

(1) Earthquake triggering by stress changes: Observations and modelling using the rate and state friction law

(2) Seismic monitoring of Sechilienne rockslide, french Alps

Earthquake triggering by stress changes: Observations and modelling using the rate and state friction law

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Earthquake triggering

Observations of aftershocks:

when? where? scaling with mainshock size?

Rate and state model :

relation between stress change and seismicity

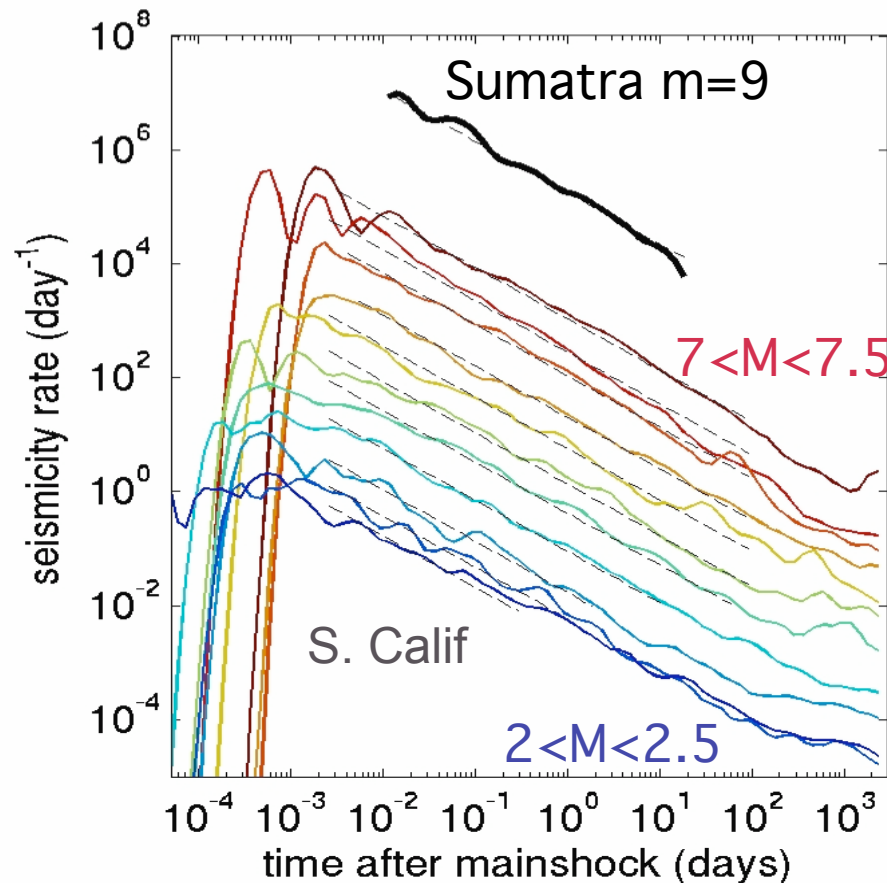
Applications:

triggering by heterogeneous static stress change

afterslip and slow earthquakes triggered by stress changes

aftershocks triggered by afterslip

Observations of aftershock sequences



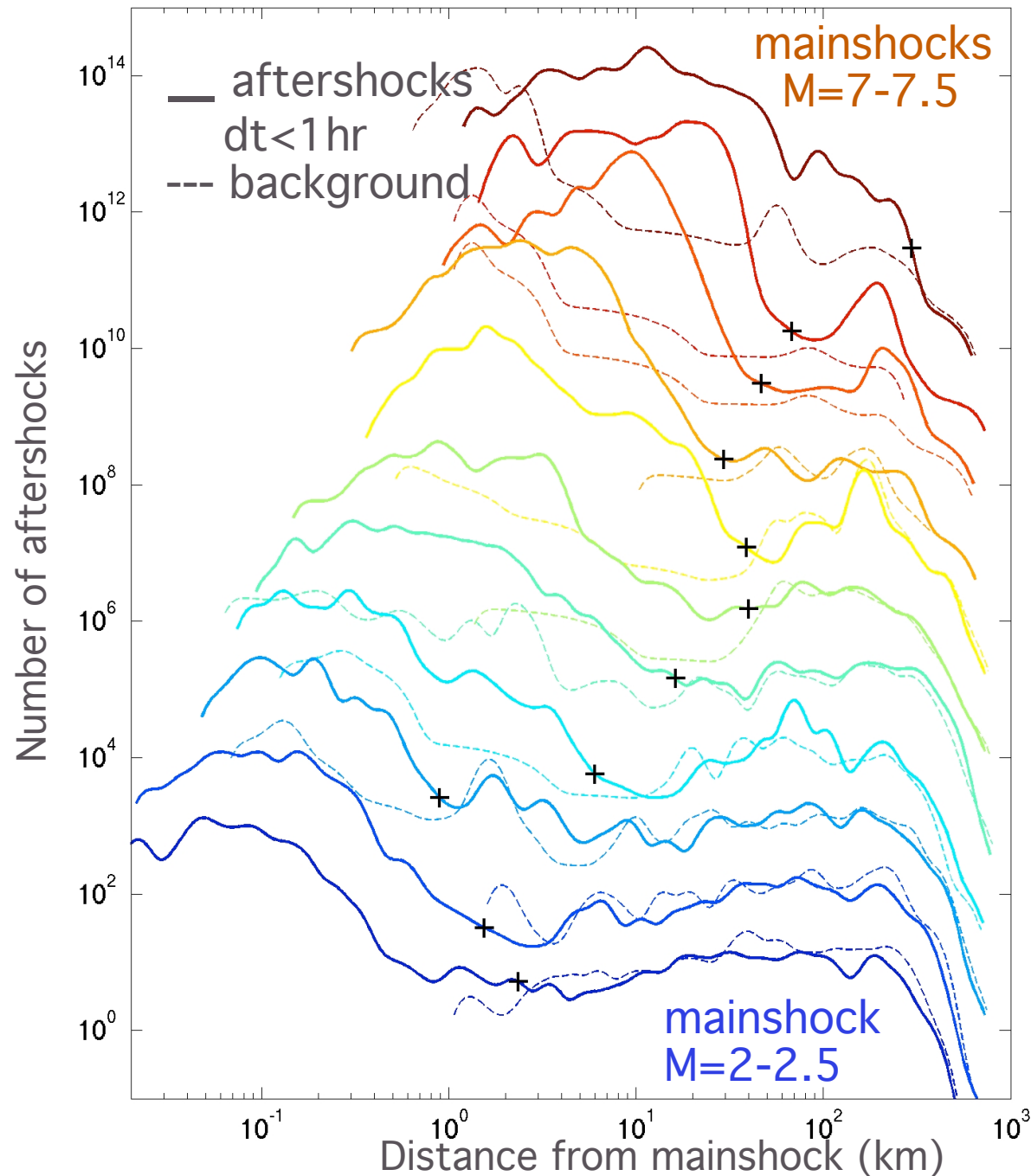
[Helmstetter
et al. 2005]

- aftershock rate :

$$R(t, M) \sim \underbrace{10^M}_{\text{rupture area}} / \underbrace{t^p}_{\text{Omori law, } p \approx 0.9}$$

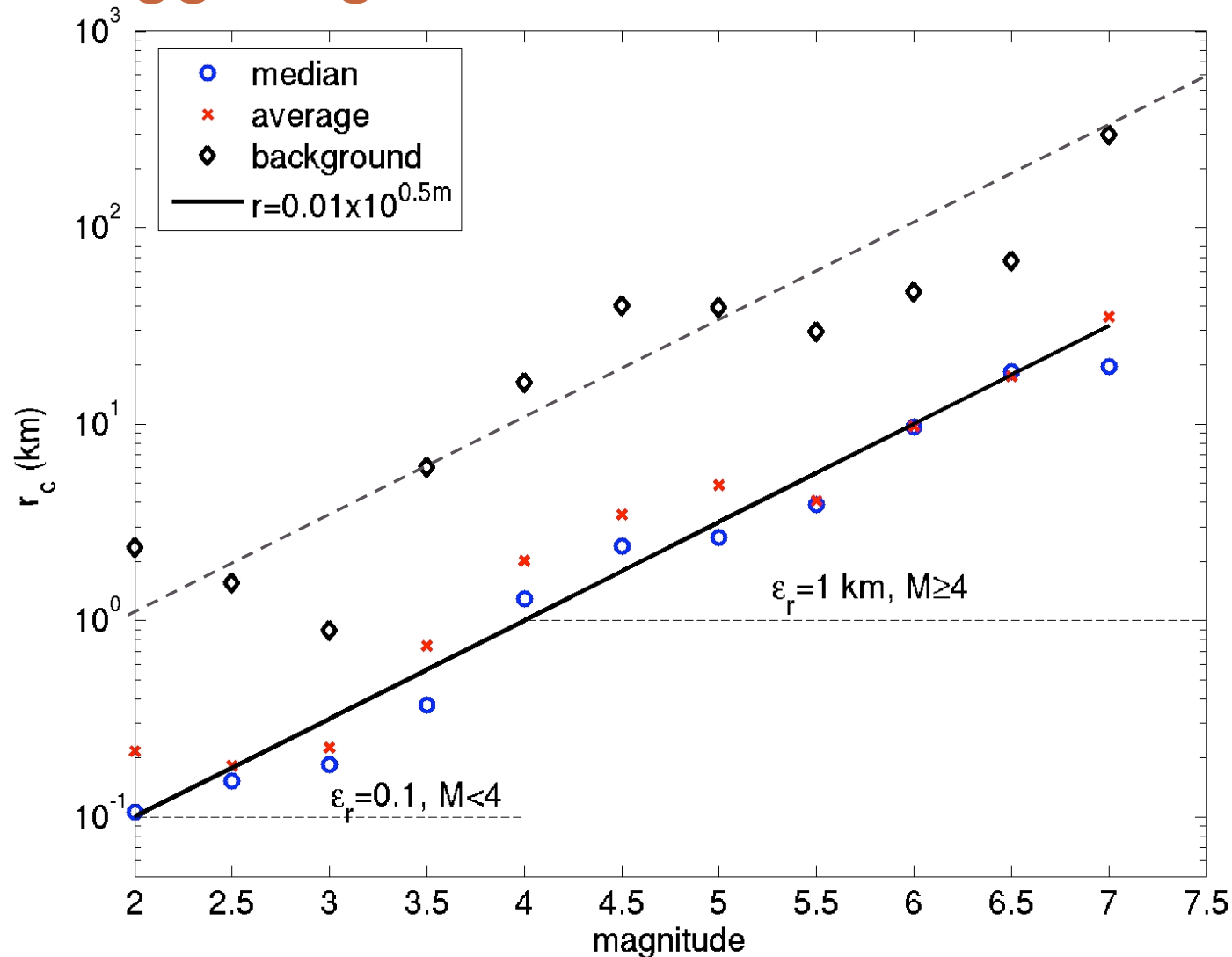
- duration ≈ 10 yrs indep of M
- short-time cutoff for $t \approx 1$ mn = catalog incompleteness?

Spatial distribution of aftershocks



- relocated catalog for Southern California [Shearer et al., 2004]
- triggering distance increases with M
- max triggering distance:
 $R \sim 7$ rupture lengths
 $\sim 0.07 \times 10^{m/2}$ km

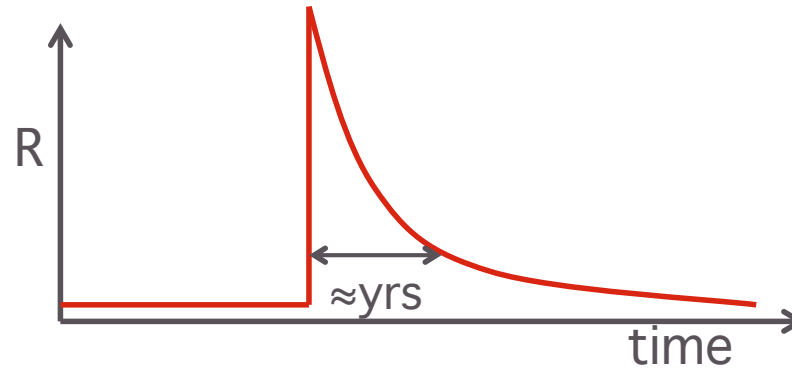
Triggering distance as a function M



- mean triggering distance $d(m) \approx 0.01 \times 10^{0.5m} \text{ km} \sim$ rupture length
- max distance $\approx 7L$

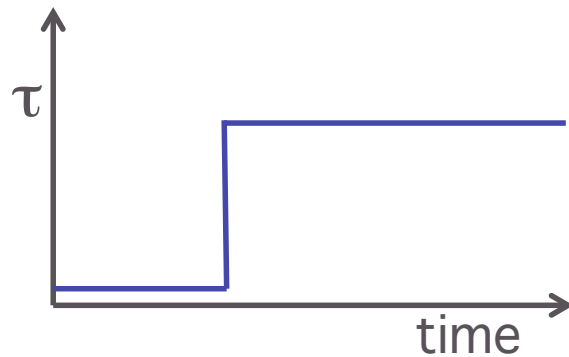
Earthquake triggering by stress changes

seismicity rate
after a mainshock

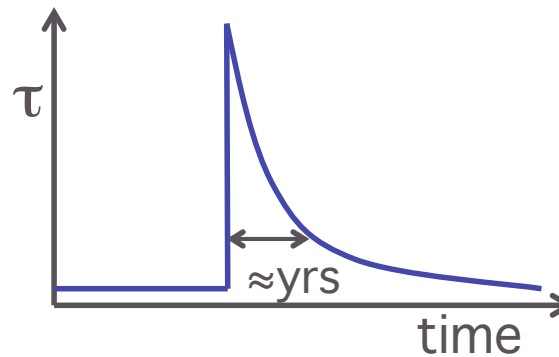


Aftershocks triggered by:

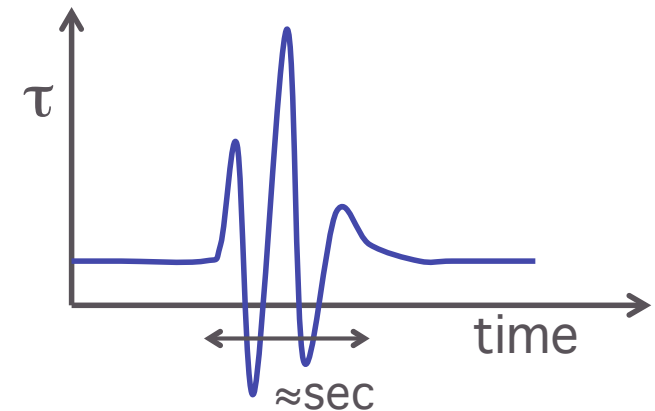
Static stress changes?



Afterslip?



Dynamic stress changes?



Earthquake triggering by stress changes

Static stress change

- ☺ permanent change → easy to explain long time triggering
- ☹ fast decay with distance $\sim 1/r^3$ → how to explain distant aftershocks?

Dynamic stress change

- ☺ slower decay with distance $\sim 1/r$ → better explains distant aftershocks
- ☹ short duration → how to explain long time triggering?

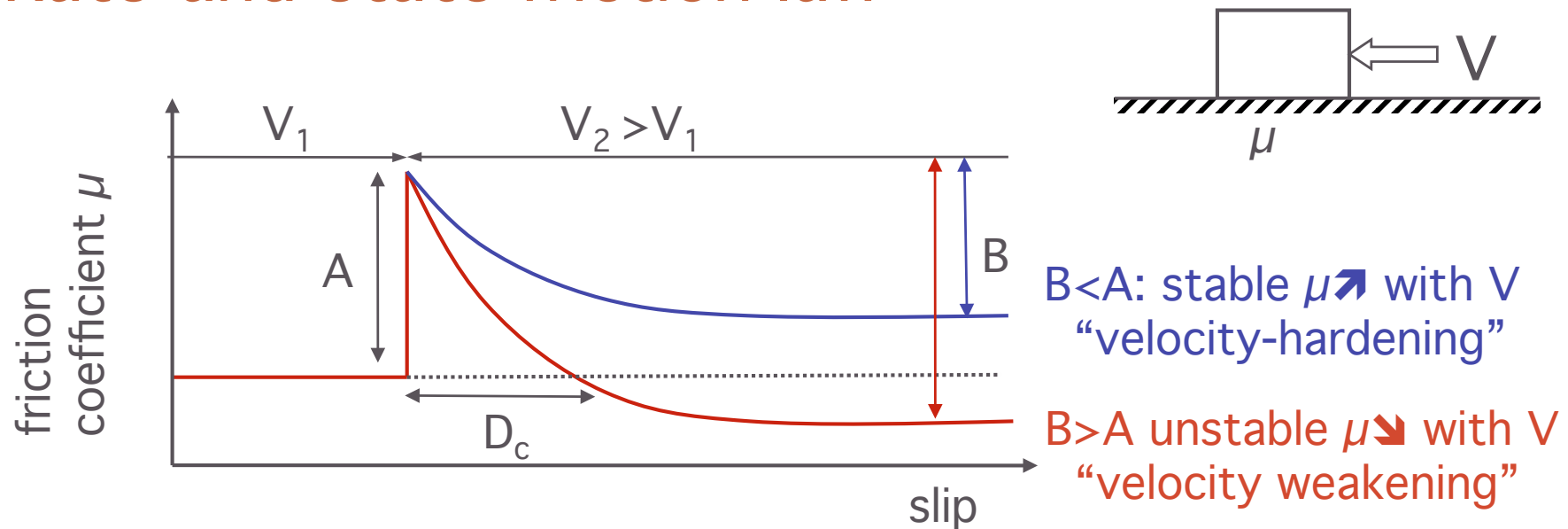
Secondary aftershocks or permanent change in permeability? (Broodsky and others)

Postseismic relaxation

Afterslip, fluid flow, viscoelastic relaxation ...

