Long-Time Tails and Cage Effect in Driven Granular Fluids

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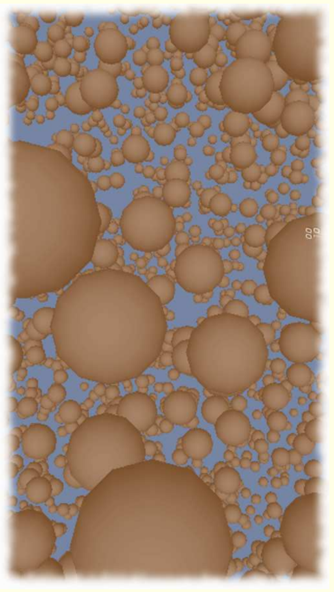
Driven Granular Fluids



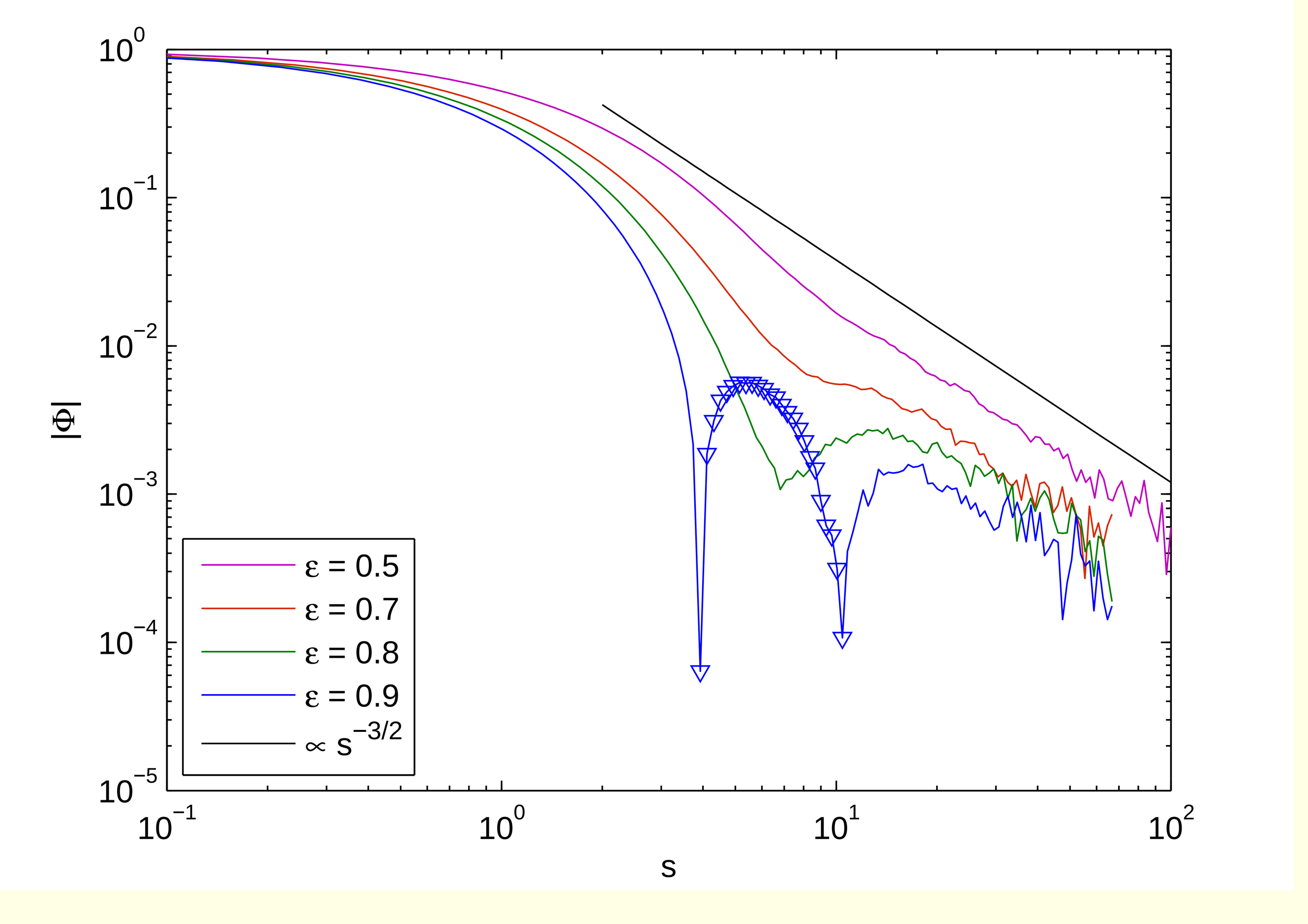
- Hard spheres of diameter a suffering inelastic collisions with a constant coefficient of restitution ε ($\varepsilon = 1$ corresponds to elastic spheres)
- No walls, no air, no gravity
- Volume driving conserves momentum on local scales: Gaussian white noise, such that the Langevin equation becomes

