution

rescaled by average rate

A = 6.1 10^{-5} sec^{-1}

g = 0.1

Spatio-temporal formulation

The probability to have the next earthquake in $[m, m+dm]$ $[t, t+dt]$ $[\vec{r}, \vec{r}+d\vec{r}]$

$$\mathsf{P}\,(t, \vec{r}, m) = \sum_j P(t - t_j, |\vec{r} - \vec{r}_j|, m, m_j)$$

f $\Delta t \rightarrow l\,\Delta t$

statistical properties are invariant provided that

$$\Delta r \rightarrow l^H \Delta r$$

$$\Delta m \rightarrow \Delta m + (1/b)\log\, l$$

We introduce **two characteristic time scales**

$$t_{ij} = k_t 10^{b(m_j - m_i)} \qquad r_{ij}^{1/H} = k_r \, | \vec{r}_i - \vec{r}_j |^{1/H}$$

leading to the scaling behavior with $\Delta t_{ij} = t_i - t_j$

$$P(\Delta t_{ij}, \Delta r_{ij}, m_i, m_j) = \Delta t_{ij}^{-H} G\left(\frac{t_{ij}}{\Delta t_{ij}}, \frac{\Delta r_{ij}}{\Delta t_{ij}^{H}} \right)$$

$$\approx \Delta t_{ij}^{-H} G_1\left(\frac{t_{ij}}{\Delta t_{ij}} \right) G_2\left(\frac{\Delta r_{ij}}{\Delta t_{ij}^{H}} \right)$$

where

$$G_1(x) = \frac{A}{e^{1/x} - 1 + g_1} \qquad G_2(y) = \frac{B}{y^m + g_2}$$

- **At t=0 choose a random event with *m* in [m_{inf}, m_{sup}] at random epicenter** $\vec{r}_{j=0}$ **on a square lattice**

- **t $\longrightarrow$ t+1 choose a random *m***

- **Evaluate the probability of the event *m(t)* by summing contributions of all rates due to previous events $m_j(t_j)$ and constant rate of independent sources** m

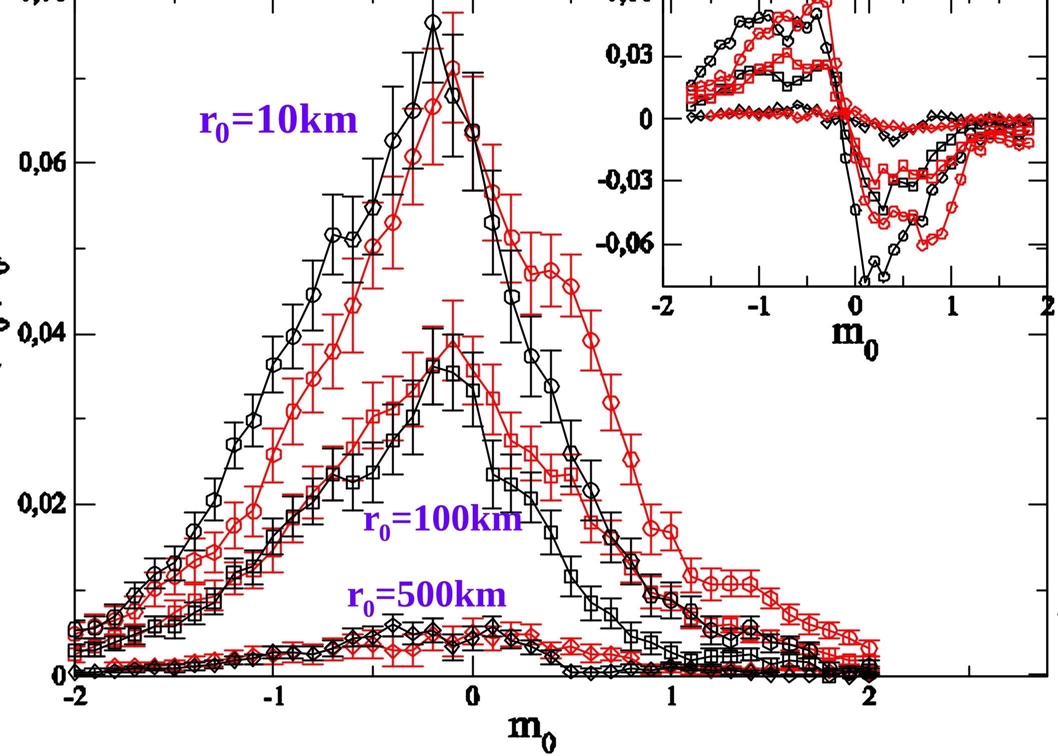
- **Compare probability with random number to select event**

