# Rheology of granular flows: Role of the interstitial fluid


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Motivations :
debris flows, landslides, avalanches, silo

Not Fault!!!

Low level of pressure:

10-100 kPa in natural events
0.1-1 kPa in the experiments

=> Rigid and non breakable particles

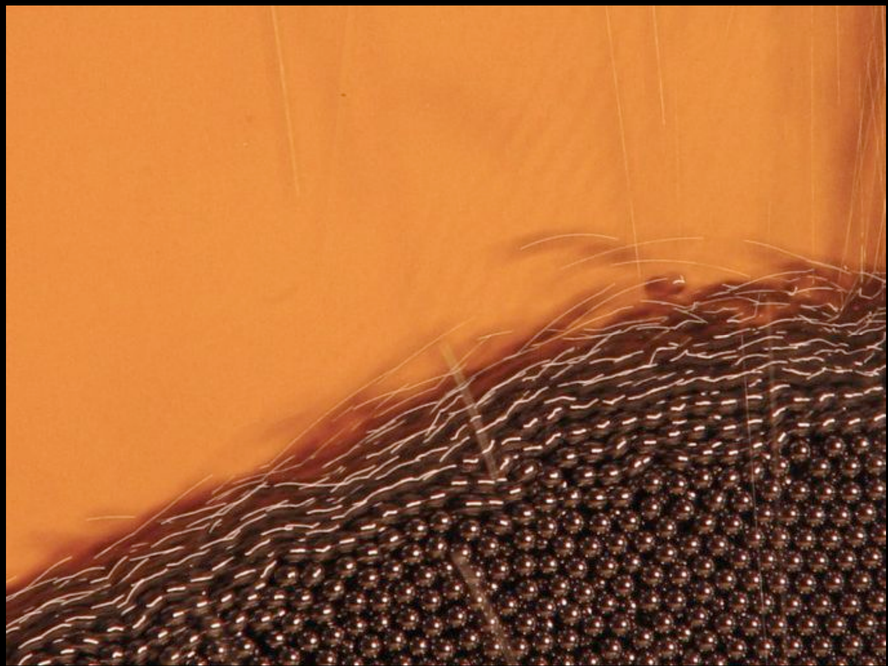
Particles of different sizes
+ liquid (non newtonian)
+ complex topography
+ unsteady flows ...

? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?

In this talk :

1) rheology of dry granular flows ?...

2) What happen when coupling with the interstitial fluid matters ...?