- ☺ Slow decay with time, $\sim$ seismicity rate → easy to explain Omori law
- ☹ Smaller amplitude than coseismic stress change

Rate-and-state friction law



- friction law

$$\frac{\tau}{\sigma} = \mu = \mu_0 + A \ln\left(\frac{V}{V^*}\right) + B \ln\left(\frac{\theta}{\theta^*}\right)$$

[Dieterich, 1979]

- state variable $\theta \approx$ age of contacts

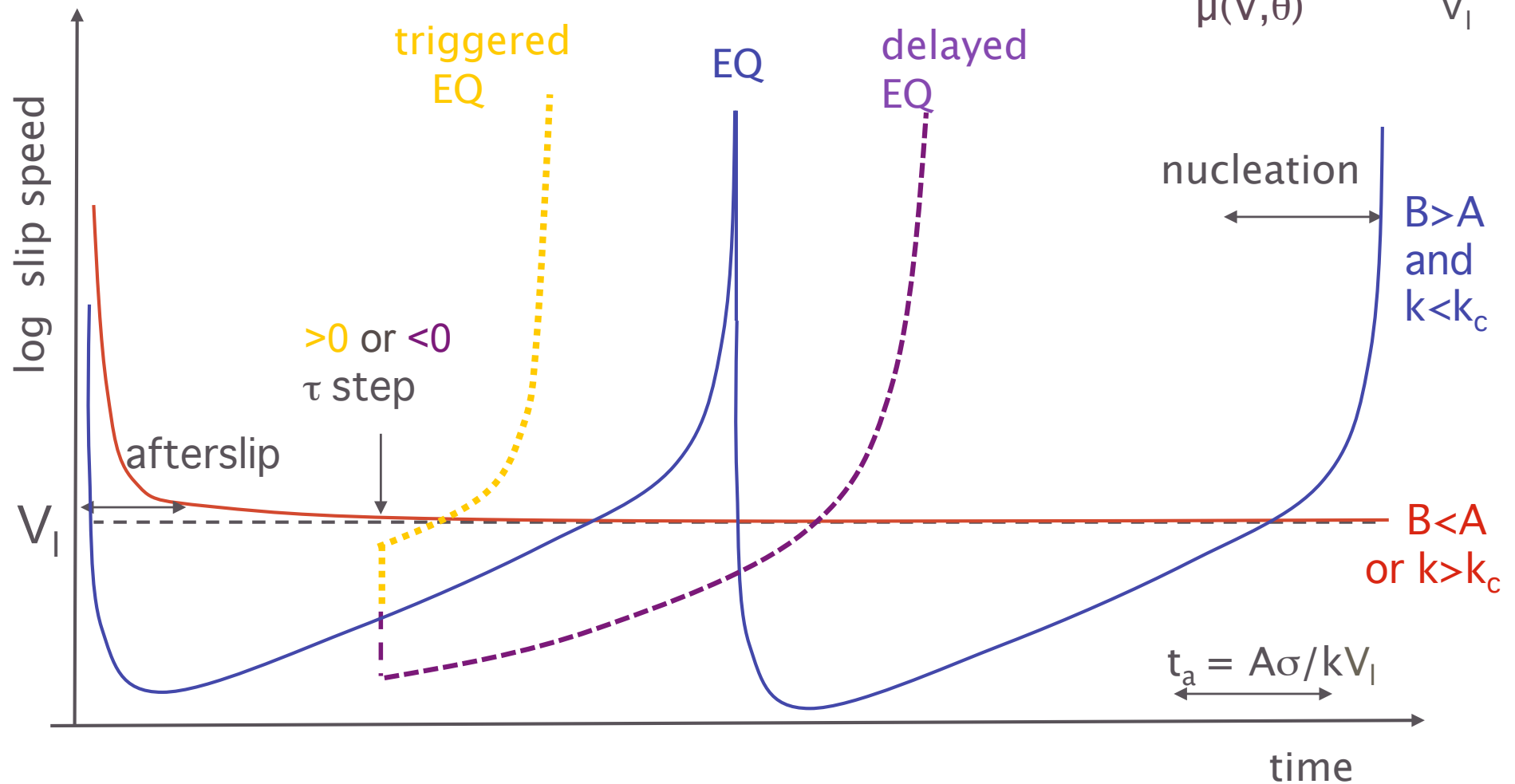
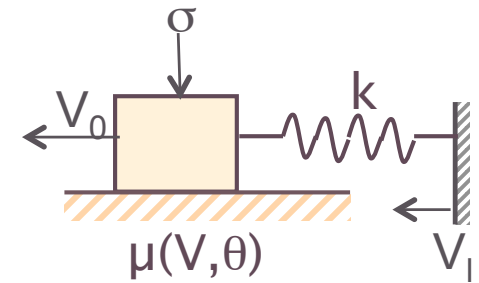
$$d\theta/dt = 1 - V\theta/D_c$$

- lab :

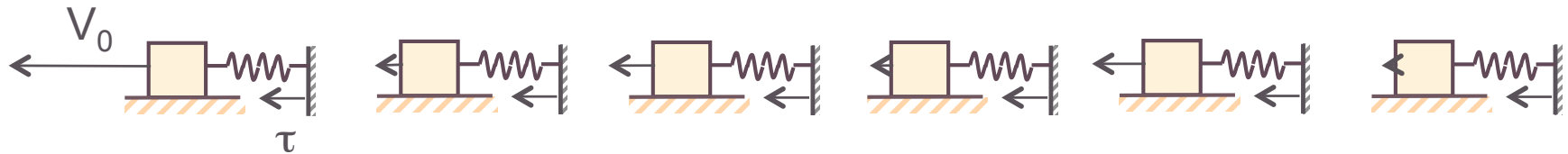
- $A \approx B \approx 0.01$, depend on T , stress, gouge thickness, strain...
- $D_c \approx 1-100 \mu\text{m}$, depends on roughness and gouge thickness

Rate-and-state friction law and EQs

Slip speed for a slider-block with a constant loading rate



Relation between stress and seismicity



- rate & state friction law
- 1 fault = slider block, stick slip regime
- infinite population of independent faults
- stress changes modify the slip rate and advance or delay the failure time
- time advance/ delay function of stress change and initial slip rate
- relation between seismicity rate and any stress history [*Dietrich, 1994*]

Relation between stress and seismicity

- *Dieterich* [2004] model is equivalent to

R : seismicity rate
 $R_0 = R(t=0)$
 $N(t) = \int^t R(t) dt$
 r : ref seismicity rate
 for $d\tau/dt = \tau'_r$
 τ : coulomb stress
 change (=0 at $t=0$)
 t_a : nucleation time
 $= A\sigma / \tau'_r$

$$A\sigma \ln \left(\frac{R}{R_0} \right) + \frac{N \dot{\tau}_r}{r} = \tau$$

short-times regime

for $T \ll t_a$

$R \sim R_0 \exp(\tau / A\sigma)$

(tides, ...)

long-times regime

for $T \gg t_a$

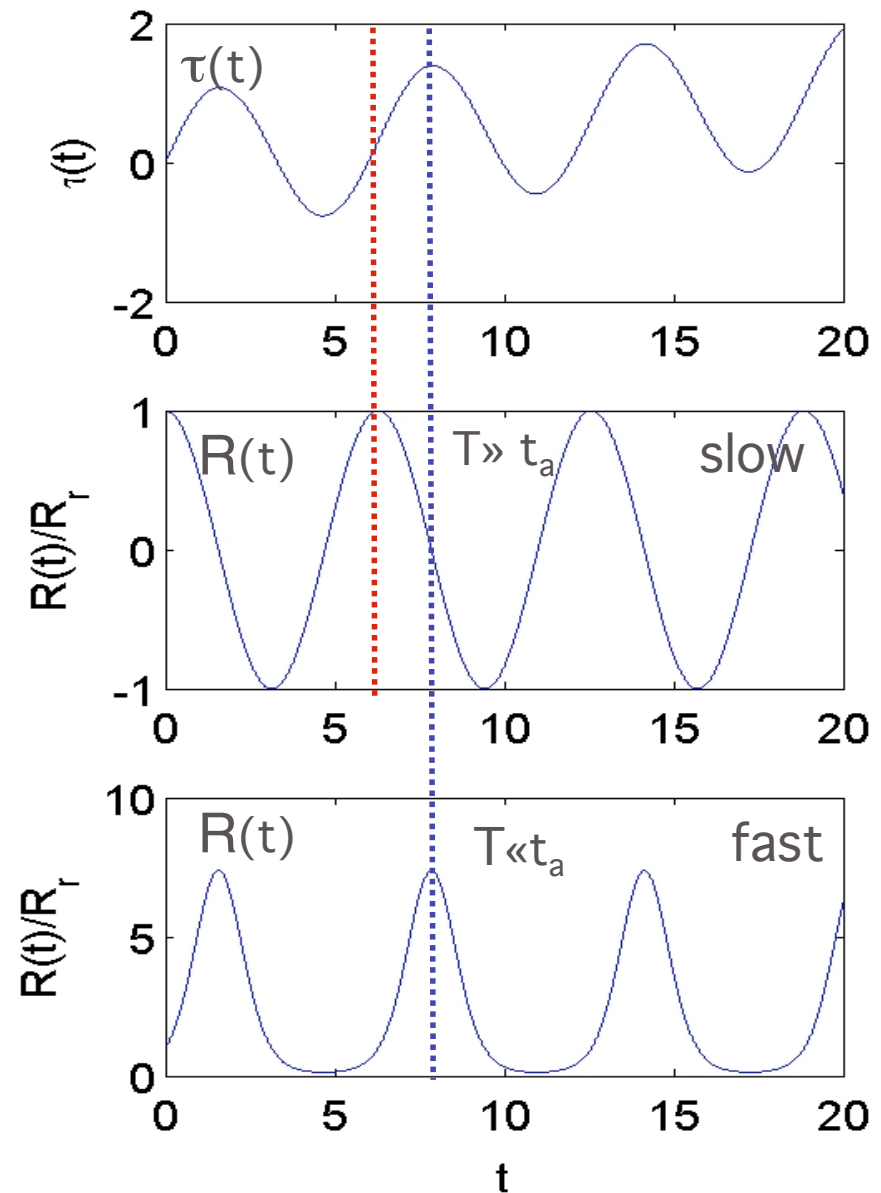
$R \sim d\tau/dt$

(tectonic loading, ...)

Example periodic stress change

$$A\sigma \ln \left(\frac{R}{R_0} \right) + \frac{N\dot{\tau}_r}{r} = \tau$$

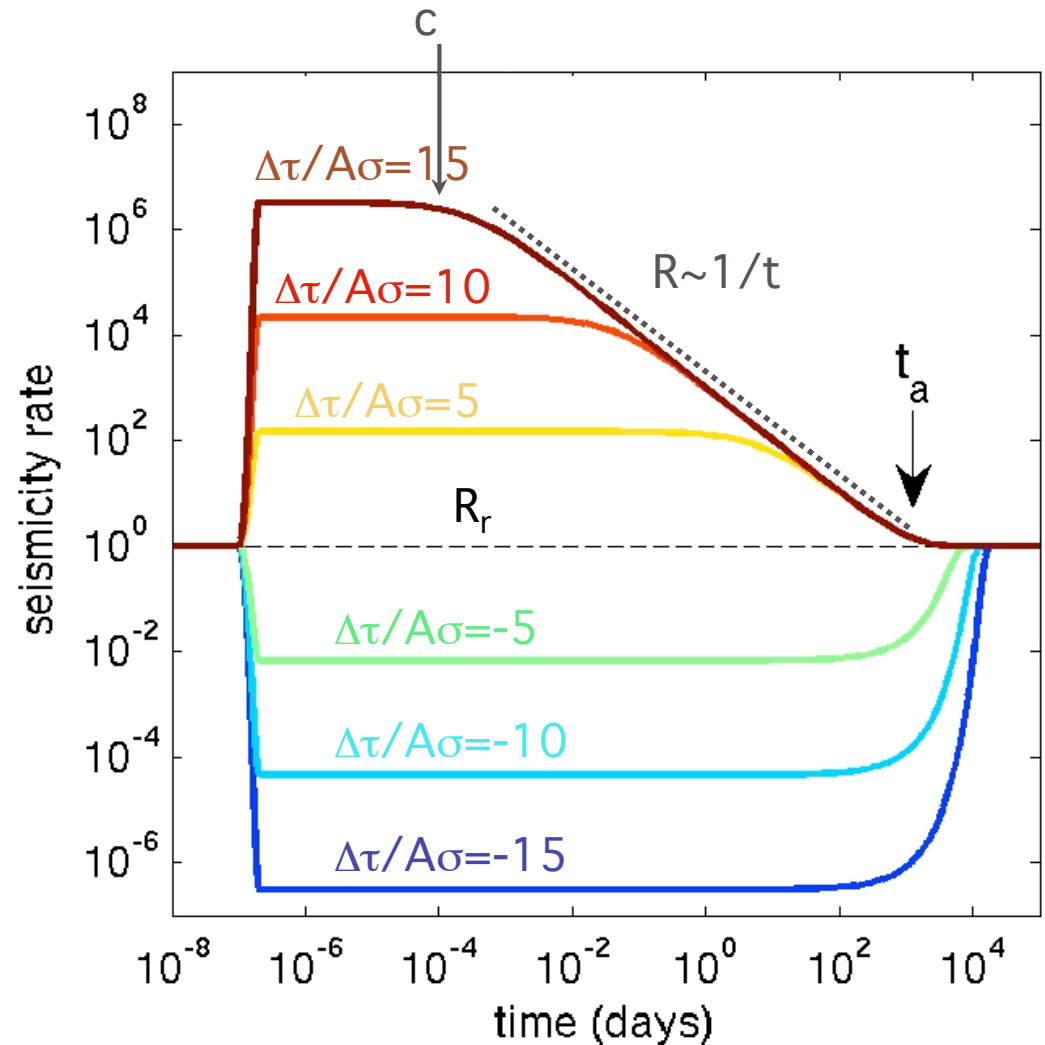
- $\tau(t) = \cos(2\pi t/T) + \tau'_r t$
- If $T \gg t_a$ $R(t) \sim d\tau/dt$
- If $T \ll t_a$ $R(t) \sim \exp(\tau/A\sigma)$
- In general there is not « simple » relation between stress change and seismicity!