- **Choose a mother among all previous events according to the probability** $G_1\left(\frac{t_{ij}}{\Delta t_{ij}}\right)$

- **Given the mother** $m^*(t^*)$ **at** $\vec{r}^{\,*}$**, determine the epicenter** $\vec{r}_j$ **from**

$$(t_j - t^*)^{-H} G_2\left(\frac{|\vec{r}_j - \vec{r}^{\,*}|}{(t_j - t^*)^H}\right) \longrightarrow H \approx 0.5$$

diffusion process



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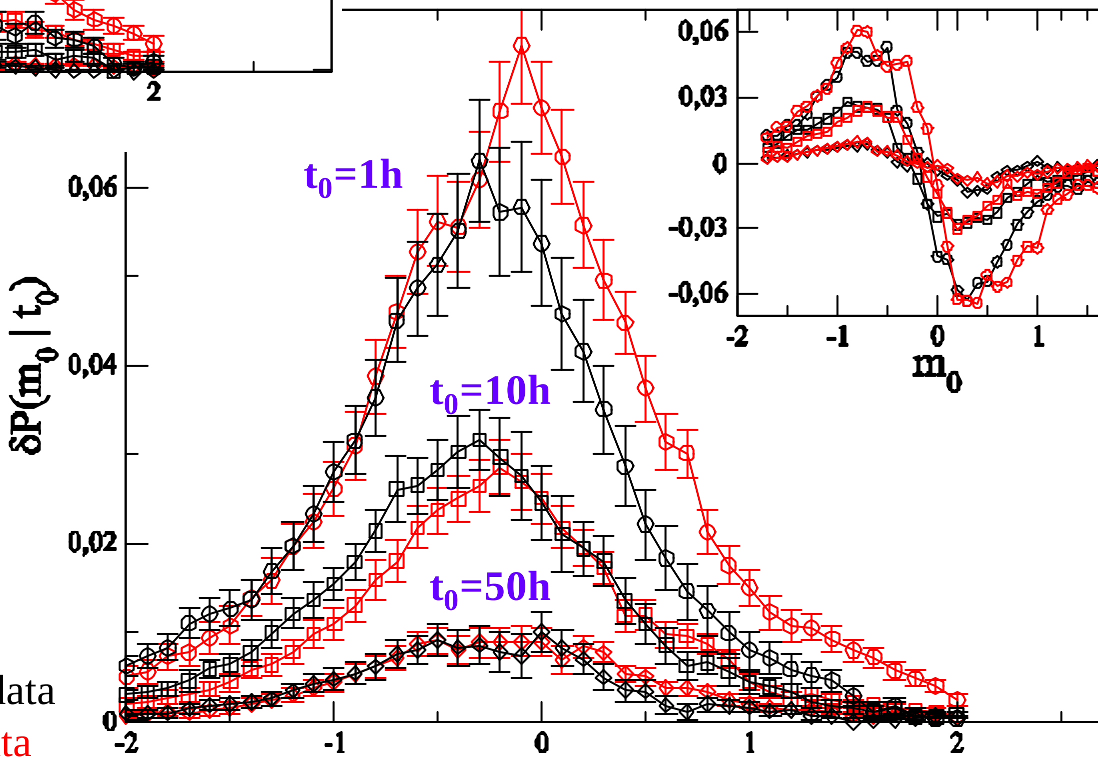
Maximum for m_0 in [-1,-0.5]