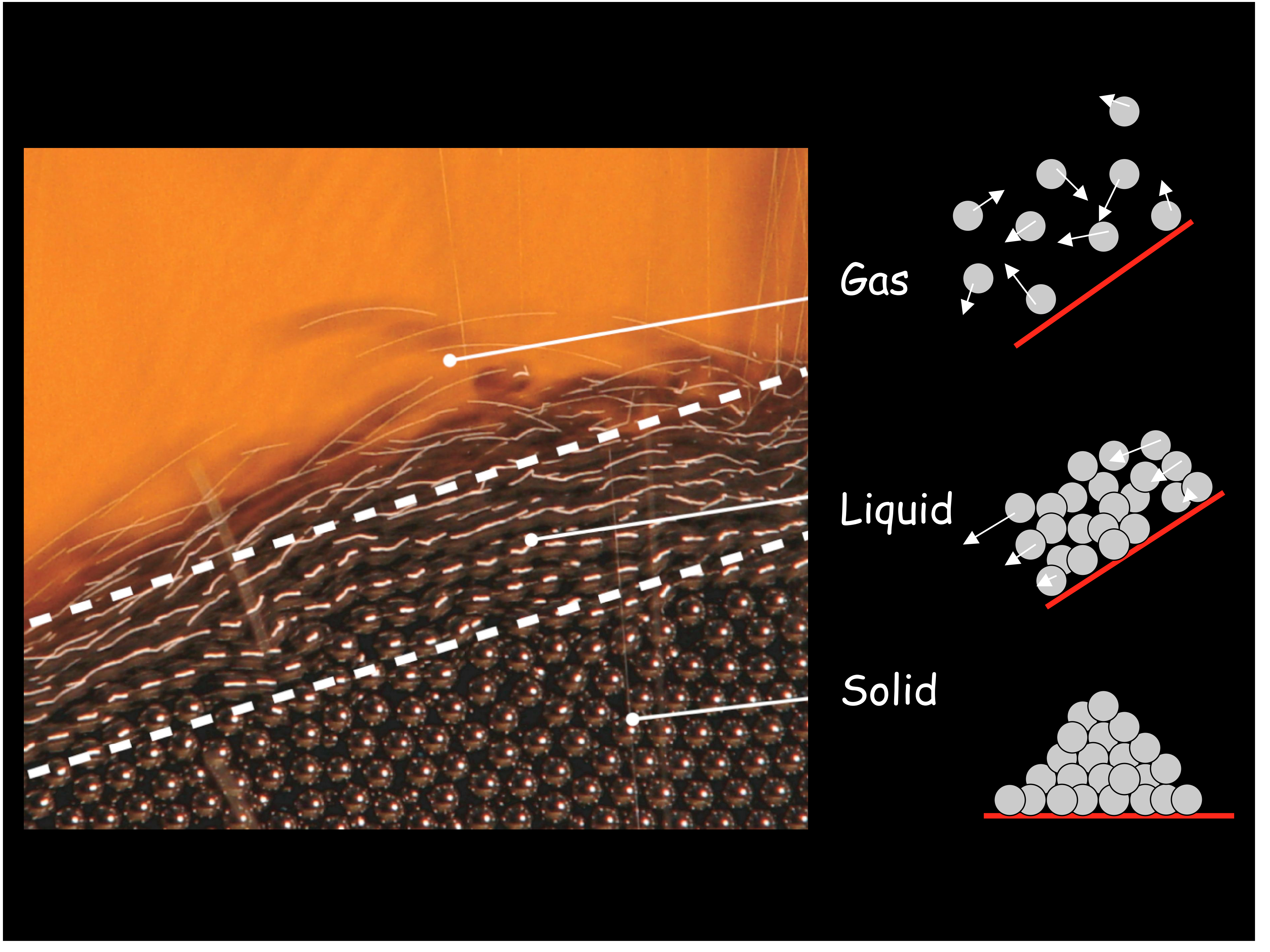


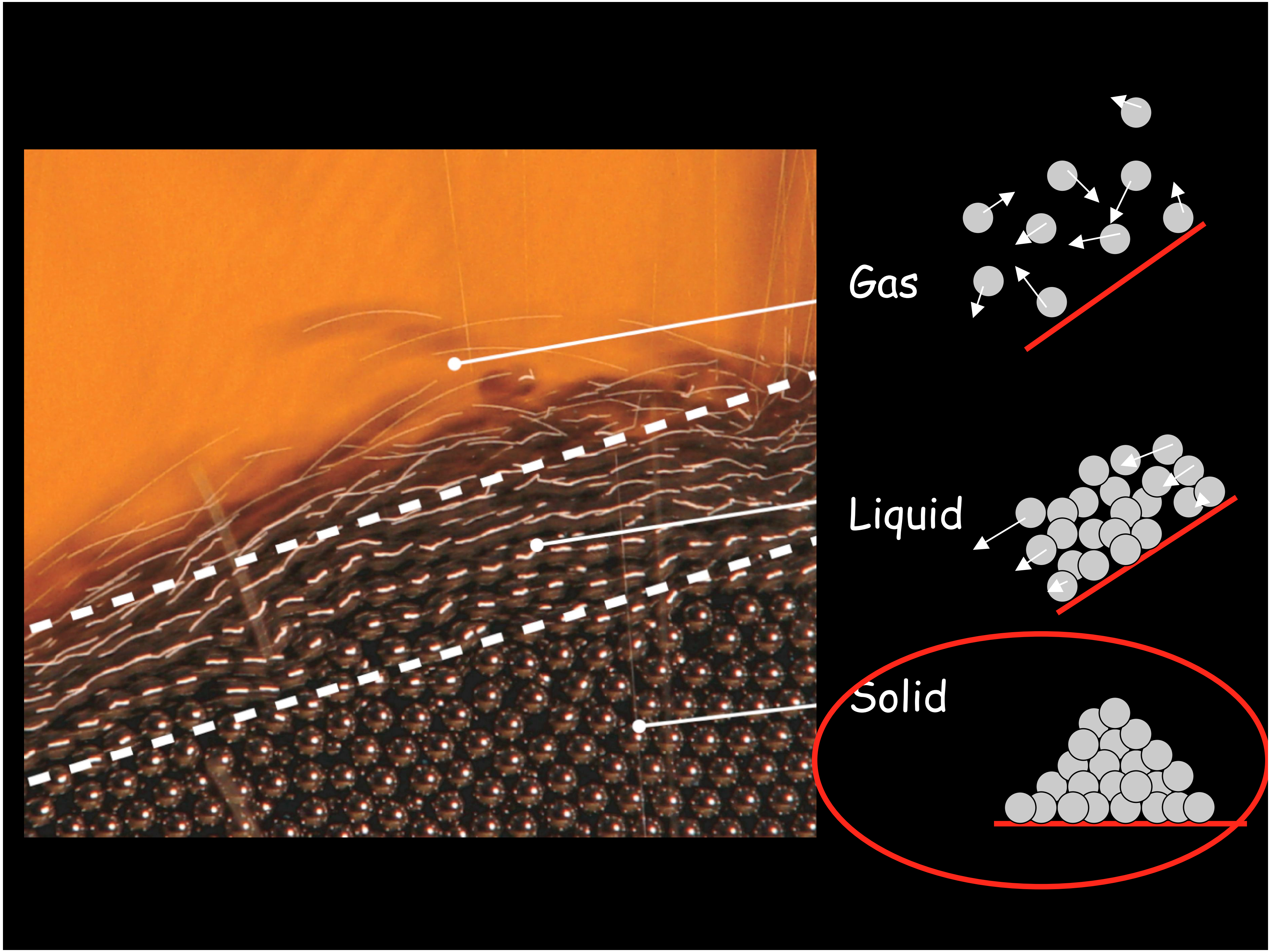
Dry granular material:
Collection of grains

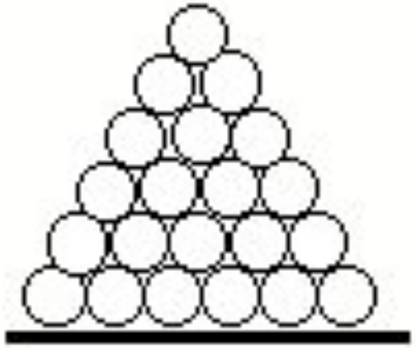
No cohesion
No brownian motion
No fluid interaction

... Only contact interactions

But not so easy...