Seismicity rate following a static stress change

[Dieterich, 1994]

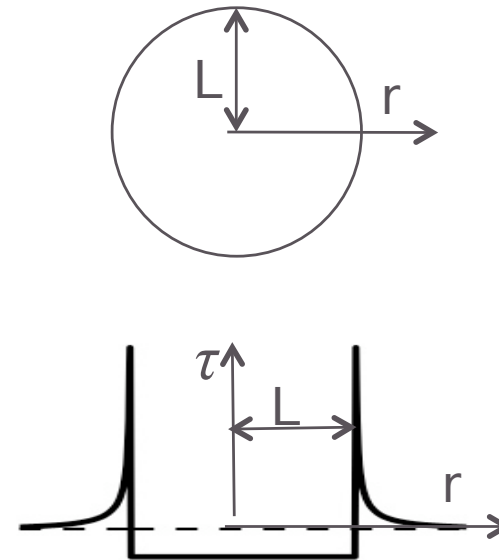
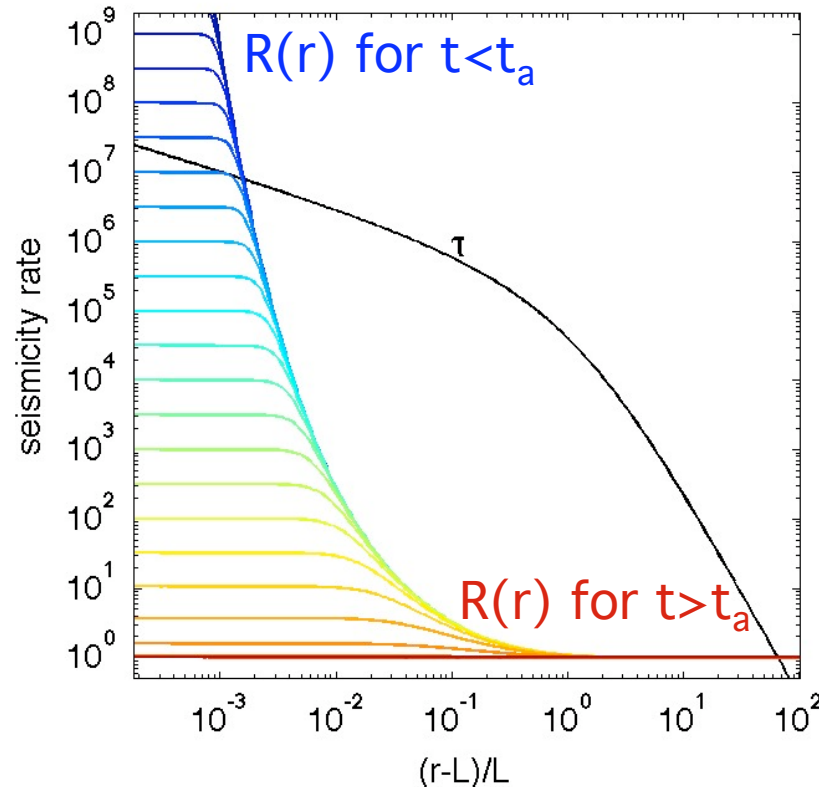
- For a stress increase
Omori law for $c \ll t \ll t_a$
 $R \sim$ rupture area $\sim 10^M$
realistic aftershock duration
- Requires very large stress !
 $\sigma = 100 \text{ MPa}$
 $A = 0.01$ (lab)
 $\Delta\tau = 15 \text{ MPa} \gg \text{EQ stress drop!}$



$$t_a = \frac{A\sigma}{\dot{\tau}_r} \quad c = t_a \exp(\Delta\tau/A\sigma)$$

Static stress changes and aftershocks

- stress change dislocation of length L : $\tau(r) \sim (1 - (L/r)^3)^{-1/2} - 1$



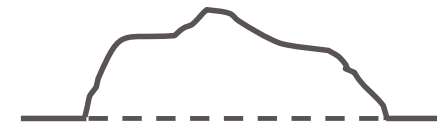
- Very few events for $r > 2L$
- «diffusion» of aftershocks with time
- Shape of $R(r)$ depends on time, very # from $\tau(r)$
- Difficult to guess triggering mechanisms from the decrease of $R(r)$

Coseismic slip, stress change, and aftershocks:

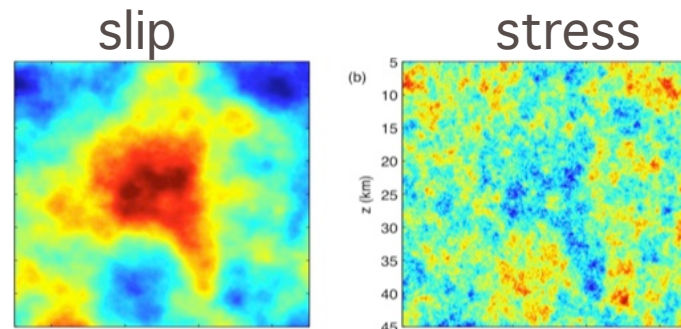
- **Model:** planar fault, uniform stress drop, and R&S model



- **Real data:** most aftershocks occur on or close to the rupture area



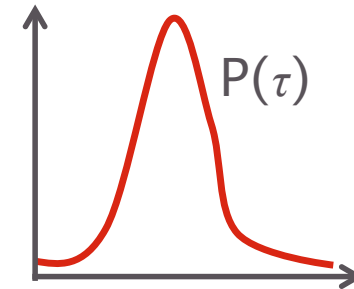
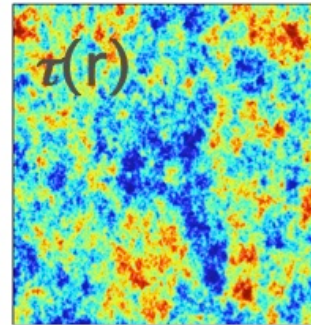
→ Slip and stress must be heterogeneous to produce an increase of stress and thus R on parts of the fault



Seismicity rate and stress heterogeneity

Seismicity rate triggered by a heterogeneous stress change on the fault

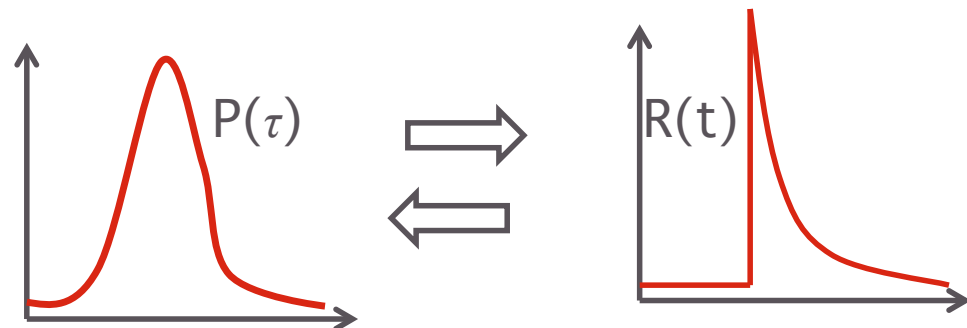
$$\begin{aligned} R(t) &= \int R(t, \tau(\vec{r})) d\vec{r} \\ &= \int_{-\infty}^{\infty} R(t, \tau) P(\tau) d\tau \end{aligned}$$



- $R(t, \tau)$: R&S model, unif stress change [Dieterich 1994]
- $P(\tau)$: stress distribution (due to slip heterogeneity or fault roughness)

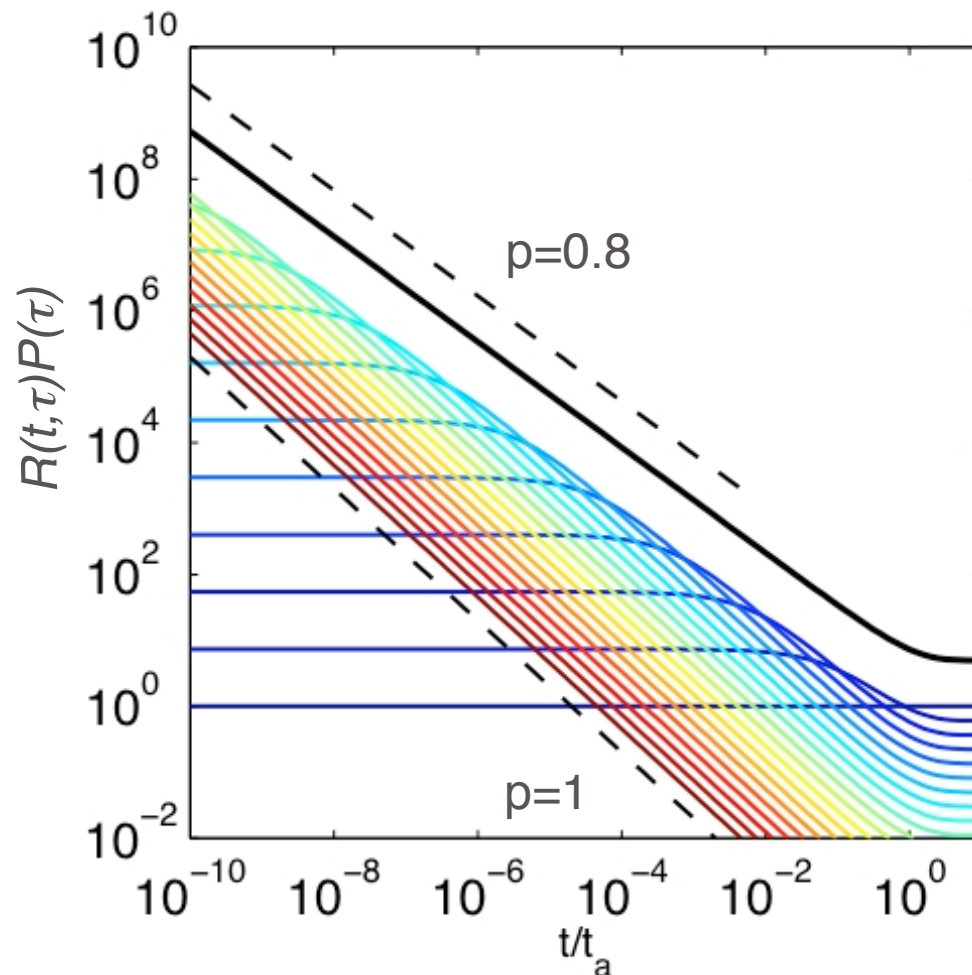
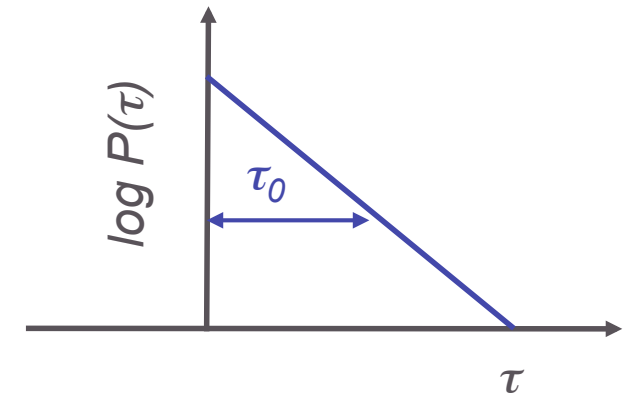
Goals

- seismicity rate $R(t)$ produced by a realistic $P(\tau)$
- inversion of $P(\tau)$ from $R(t)$



Stress heterogeneity and aftershock time decay

- For an exponential pdf $P(\tau) \sim e^{-\tau/\tau_0}$
- ➔ Omori law $R(t) \sim 1/t^p$ with $p = 1 - A\sigma/\tau_0$
- $p \leq 1$, ↗ if «heterogeneity» τ_0 ↗

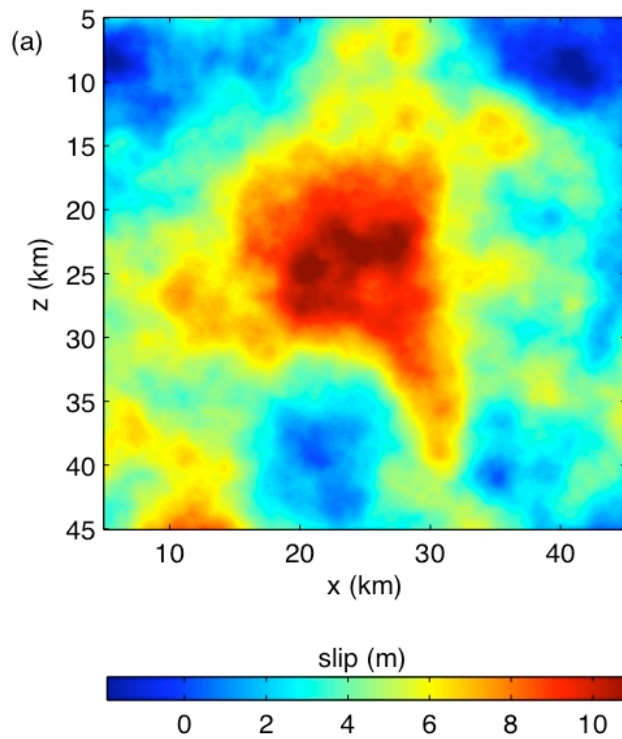


- colored lines:
EQ rate for a uniform τ
 $R(t, \tau)P(\tau)$
from $\tau=0$ to $\tau=50$ MPa
- black: global EQ rate,
heterogeneous τ :
 $R(t) = \int R(t, \tau)P(\tau)d\tau$
with $\tau_0/A\sigma=5$

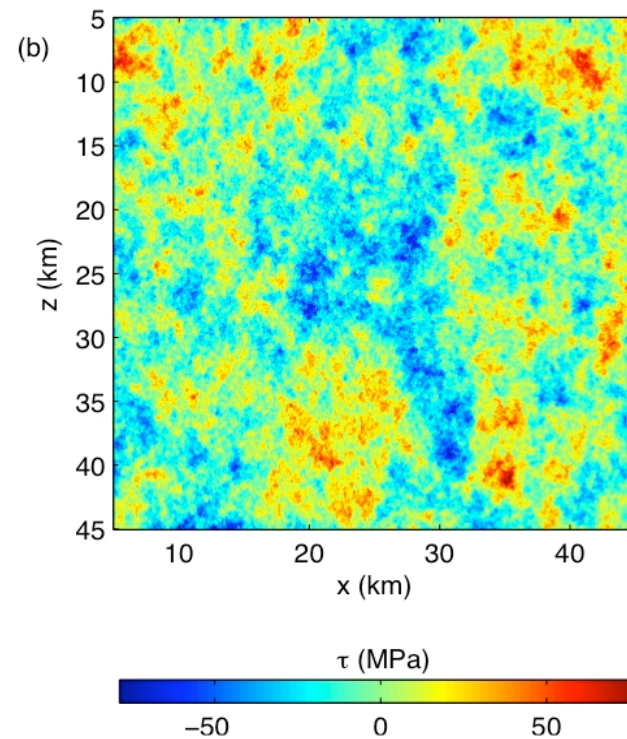
Slip and shear stress heterogeneity, aftershocks

Modified « k^2 » slip model: $U(k) \sim 1/(k+1/L)^{2.3}$ [Herrero & Bernard, 1994]

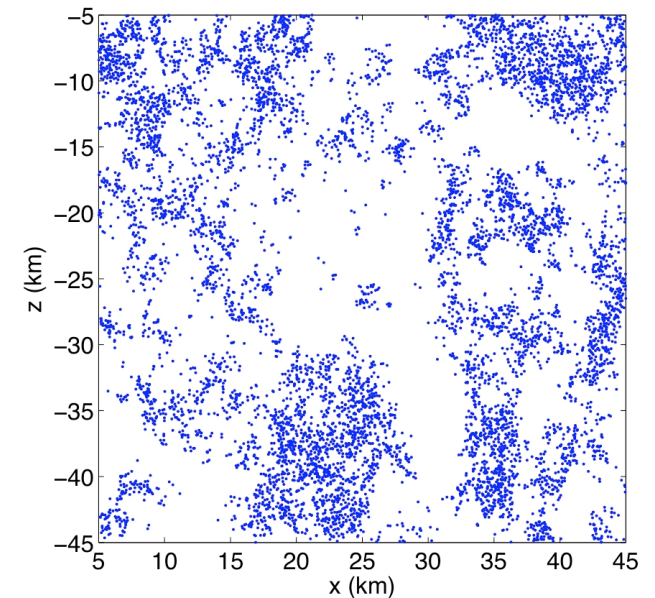
slip



shear stress
stress drop $\sigma_0 = 3$ MPa



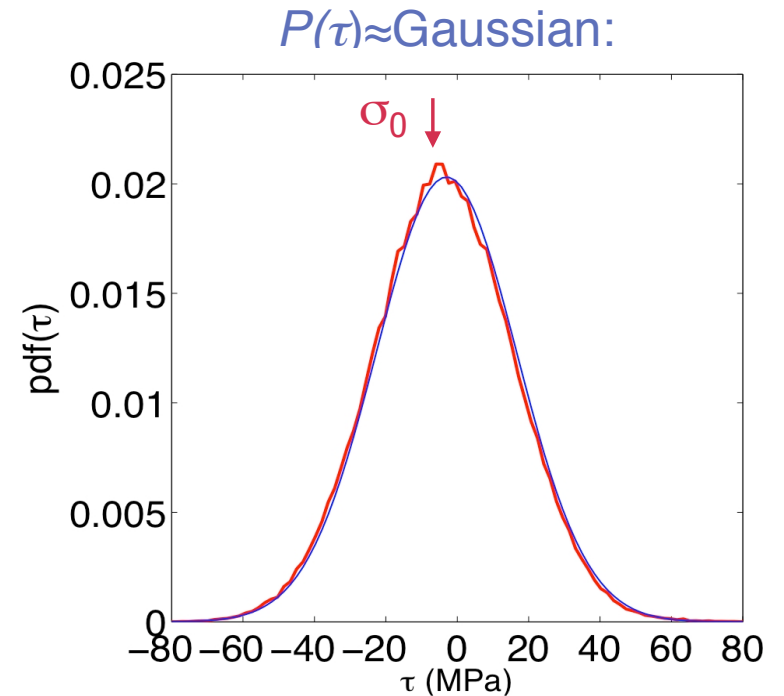
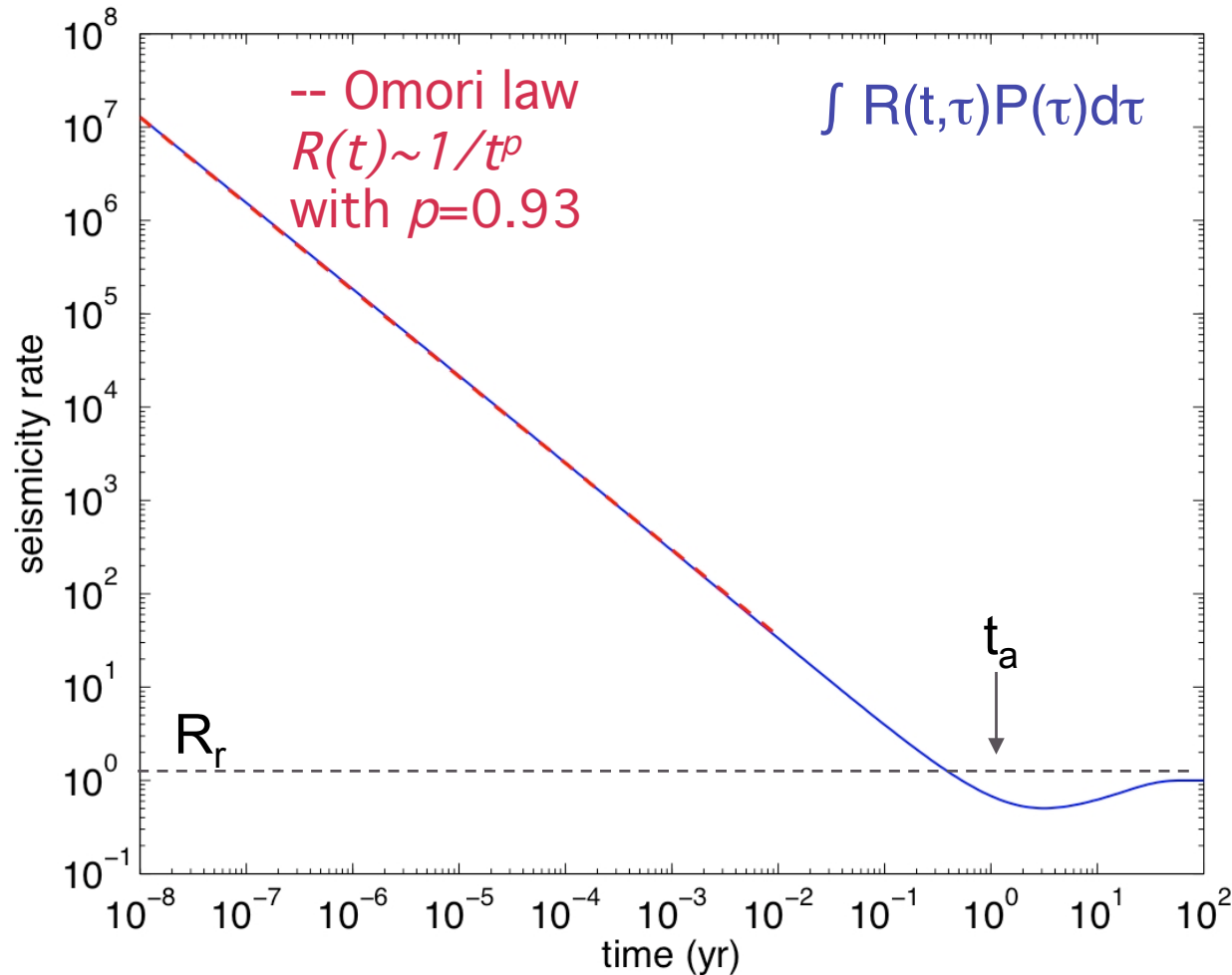
aftershock map
synthetic catalog
R&S model