Analogously for $dP(m_0 \mid t_0)$



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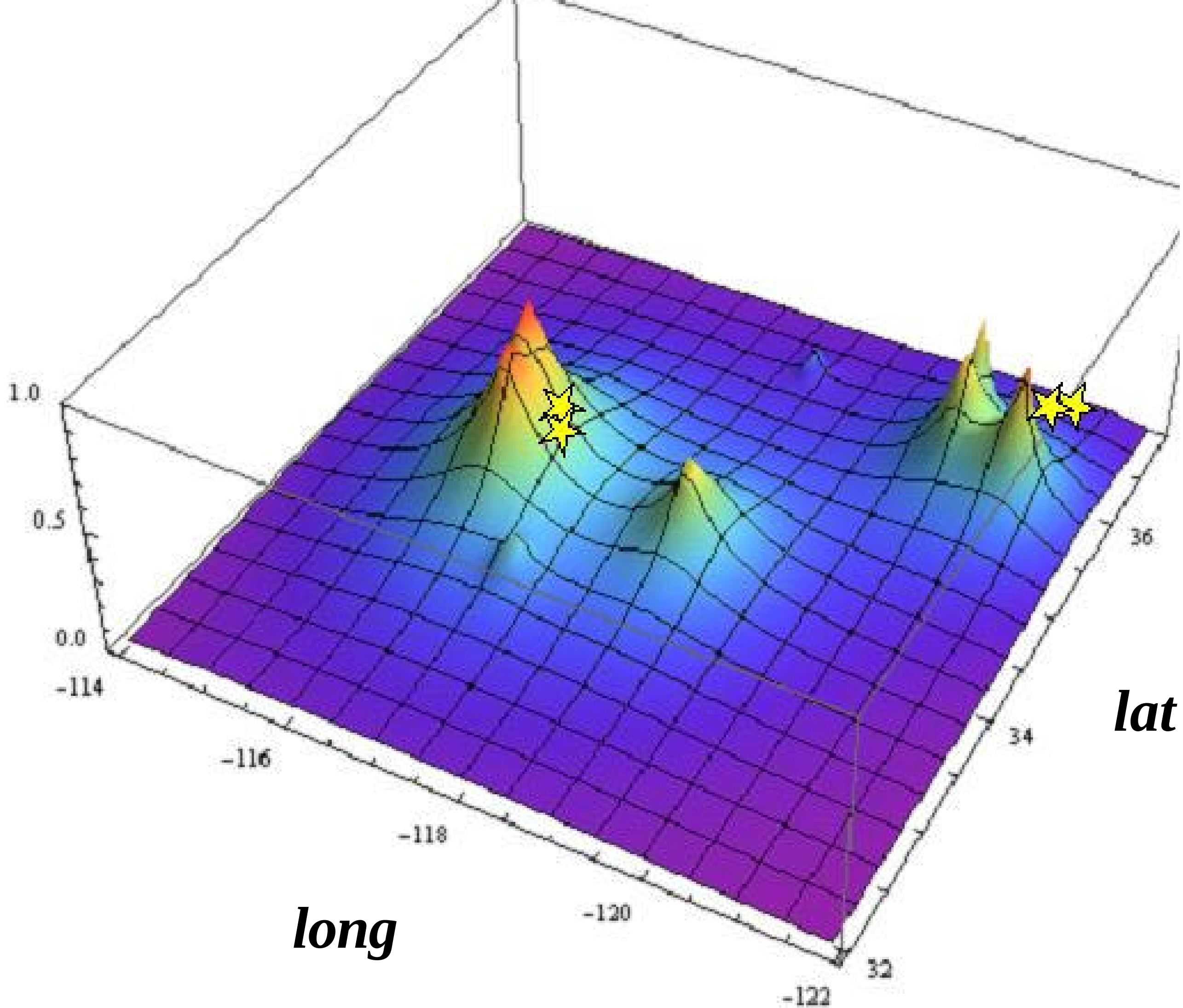


The probability *P* to have $m \geq 3$ earthquakes during January 2007 due to past seismicity. Recorded events (yellow stars) are closely located near the maximum of *P*.