Gas
Liquid
Solid


Gas
Liquid
Solid



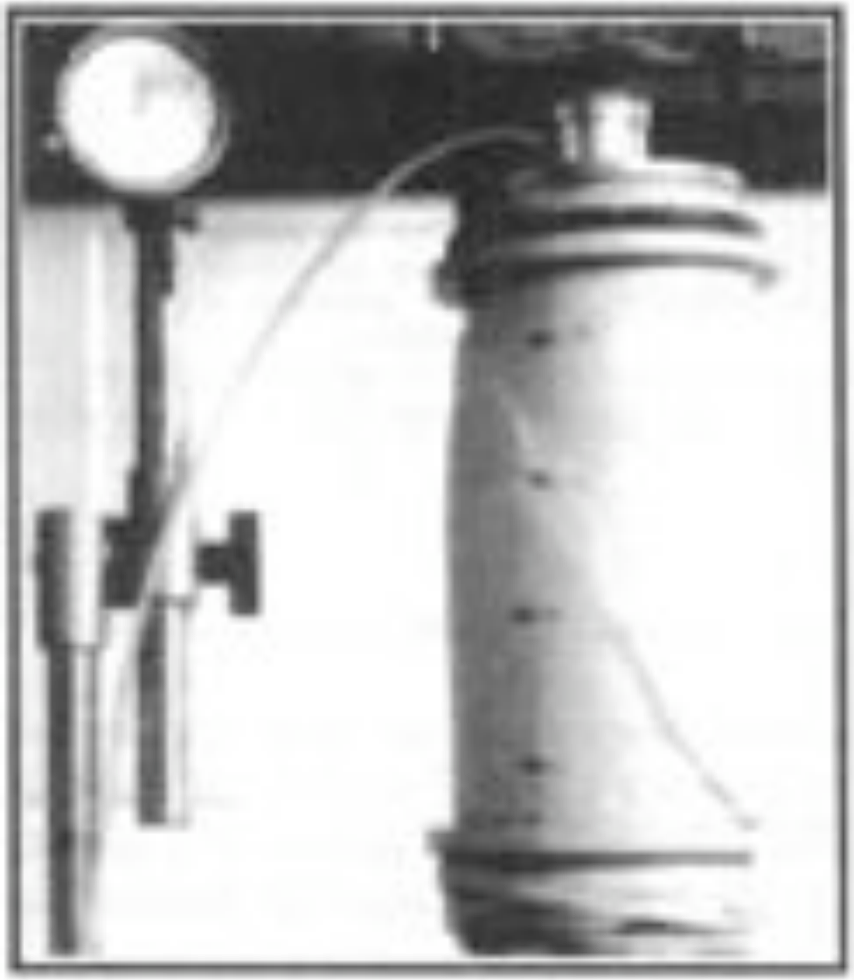

# Quasi-static deformations : Soil mechanics and plasticity

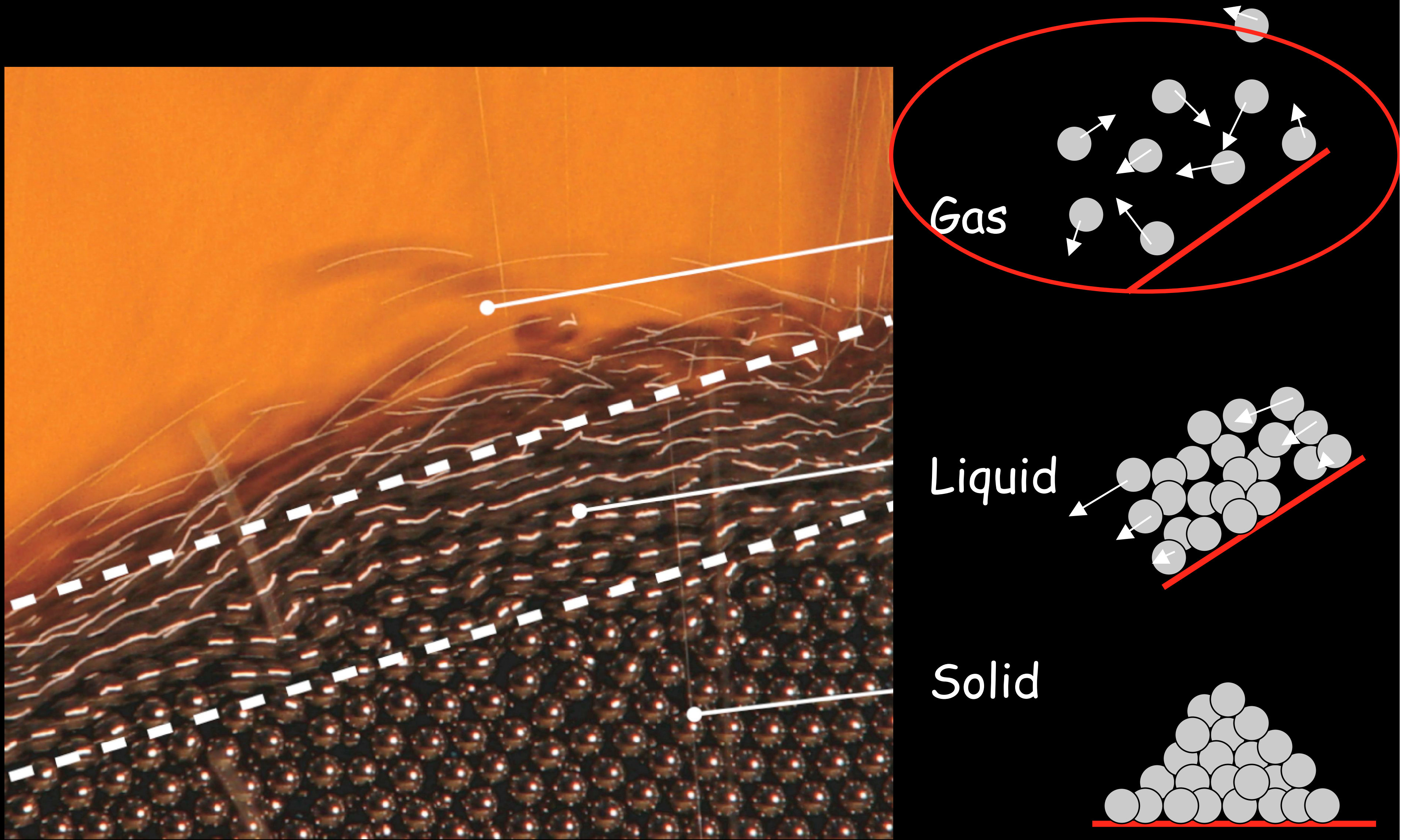
Focus on initial deformation