↑
mean stress σ_0

Stress heterogeneity and aftershock time decay

Aftershock rate on the fault with R&S model for modified k^2 slip model

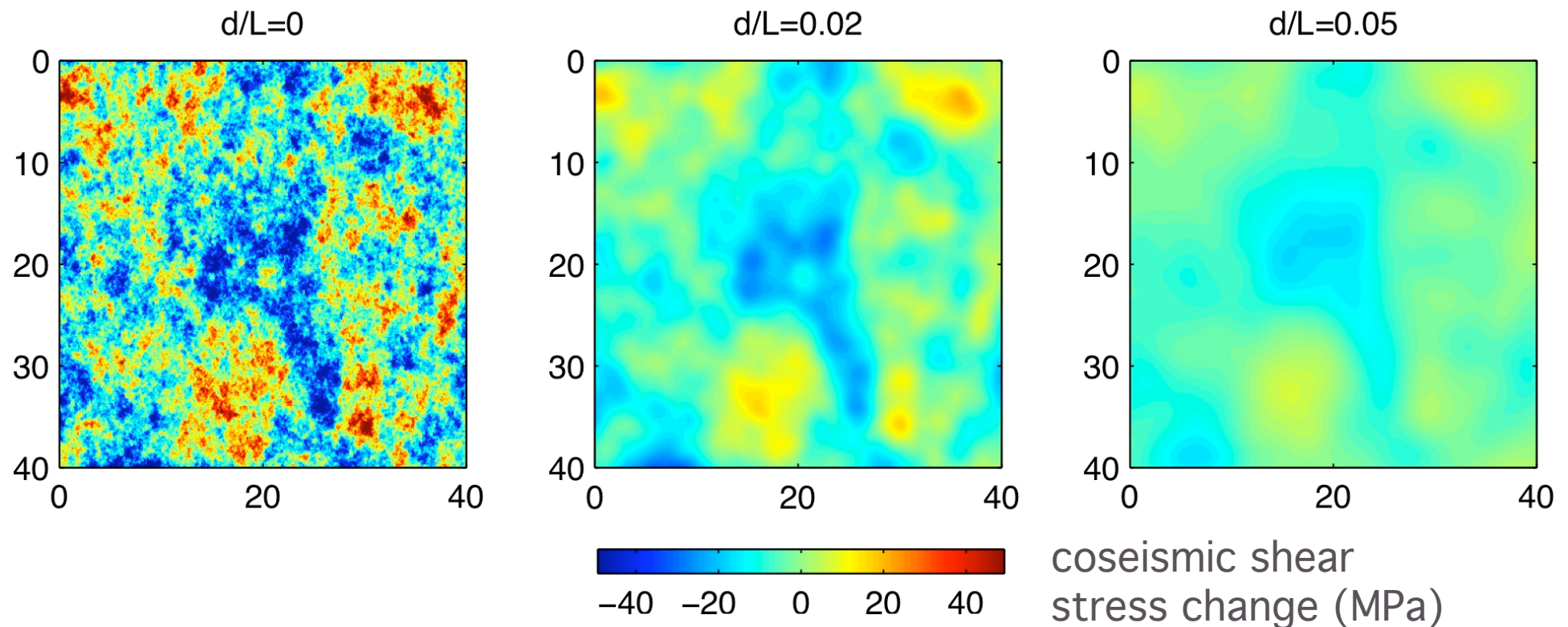


Short times $t \ll t_a$: apparent Omori law with $p \leq 1$

Long times $t \approx t_a$: stress shadow $R(t) < R_r$

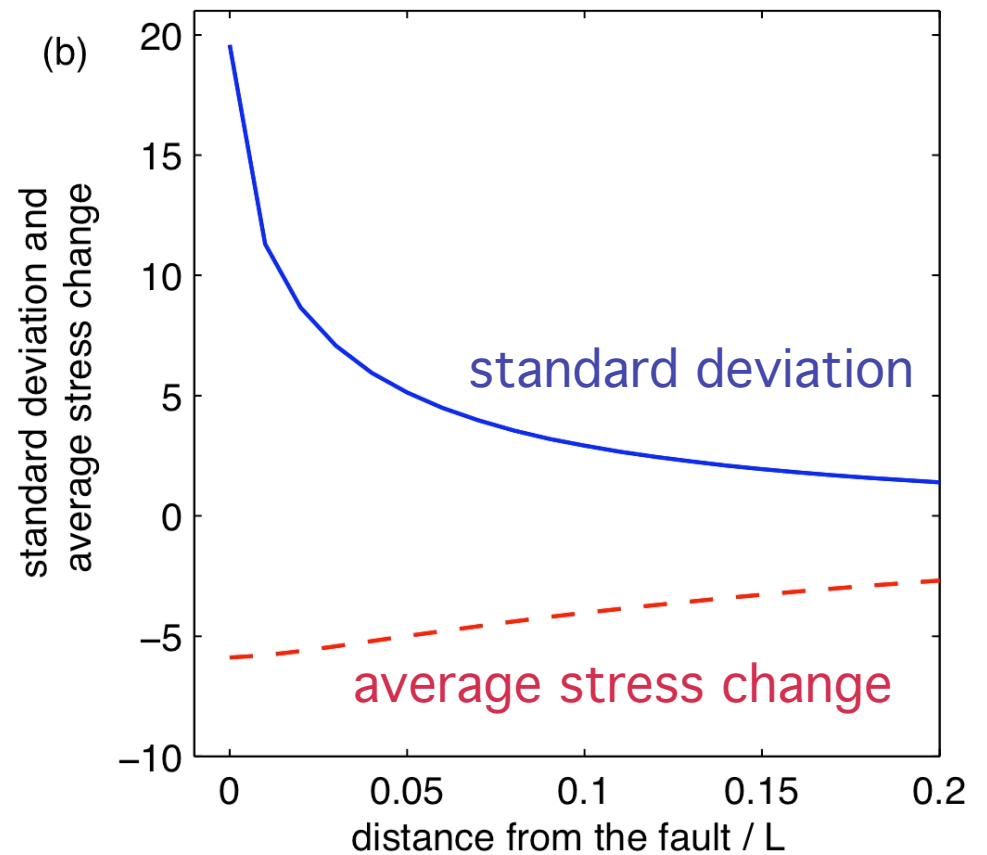
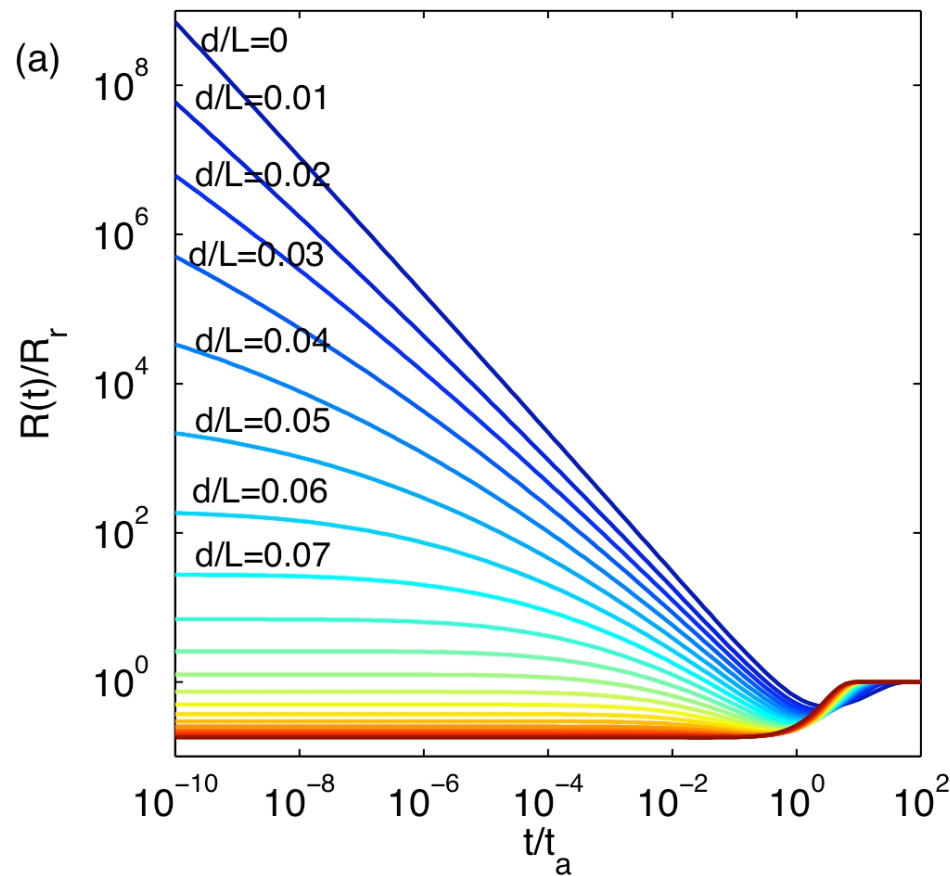
Modified k^2 slip model, off-fault stress change

- distance $d < L$ from the fault: $\tau(k,d) \sim \tau(k,0)e^{-kd}$ for $d \ll L$
- fast attenuation of high frequency τ perturbations with distance



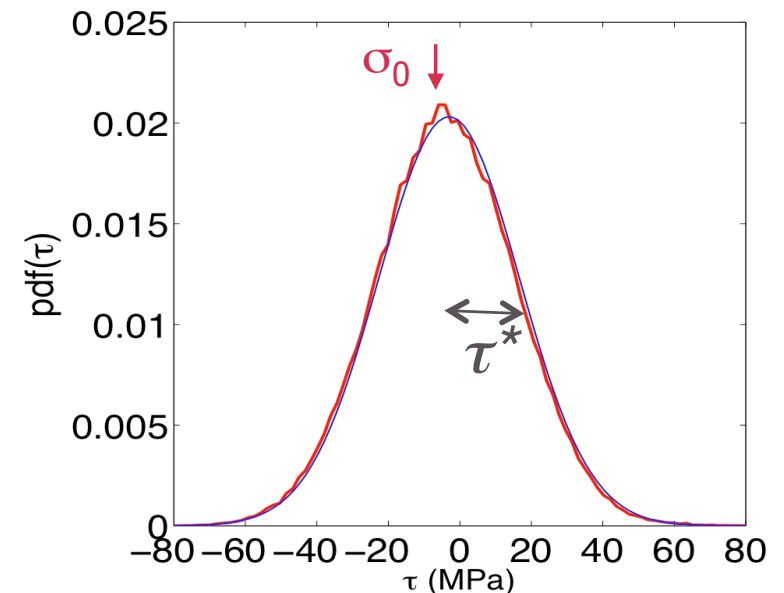
Modified k^2 slip model, off-fault aftershocks

- seismicity rate and stress change as a function of d/L
- quiescence for $d > 0.1L$

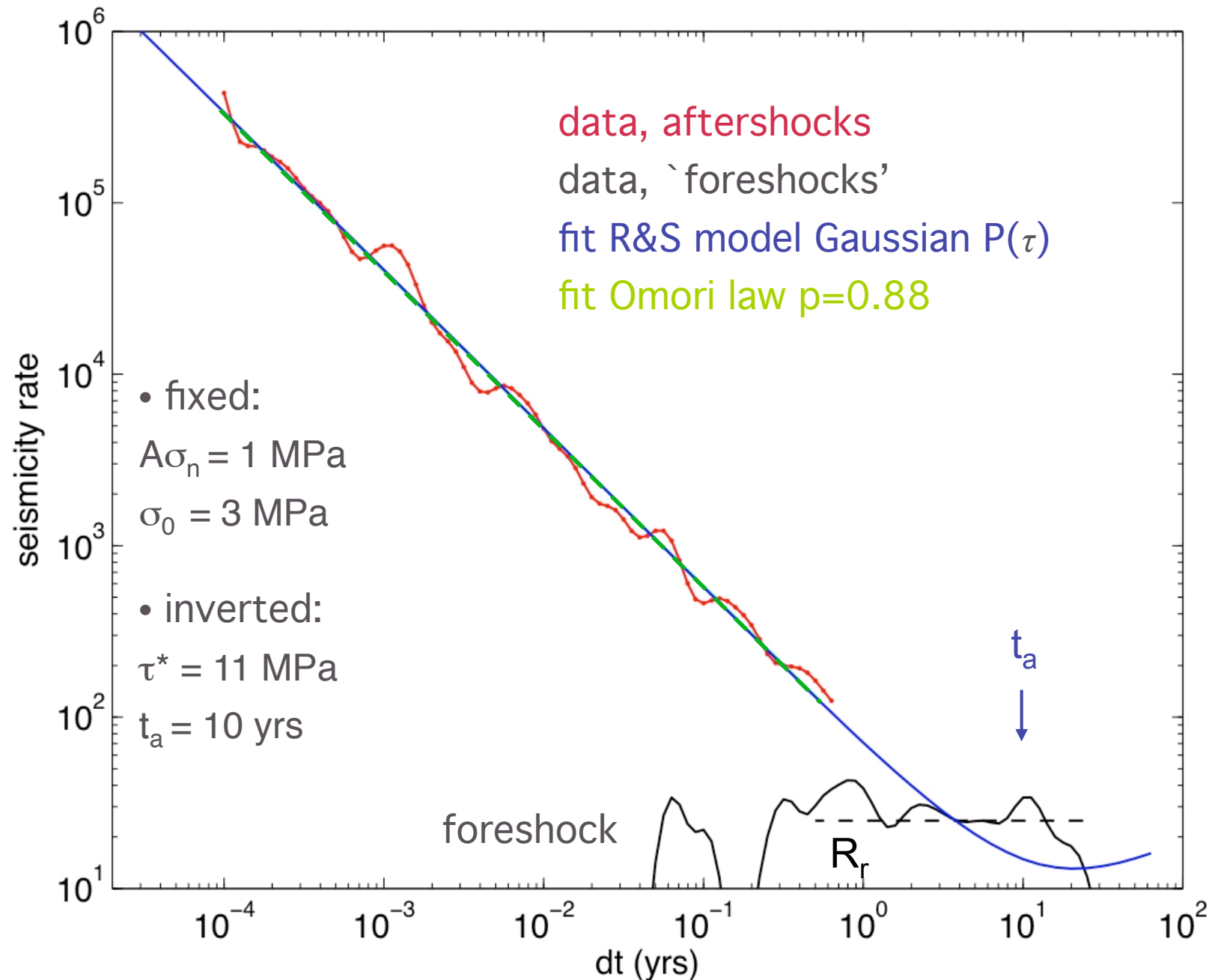


Inversion of stress distribution from aftershock rate

- We invert for $P(\tau)$ from $R(t)$ for individual aftershocks sequences in California and stacked sequences in Japan
- select aftershocks close to fault plane
- assume $P(\tau)$ is gaussian
- stress drop σ_0 fixed to 3 MPa
- $A\sigma=1$ MPa
- invert for t_a and standard deviation τ^*