Generalized Omori Law

Lippiello, Godano, LdA, GRL 2007

Given a main shock M_M at $t=t_M$, the rate of aftershocks with $M>M_I$

$$n(t) \propto (t+c)^{-p}$$

Shcherbakov et al 2004 $$c(M_I) = c^* 10^{\frac{b-b}{p-1}(M_M - \Delta M - M_I)}$$

- Kagan 2004 STAI $$c = 10^{(M_M - M_I - M_1)/d}$$ many small events close to t_M are lost

- We calculate the aftershock probability with the DS approach choosing $F(z) = \dfrac{A}{z^p+1}$

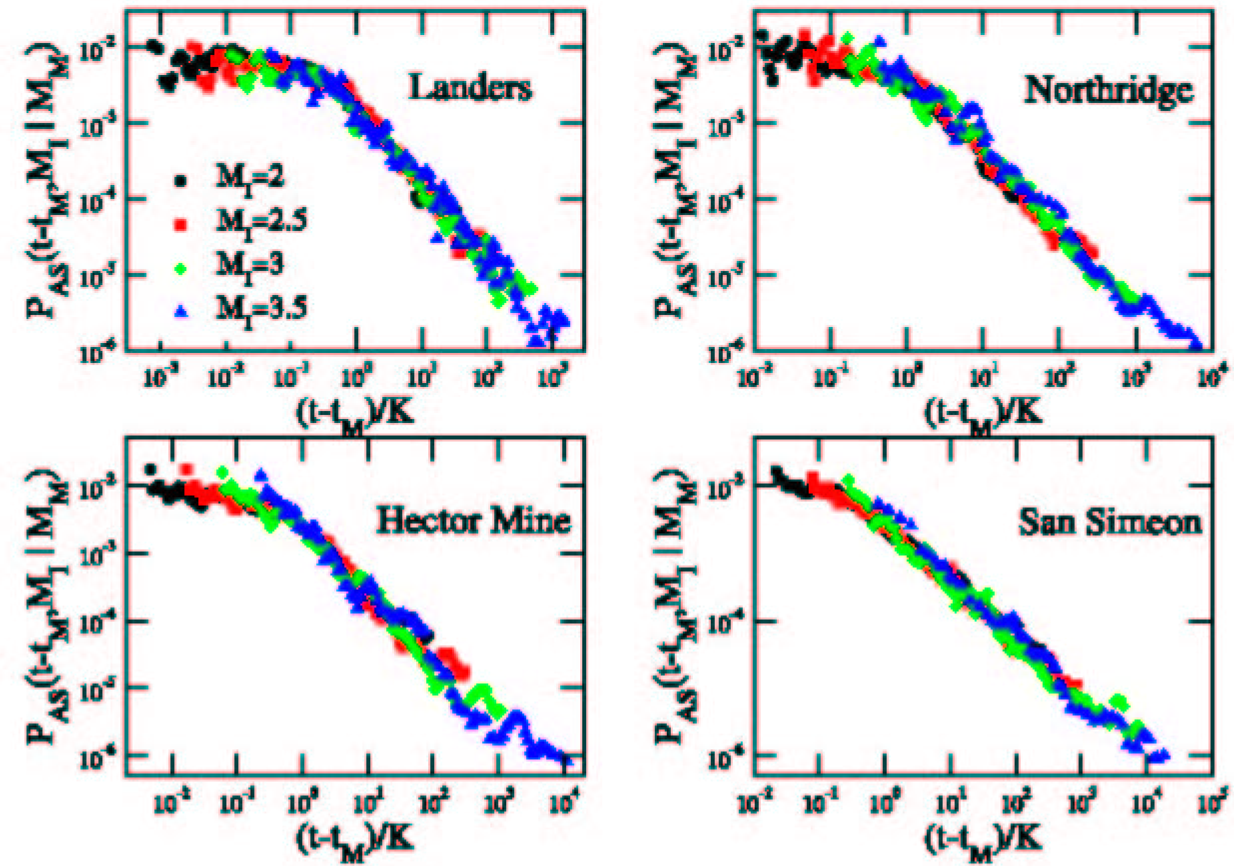
$$P_{AS}(t-t_M, M_I \mid M_M) = \int_{M_I}^{\infty} dM p(M,t \mid M_M, t_M) = \frac{A}{b}\log\left[1+\left(\frac{t-t_M}{t_0 K}\right)^{-p}\right] \qquad \forall M \geq M_I$$