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{v}_i(t) = \underbrace{\frac{\mathbf{F}_i}{m}}_{\substack{\text{systematic force}\\ \text{due to collisions}}} + \underbrace{\boldsymbol{\xi}_i(t)}_{\substack{\text{stochastic force}\\ \text{due to driving}}}$$

- Driving leads to stationary state whose velocity distribution may be approximated by a Gaussian
- We are interested in the velocity autocorrelation function (VACF) $\langle \mathbf{v}_i(t)\mathbf{v}_i(0)\rangle$ and the mean square displacement (MSD) $\left\langle (\mathbf{r}_i(t) - \mathbf{r}_i(0))^2 \right\rangle$
- Diffusion Coefficients can be obtained by the derivative of the MSD

$$D = \frac{1}{6}\frac{\mathrm{d}}{\mathrm{d}t}\left\langle (\mathbf{r}_i(t) - \mathbf{r}_i(0))^2 \right\rangle$$

Backscattering

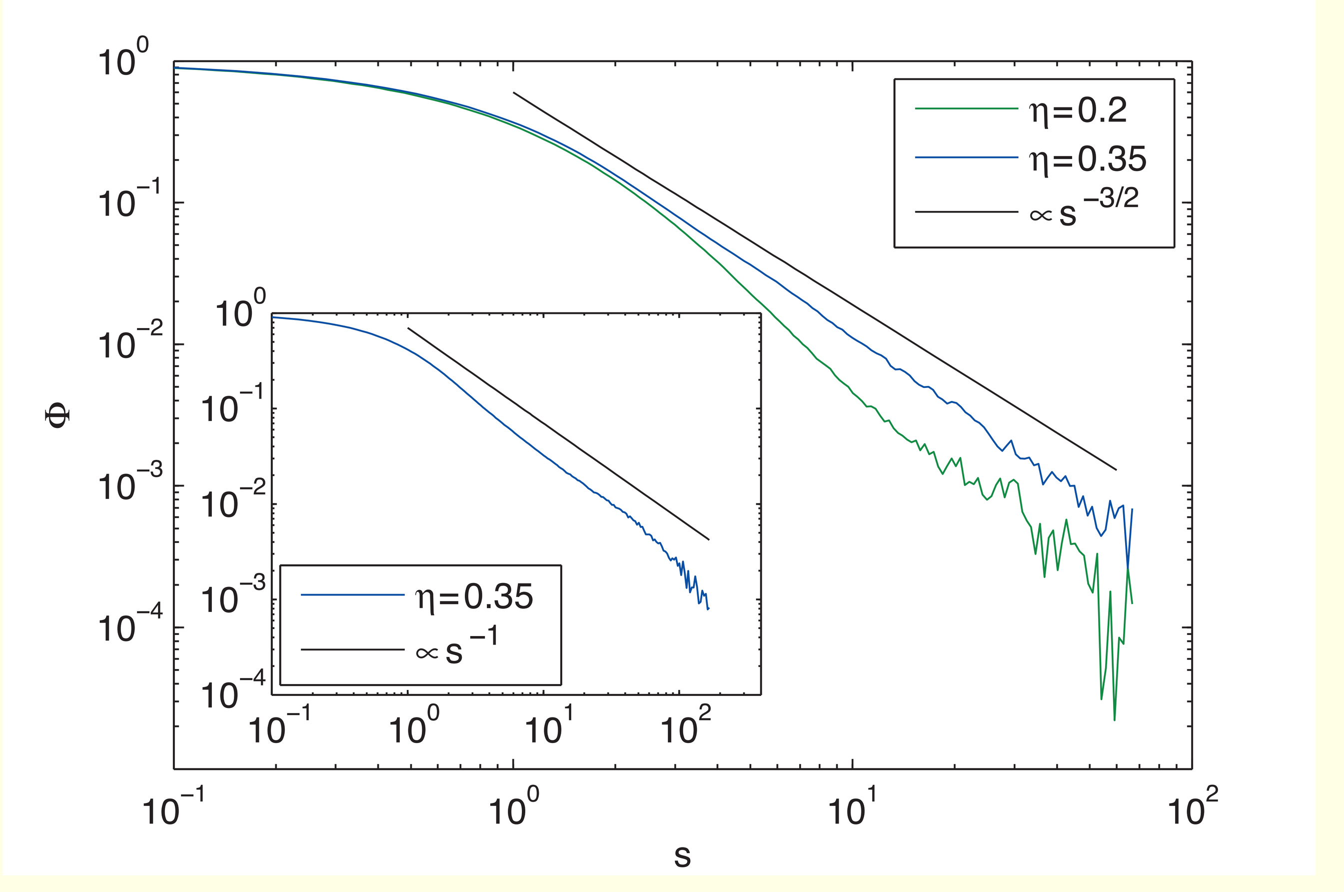


Modulus of the VACF $\langle \mathbf{v}_i(t)\mathbf{v}_i(0)\rangle$ for $\eta = 0.45$ and different values of ε. Markers indicate negative values of the VACF. (s: number of collisions per particle)

For increasing inelasticity ε, the negative range of the VACF transforms into a dint and finally disappears for $\varepsilon = 0.5$ showing that backscattering effects have vanished. Reasons:

- Less pronounced cage reflection for smaller ε;
- Increasing driving strength randomizes the system more strongly.