Parkfield 2005 M=6 aftershock sequence



Inversion of $P(\tau)$ for real sequences

Sequence	p	τ^* (MPa)	t_a (yrs)
Morgan Hill M=6.2, 1984	0.68	6.2	78.
Parkfield M=6.0, 2004	0.88	11.	10.
Stack, $3 < M < 5$, Japan*	0.89	12.	1.1
San Simeon M=6.5 2003	0.93	18.	348.
Landers M=7.3, 1992	1.08	**	52.
Northridge M=6.7, 1994	1.09	**	94.
Hector Mine M=7.1, 1999	1.16	**	80.
Superstition-Hills, M=6.6, 1987	1.30	**	**

* [Peng et al., 2007]

** we can't estimate τ^* because $p > 1$

Conclusion - triggering by heterogeneous static stress changes

R&S model with stress heterogeneity explains

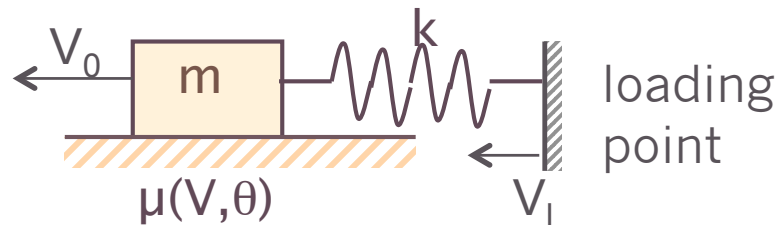
- short-times triggering
 - Omori law with $p \leq 1$
 - p decreases with stress variability
- Long times quiescence for $t \approx t_a$
- in space : clustering on/close to the rupture area

Problems:

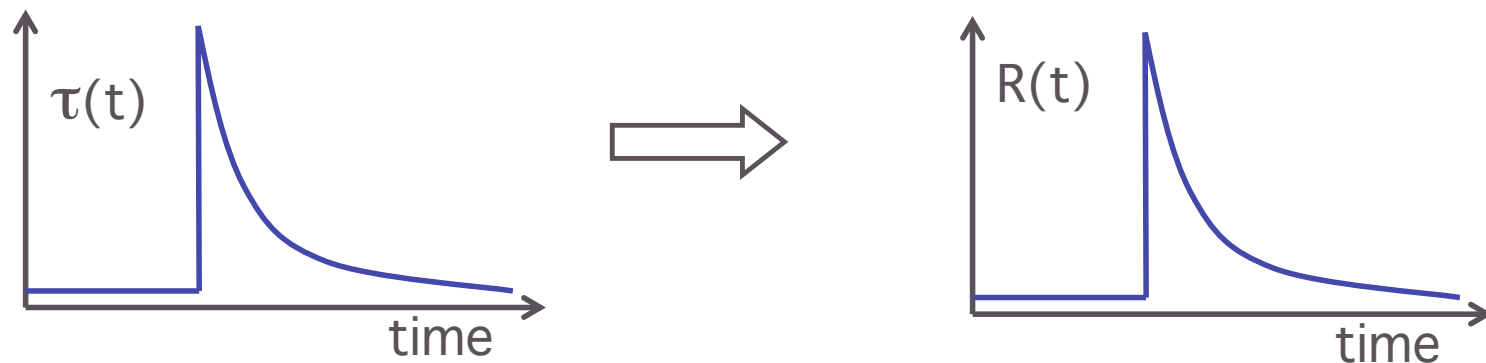
- inversion: stress drop not constrained if catalog too short
- we don't know $A\sigma$: 0.01 or 1 MPa??
- secondary aftershocks?
- can't explain $p > 1$: post-seismic stress relaxation?

Earthquakes triggering by aseismic stress changes (afterslip, viscous relaxation, fluid flow,...)

- Modelling afterslip and slow slip events with a simple slider-block model and R&S friction

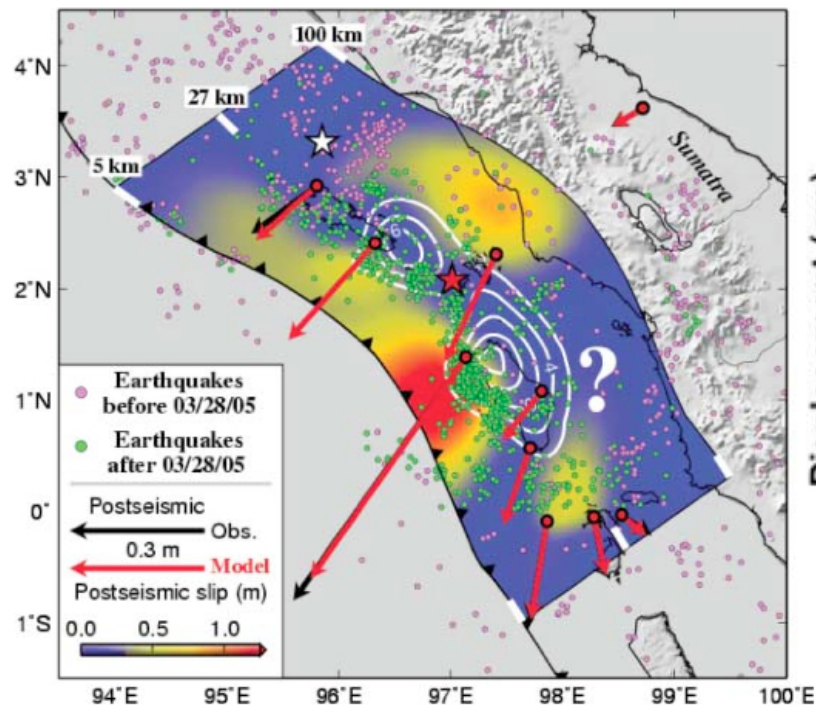


- Triggering of aftershocks by afterslip

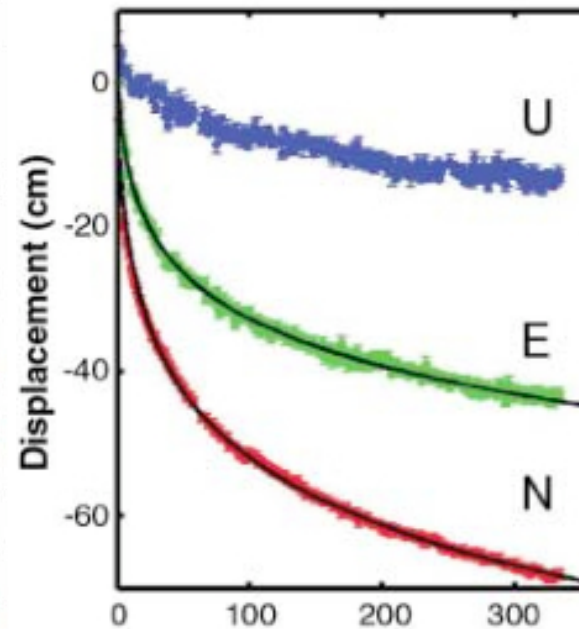


Observations: example for 2005 m=8.7 Nias EQ

Co- and after- slip

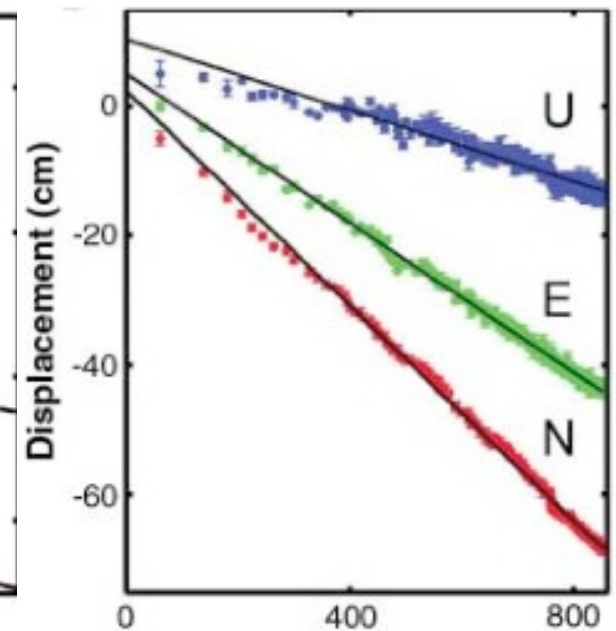


Afterslip (time)



Days after Nias earthquake

Afterslip and # of aftershocks



Cumulative number of aftershocks

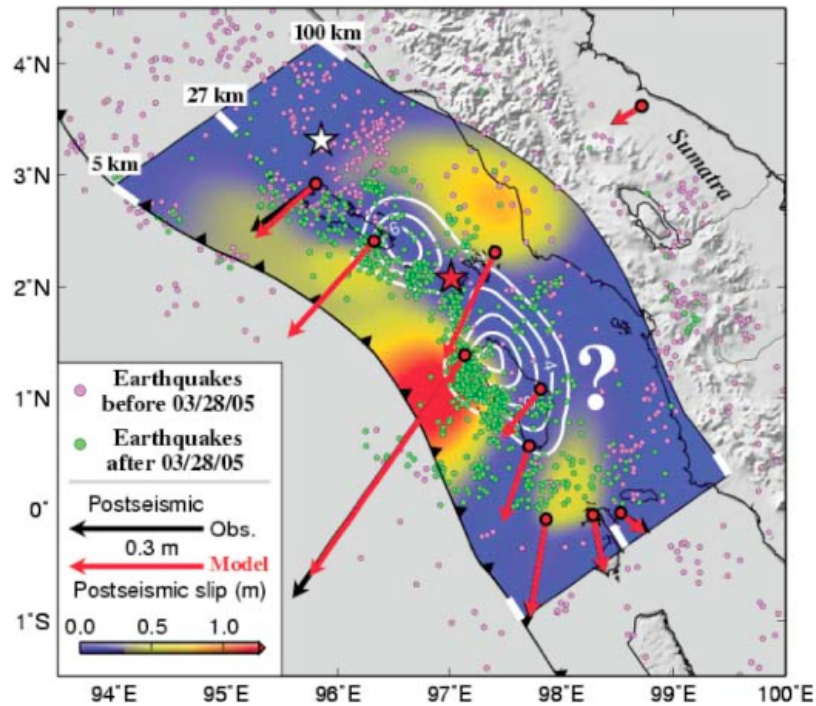
[Hsu et al, Science 2006]

Interactions between afterslip and aftershocks

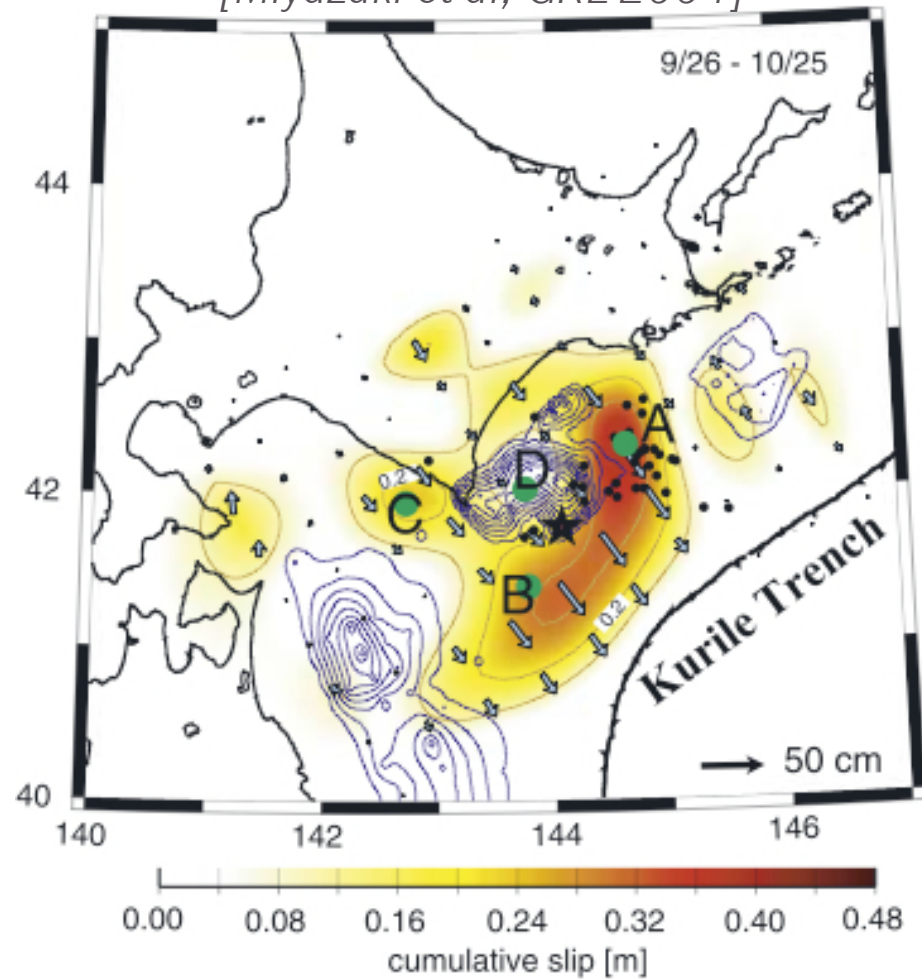
- relation between coseismic and postseismic slip?
- can we use afterslip to constrain the rheology of the crust (stable/unstable)?
- relation between afterslip and aftershocks?
- mechanisms for aftershock triggering?

Observations of postseismic behavior

2005 $m=8.7$ Nias
[Hsu et al, 2006]



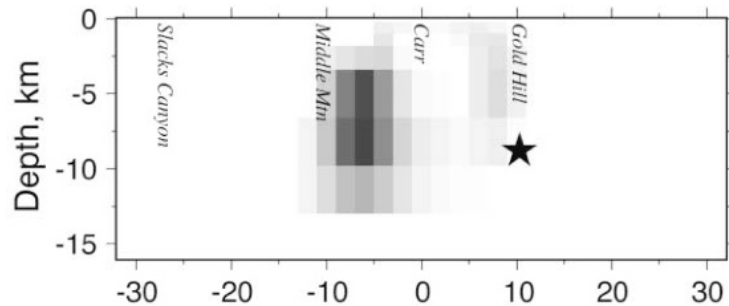
2003 $m=8$ Tokachi
[Miyazaki et al, GRL 2004]



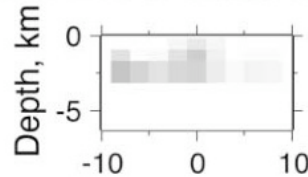
Observations of postseismic behavior

Parkfield 2004, M=6
[Langbein et al 2006]

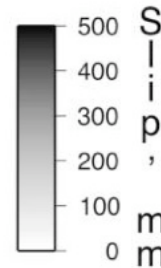
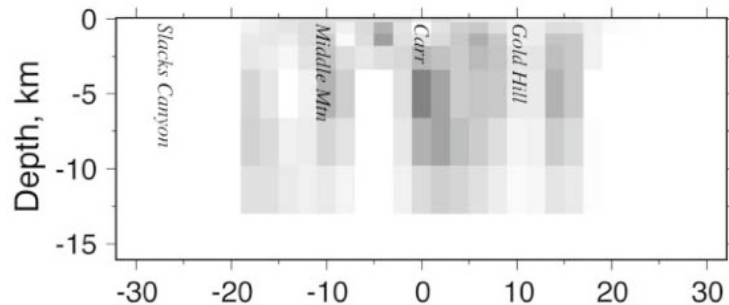
SAF co-seismic slip



SWFZ co-seismic slip

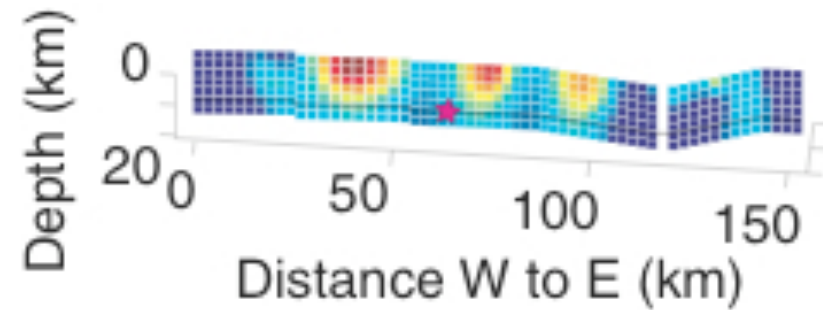


SAF post-seismic slip; day 0 to 60

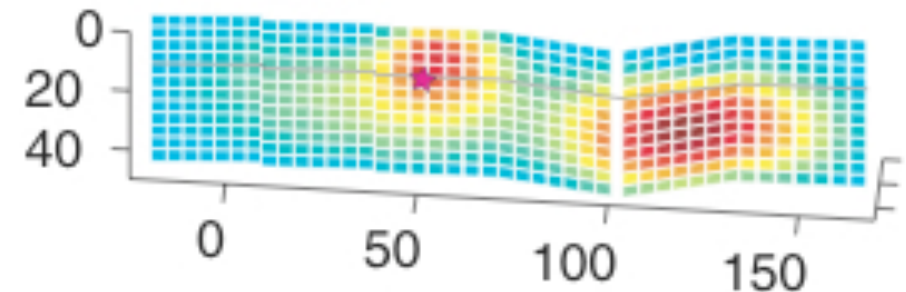


Izmit 1999, M=7.6
[Burgmann, 2002]

Coseismic (Aug. 17, 1999)