For $t - t_M \gg t_0 K$ recover $(t-t_M)^{-p}$ behaviour

$$\Longrightarrow \boxed{c(M_I) = t_0 K = t_0 10^{(b/p)(M_M - M_I)}}$$ fixes the onset of the Omori behaviour

$$K = 10^{(b/p)(M_M - M_I)}$$

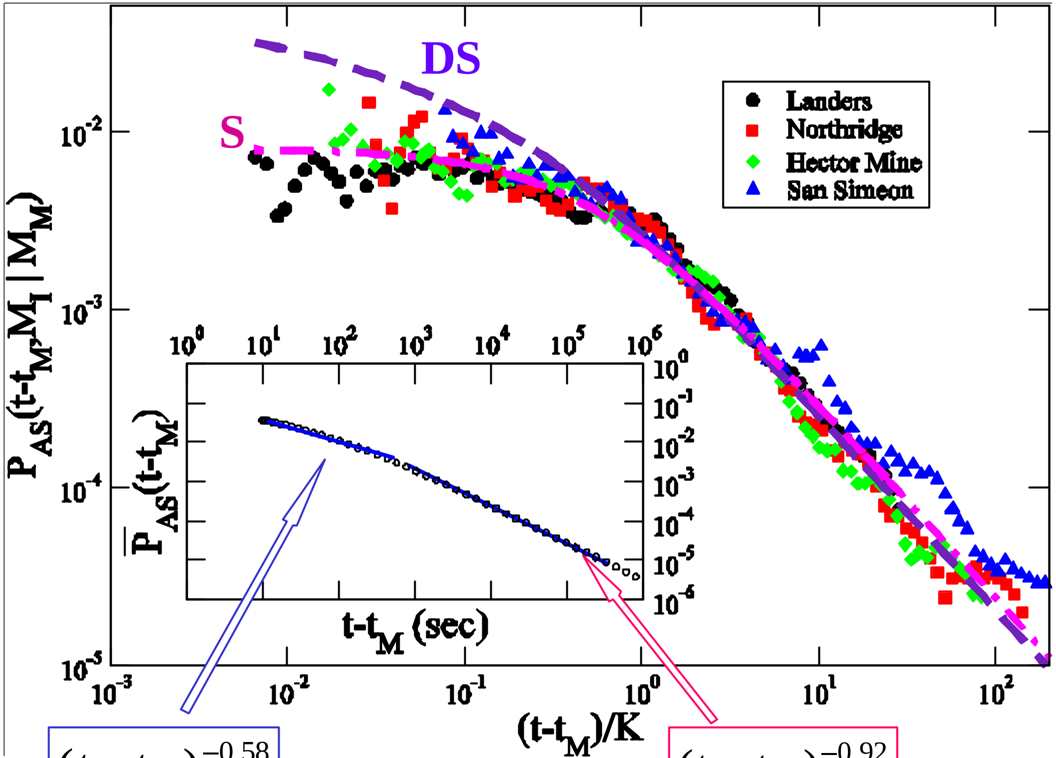
M_M=7.3 **M_M=6.7**



M_M=7.1 **M_M=6.5**

$M_I=2.5$

Peng et al made a careful analysis for shallow earthquakes in Japan and found 5 times more events in the first 200 sec after the main event.



$(t-t_M)^{-0.58}$

$(t-t_M)^{-0.92}$

$$\overline{P_{AS}}(t-t_M) = \int_3^5 dM_M P_{AS}(t-t_M, M_I \mid M_M) P(M_M)$$

Peng et al,

JGR 2007

Recent results

- Log-Likehood for the DS model with a PSRS approach (sub. JGR)
- New method for aftershock detection based on variability coefficient (JGR 2009)
- Analysis of inter-time and inter-distance distributions for sequences (characteristic spatial length scale)
- Spatial distribution of aftershocks → static stress triggering scenario (PRL 2009)
- 3d molecular dynamics simulations of granular media within rough faults

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