Long-Time Tails

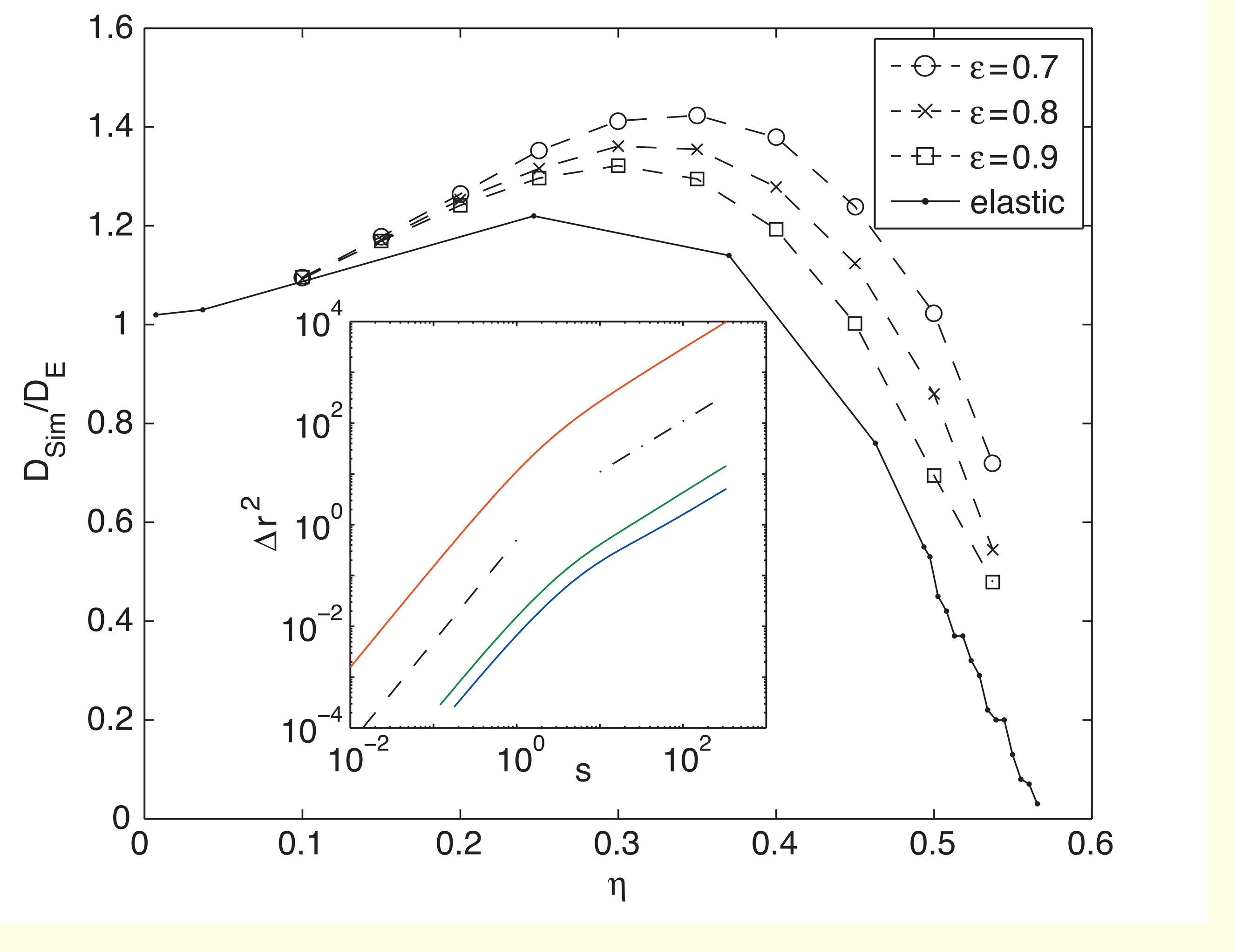


VACF $\langle \mathbf{v}_i(t)\mathbf{v}_i(0)\rangle$ for different values of η and $\varepsilon = 0.7$. Inset: Driving without conservation of momentum.

In order to conserve momentum on local scales we kick neighbouring particles pairwise. This kind of driving yields the $t^{-3/2}$ long-time tail as known from elastic systems. If we kick single particles, momentum is not conserved and we find a different decay in the VACF $\propto t^{-1}$.

MSD and Relative Diffusion Coefficients

Diffusion Coefficients within Enskog approximation can be calculated using kinetic theory. Quantifying deviations from this approximation is done by comparing D_E with diffusion coefficients from our simulations (obtained by the derivative of the MSD).



Relative Diffusion Coefficients as a function of η, values for elastic spheres from Speedy, R. J.: Mol. Phys. **62** *509, 1987; Inset: MSD for $\varepsilon = 0.7$.*

- Relative Diffusion Coefficients $\frac{D}{D_\mathrm{E}}$ are larger for smaller ε
- Maximum is shifted to higher volume fractions for smaller ε

Simulation Details

- Event Driven Molecular Dynamics Simulation with periodic boundary conditions and sub-boxes
- Coefficients of Restitution $\varepsilon = 0.5 \ldots 0.9$
- Volume fractions $\eta = 0.1 \ldots 0.53725$
- $N = 10^4$ particles
- average over 10^3 runs

Conclusion

- The VACFs exhibit evidence for long time tails in the decay, depending on the driving mechanism. The long-time tails are most pronounced for intermediate densities η and small inelasticities ε.
- Backscattering effects occur for higher densities and are less visible for smaller ε.
- As for elastic hard spheres, the relative diffusion coefficient $\frac{D}{D_\mathrm{E}} = f(\eta)$ is not monotonic. Deviations from Enskog approximation are more pronounced for smaller ε.

Outlook

- Mode Coupling Theory: analytic theory for long-time tails and decrease of diffusion coefficients leading to a glass transition in inelastic fluids.
- Study of two-dimensional systems.
- Study of similar systems including Stokes friction, to compare with experiments on air-fluidized granular beds.

Literature

More detailed information can be found in PRL **102**, 098001 (2009).