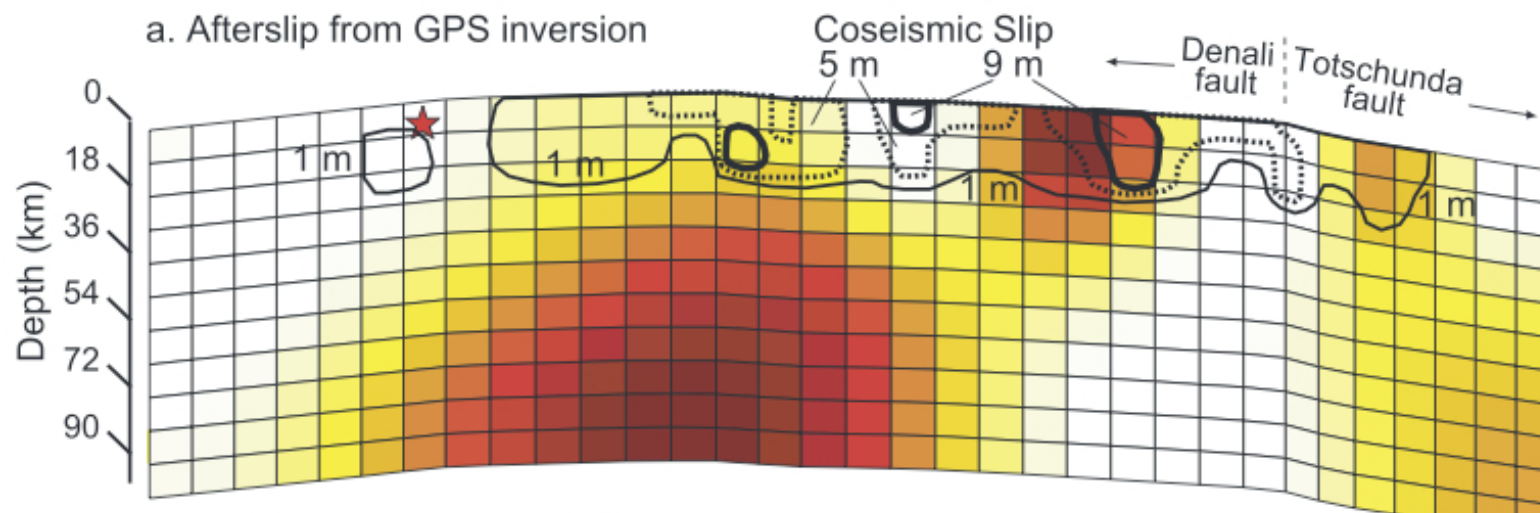


Day 10



Spatial distribution of afterslip and aftershocks

2002 $m=7.8$ Denali [Freed et al, JGR 2006]



Observations of afterslip

- afterslip on average scales with co-seismic slip
- afterslip moment is usually a few % of coseismic
- But it may be larger than coseismic moment (eg, Parkfield 2004)
- Slip rate usually decays as $1/t$

... but hard to distinguish from exponential decay

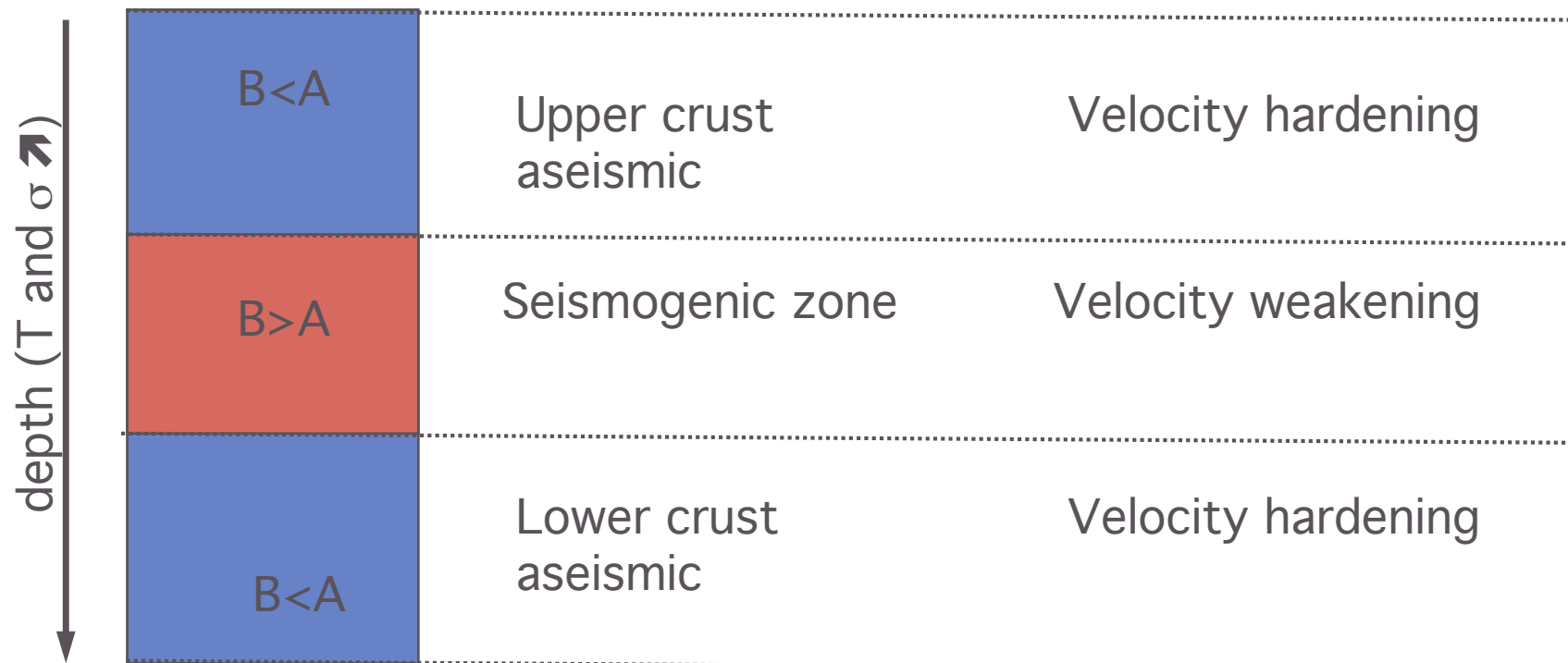
- Afterslip is usually associated with velocity-hardening faults and earthquakes associated to velocity weakening?

[Marone et al 1991, Perfettini and Avouac 2003, ...]

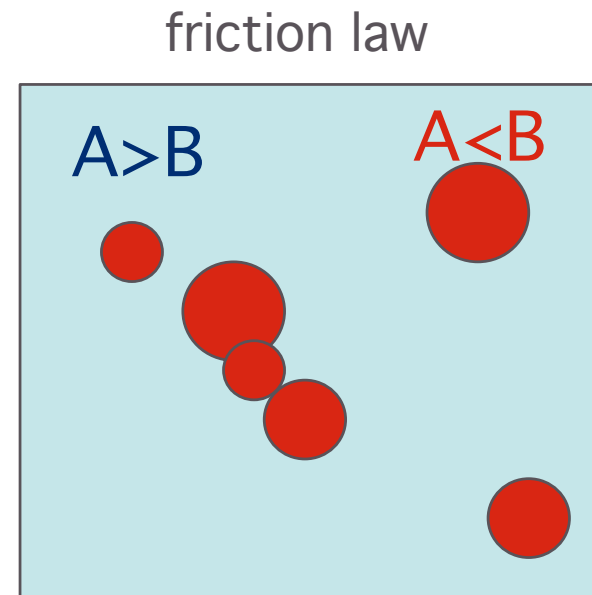
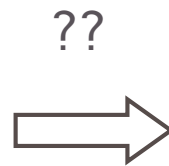
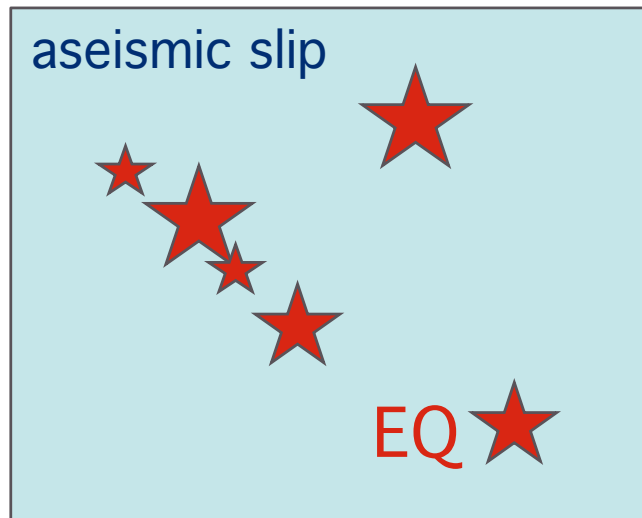
- Some overlap between aftershocks, co- and post-seismic slip
 - ➔ temporal or spatial changes in the friction law parameters?

Rate-and-state friction and fault dynamics

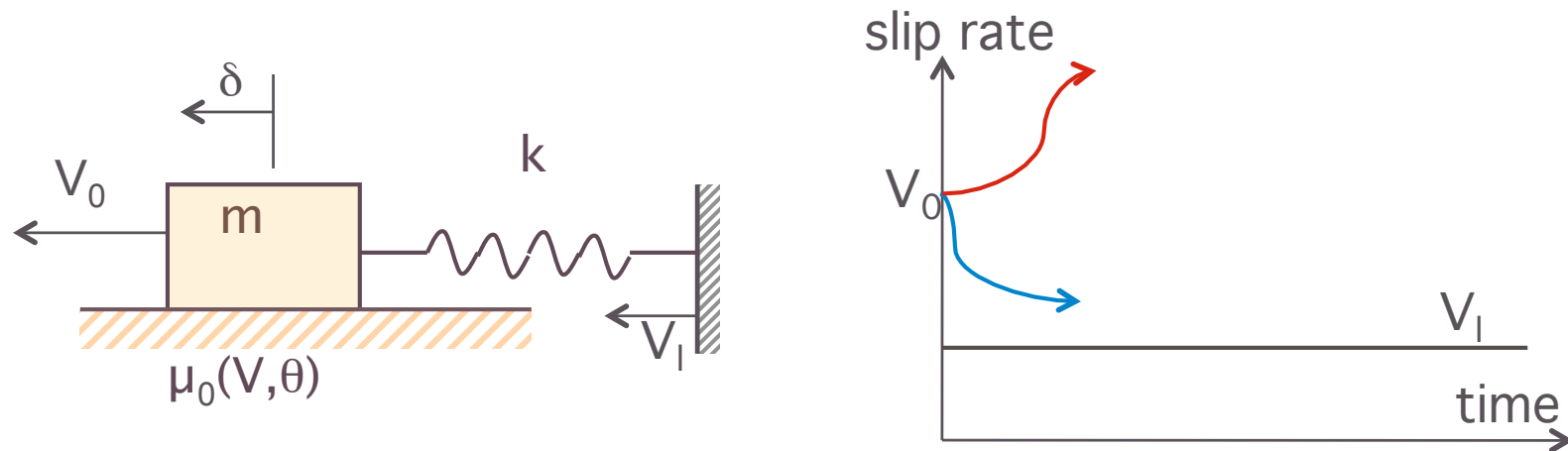
From lab experiments, A and B are expected to vary with T and σ
Variations of B/A can explain the distribution of seismicity with depth



Rate & state friction and fault behavior



Rate-and-state friction law and afterslip



- friction law [*Dieterich, 1979*]

$$\mu = \mu_0 + A \log(V/V_0) + B \log(\theta/\theta_0) = \mu_0 - k(\delta - V_l t)/\sigma$$

$$d\theta/dt = 1 - V\theta/D_c$$

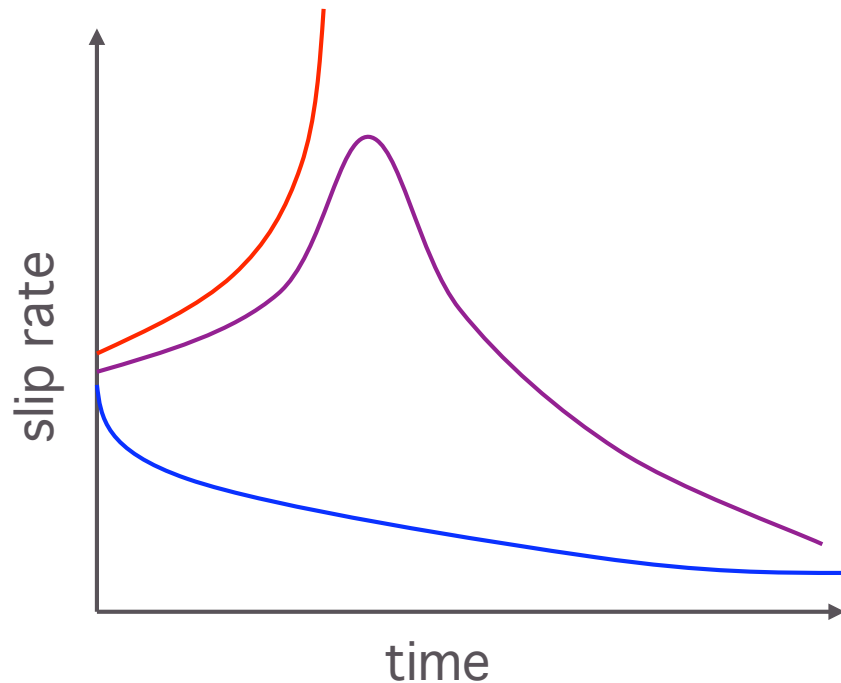
- relaxation or nucleation of a slip instability after a stress step
- inertia and tectonic loading negligible:

$$V_l \ll V \ll \text{coseismic slip rate}$$

Numerical & analytical analysis

Fault behavior after a stress step

Different behaviors are observed in numerical simulations as a function of friction parameters B/A , stiffness k/k_c and stress μ :



Aftershock:

Slip instability triggered by stress change

Slow EQ

Slip rate increase followed by relaxation

Afterslip

Relaxation toward background rate

Fault behavior – phase diagram

Fault behavior controlled by B/A , stiffness k and stress ($V \gg V_l$) [Helmstetter and Shaw, 2009]

- slip accelerations

if $k < k_B$ and $\mu > \mu_a > \mu_{ss}$

- slip instabilities

if $k < k_c$ and $\mu > \mu_l > \mu_{ss}$

- steady-state

$$d\theta/dt = 0$$

$$V = D_c / \theta = \text{const}$$

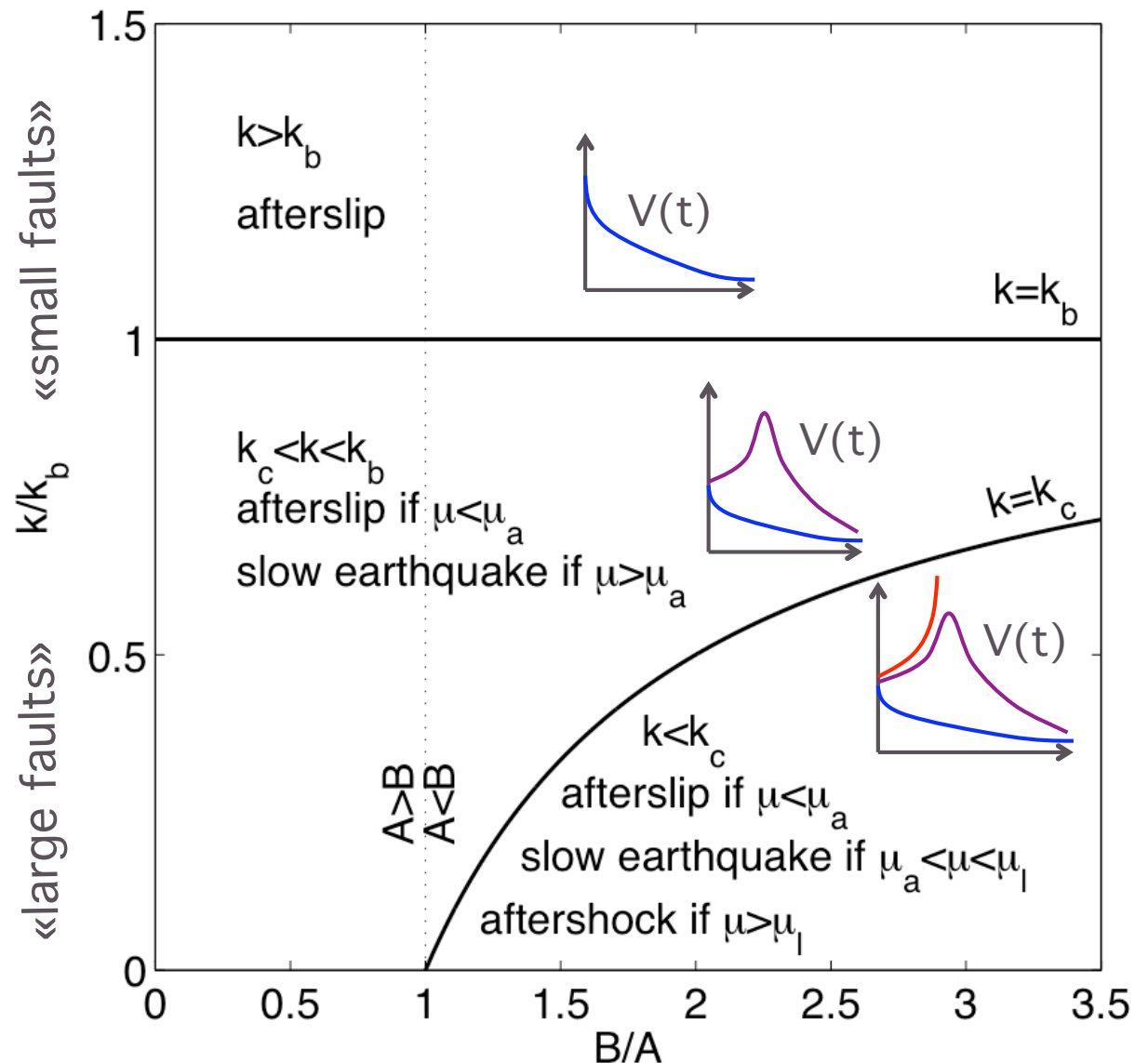
$$\mu_{ss} = \mu_0 + (B-A) \ln(V/V_0)$$

$$k_B = B\sigma / D_c$$

$$k_c = (B-A)\sigma / D_c$$

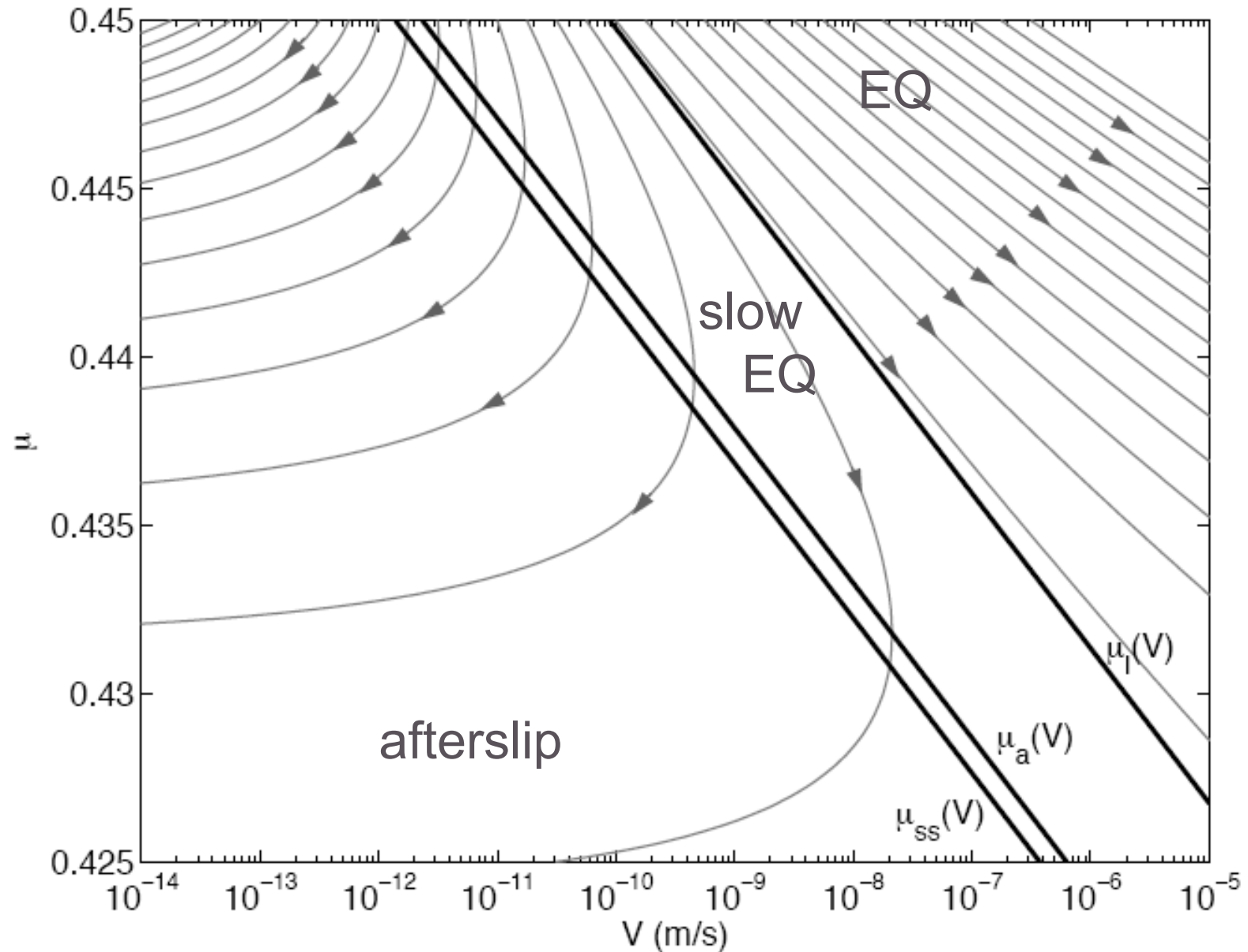
$$\mu_l = \mu_{ss} - B \ln(1 - k/k_c)$$

$$\mu_a = \mu_{ss} - B \ln(1 - k/k_B)$$



Fault behavior

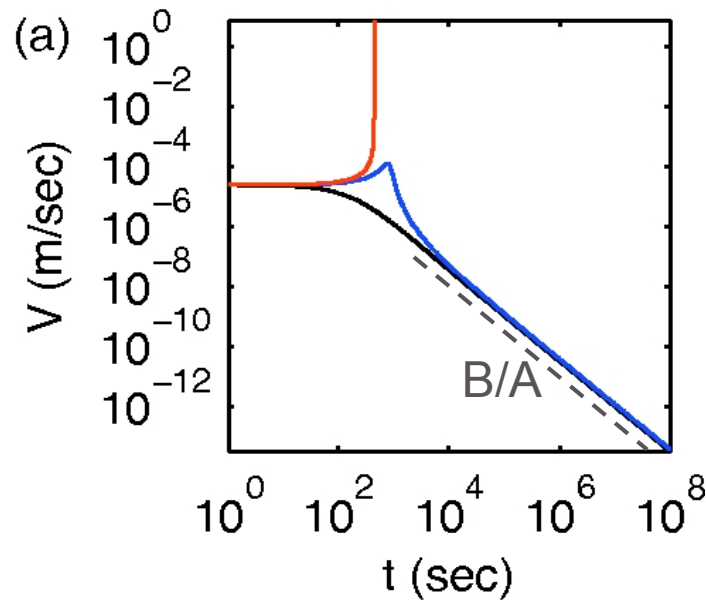
Fault behavior for $B > A$ and $k < k_c$ ($V \gg V_l$)



Slip rate history

- # behaviors: aftershocks, slow EQ, and afterslip
- # afterslip regimes, with slope exponent= B/A or 1
- # characteristic times t^*
- analytical solutions for $\mu \gg, \ll$ or $\approx \mu_{ss}$

[Helmstetter and Shaw, 2009]



Unstable case:

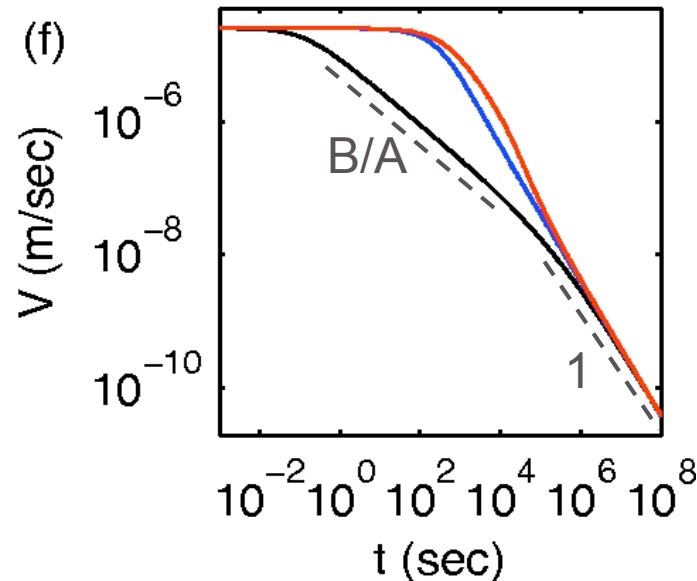
$$B=1.5A$$

$$k=0.8k_c$$

$\mu_0 > \mu_l$: aftershock

$\mu_l > \mu_0 > \mu_a$: slow EQ

$\mu_0 < \mu_a$: afterslip



Stable case

$$B=0.5A$$

$$k=2.5|k_c|$$

only afterslip

$\mu_0 > \mu_{ss}$

$\mu_0 = \mu_{ss}$

$\mu_0 < \mu_{ss}$

Slip history - 1 D model and afterslip data

Data:

- GPS and creep-meter for 2004 $m=6$ Parkfield [Langbein et al , 2006]
- GPS data for 2005 Nias $m=8.5$ [Hsu et al , 2006]
- GPS data for 2002 Denali $m=7.8$ [Freed et al, 2006]

Models : each dataset fitted individually with

- Omori law: $V=V_0/(t/c+1)^p + V_l$
- Rate-dependant friction law : $\mu=\mu_0 + (B-A) \ln(V/V_0)$

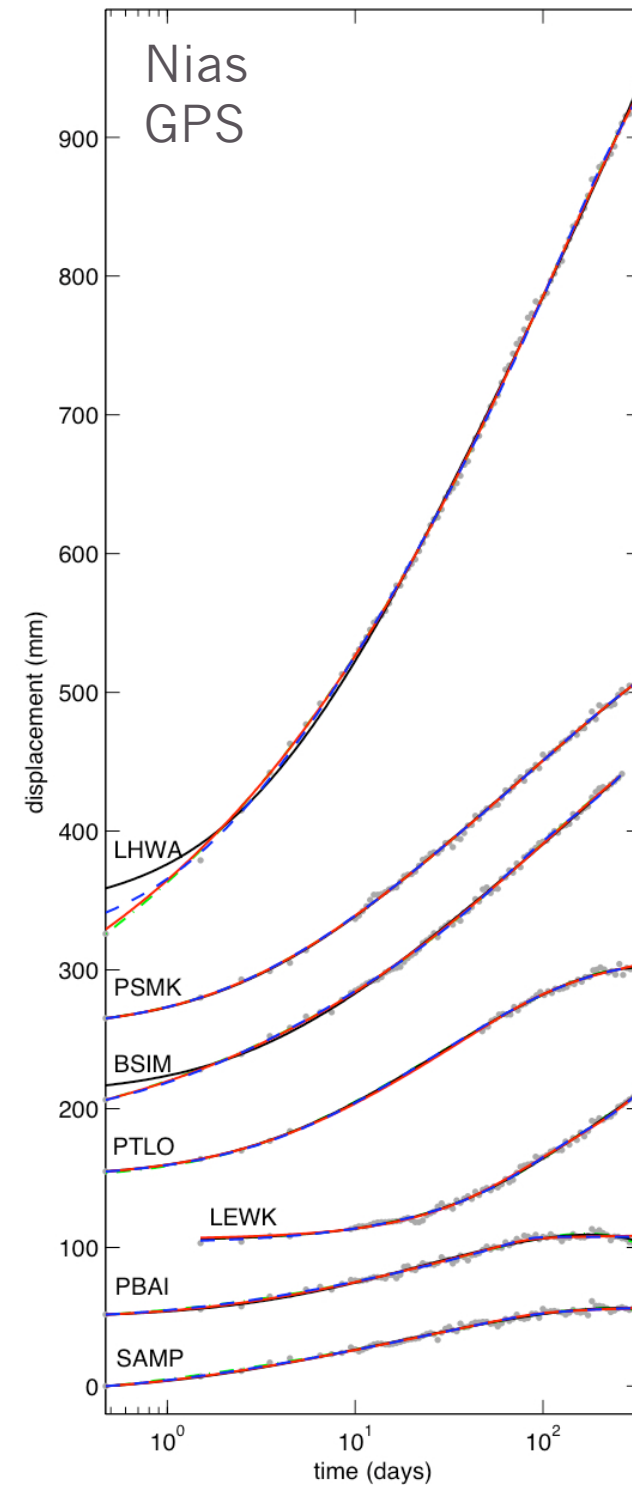
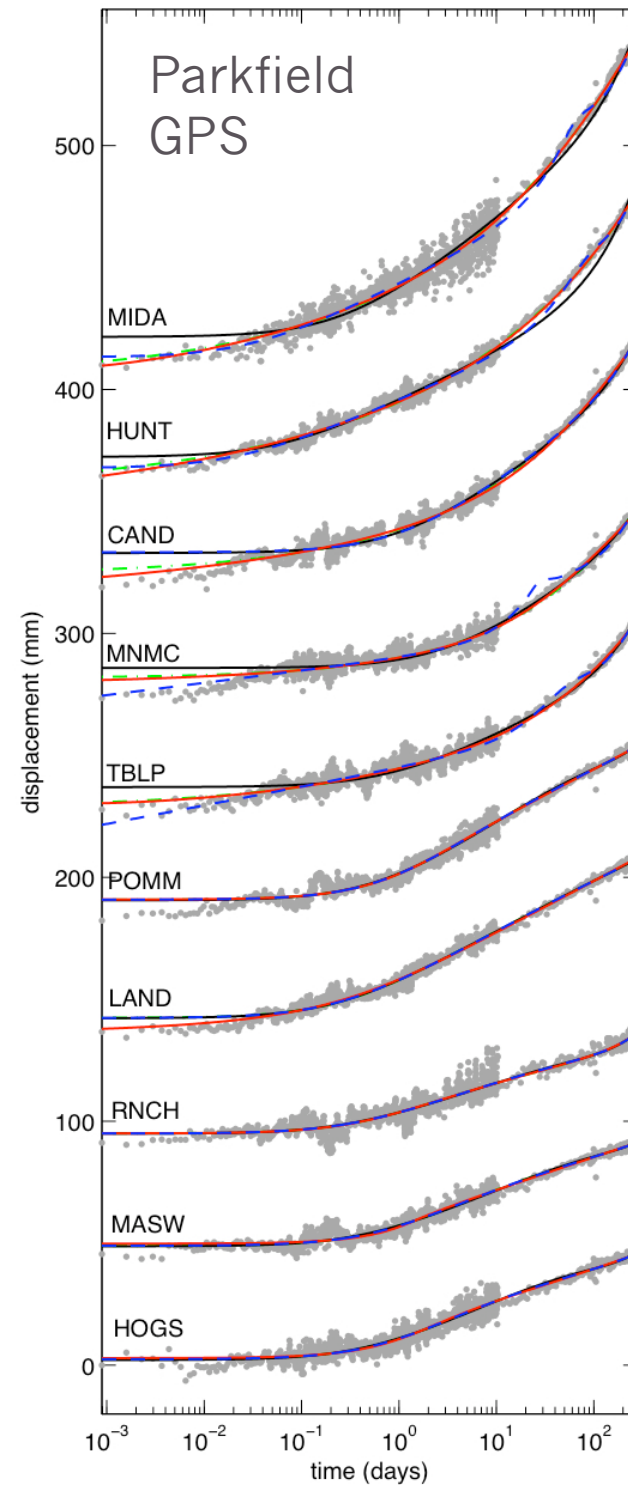
[Marone et al., 1991; Hsu, 2006; Perfettini et al, 2004, 2007, ...]

$$V= V_0/[1+\exp(-t/t_r)(1/d-1)] + V_l$$

- Full R&S friction law with constant tectonic rate :

invert for A, B, k, D_c, V_l, V_0 and μ_0

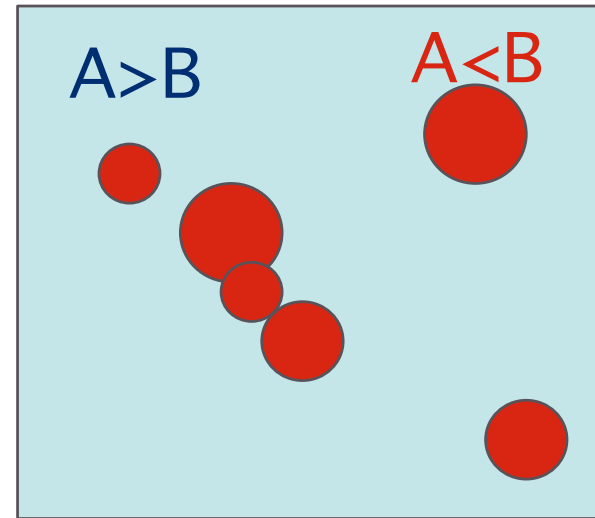
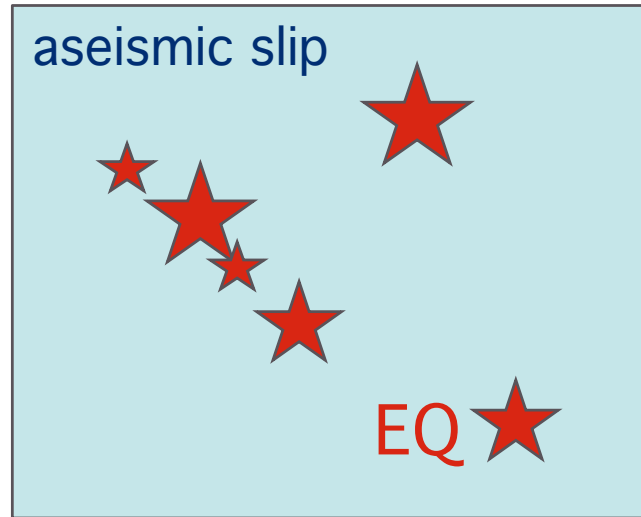
- Omori
- _ R friction
- _ R&S friction with $A > B$
- R&S friction with $A < B$



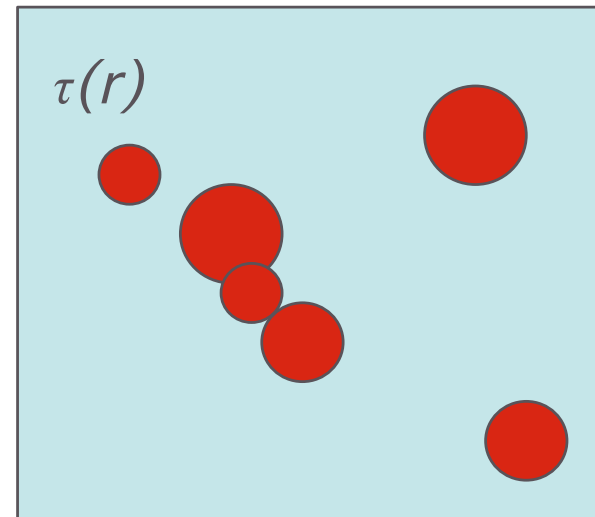
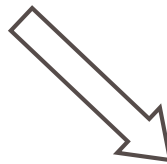
Results - 1 D model and fit of afterslip data

- All models provide a good fit to the data for the 3 EQs
- full R&S friction law usually gives a better fit than rate-dependant friction or than Omori law, but with more inverted parameters
- Inversion is not constrained: many very # models give similar slip history and very good fits, but sometimes unphysical values ($A=100000$, $D_c=1\text{ km}$, ...)
- Models with $A>B$ or $B>A$ often provide similar fit

Conclusions : rate & state and fault behavior



friction law



stress

fault
roughness

Afterslip and aftershocks

- mainshock $\Rightarrow$ coseismic stress change $\Rightarrow$ afterslip $\Rightarrow$ postseismic reloading $\Rightarrow$ aftershocks?

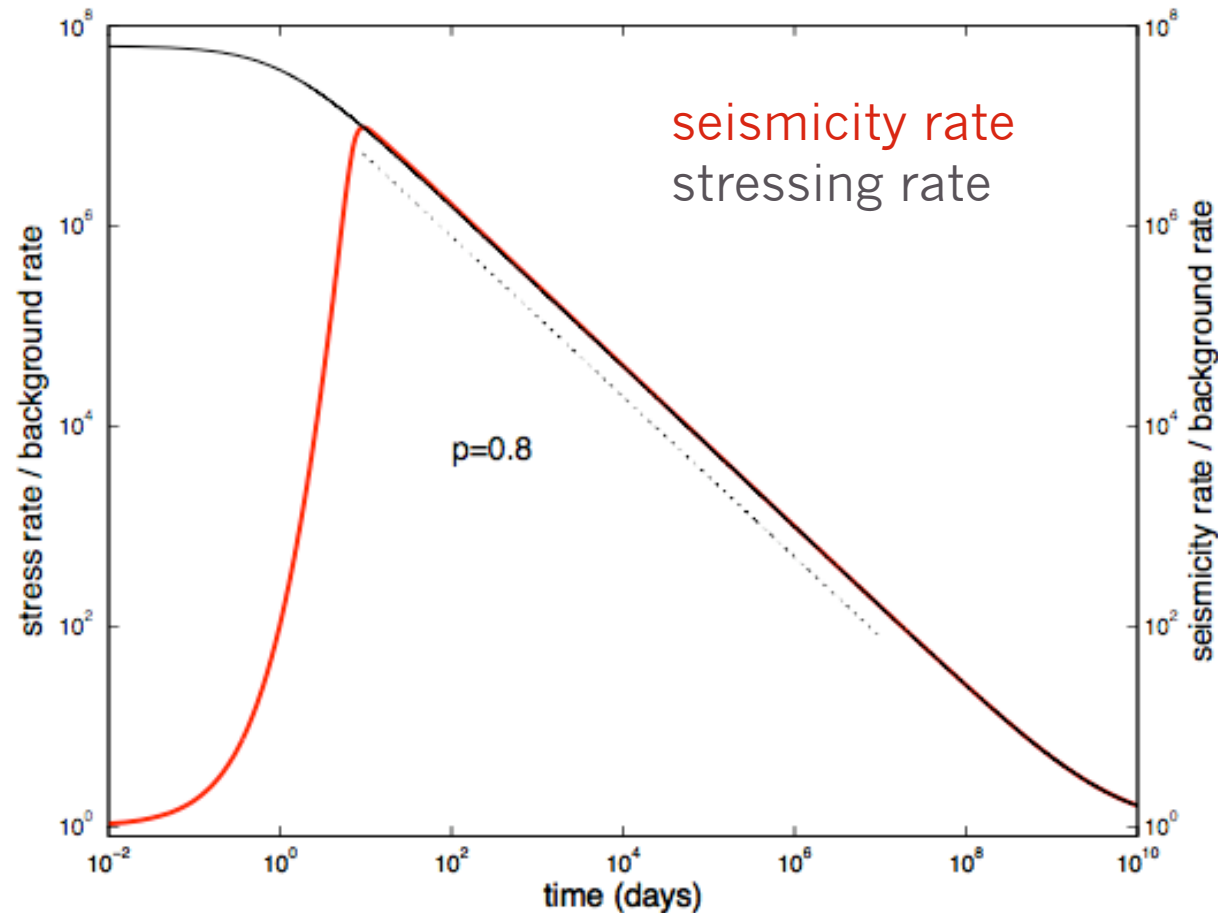
[Rice and Gu, 1983, Dieterich 1994, Schaff et al 1998, Perfettini and Avouac 2004, 2007; Wennerberg and Sharp 1997, Hsu et al 2006, Savage 2007a,b, ...]

- Afterslip also unloads other parts of the fault and modifies aftershock time decay
- we use the R&S model of *Dieterich [1994]* to model triggering due to afterslip $\sim d\tau/dt$

$$A\sigma \ln \left(\frac{R}{R_0} \right) + \frac{N\dot{\tau}_r}{r} = \tau$$

Aftershocks triggered by afterslip

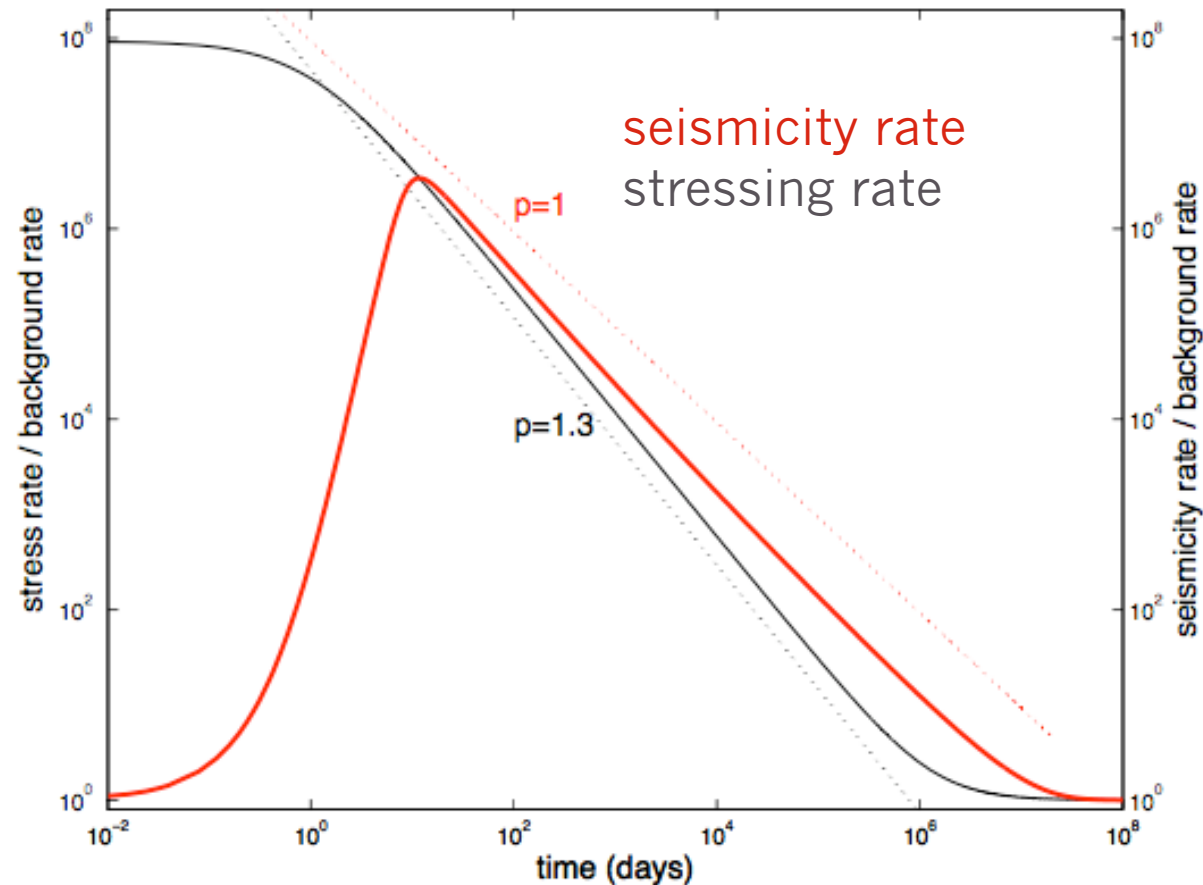
- numerical solution of R - τ relation assuming reloading by afterslip
- Stress rate $d\tau/dt \sim 1/(1+t/t^*)^q + \tau'_r$ with $q=0.8$



- when $p < 1$, $R(t) \sim d\tau/dt$ for «large times»

Aftershocks triggered by afterslip

- Afterslip reloading $d\tau/dt \sim \tau'_0/(1+t/t^*)^q$ with $q=1.3$



- apparent Omori exponent $p(t)$ decreases from 1.3 to 1

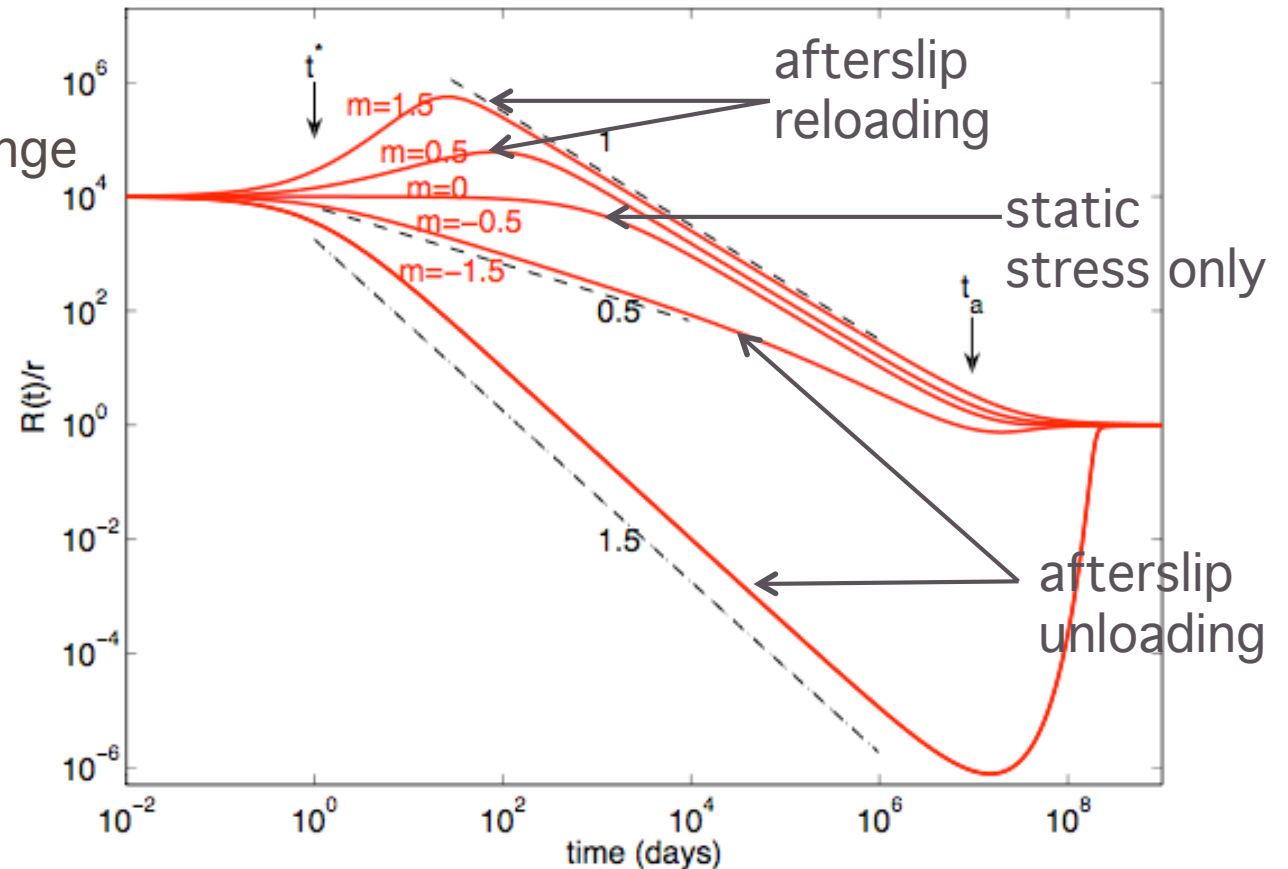
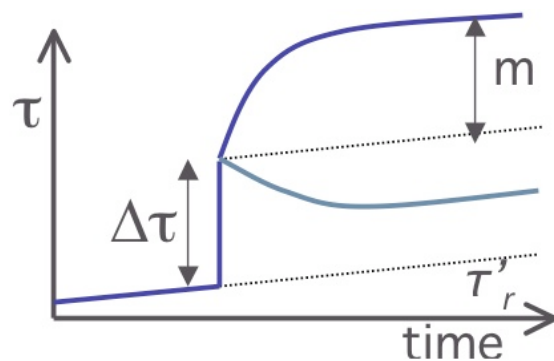
Aftershocks triggered by afterslip

- coseismic stress step + unloading or reloading by afterslip
- afterslip stress rate

$$d\tau/dt \sim \tau'_0/(1+t/t^*)$$

- total afterslip stress change

$$m = \tau'_0 t^* / A\sigma$$



- afterslip reloading ($m > 0$) : $p=1$
- afterslip unloading ($m < 0$) : $p=-m$

[Dieterich 1994]

[Dieterich 1994]

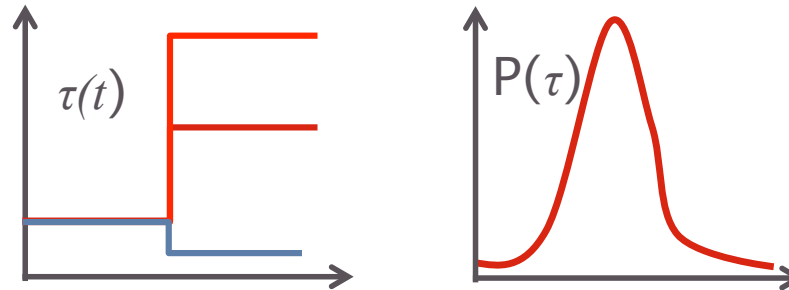
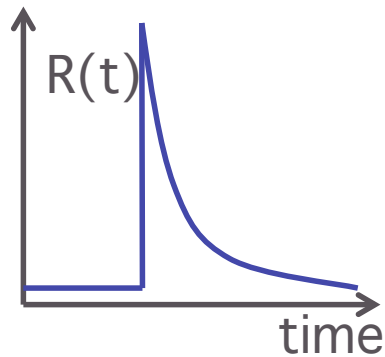
Conclusions: aftershocks triggered by afterslip

- R&S friction law can be used to model aftershock rate
- afterslip is likely a significant mechanism for aftershock triggering
- but less important than static stress changes, because slip (and σ) is smaller

! EQ rate does not scale with stress rate

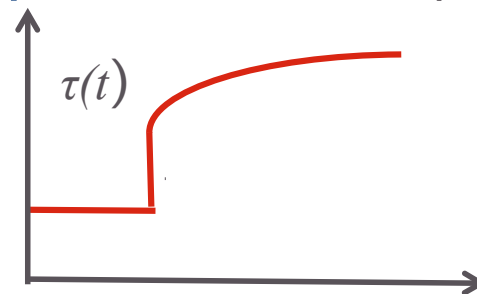
Conclusions: EQ triggering and R&S model

heterogeneous stress step



- short-time triggering $p < 1$, depends on stress heterogeneity
- long time quiescence

afterslip (+ coseismic step)



- Triggering or quiescence
- Omori law decay with $p < 0$ or $p > 1$, depends on amplitude and time decay of stress-rate

Conclusions

☺ Rate & state fits well observations

aftershock rate (t, r), afterslip, slow slip events

☹ little constrain on mechanisms and parameters

static / afterslip triggering?

stress change?

velocity weakening / strengthening?

☹ complex behavior, yet very simplistic model

1 slider block / continuous model

no inertia

no heterogeneity of friction law parameters

no secondary aftershocks