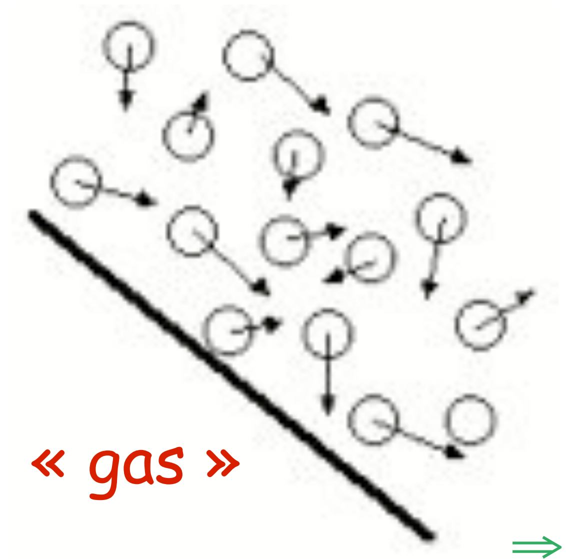
What happens :

⇒ at large deformations ?

⇒ for fast deformations ?



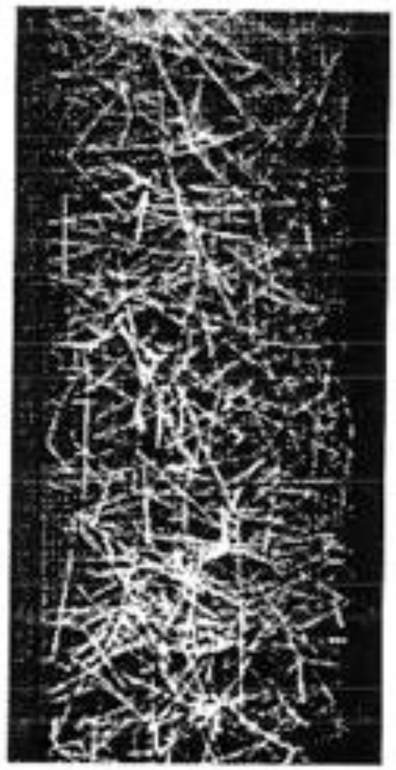

Gas
Liquid
Solid




# Kinetic theory for rapid granular gases

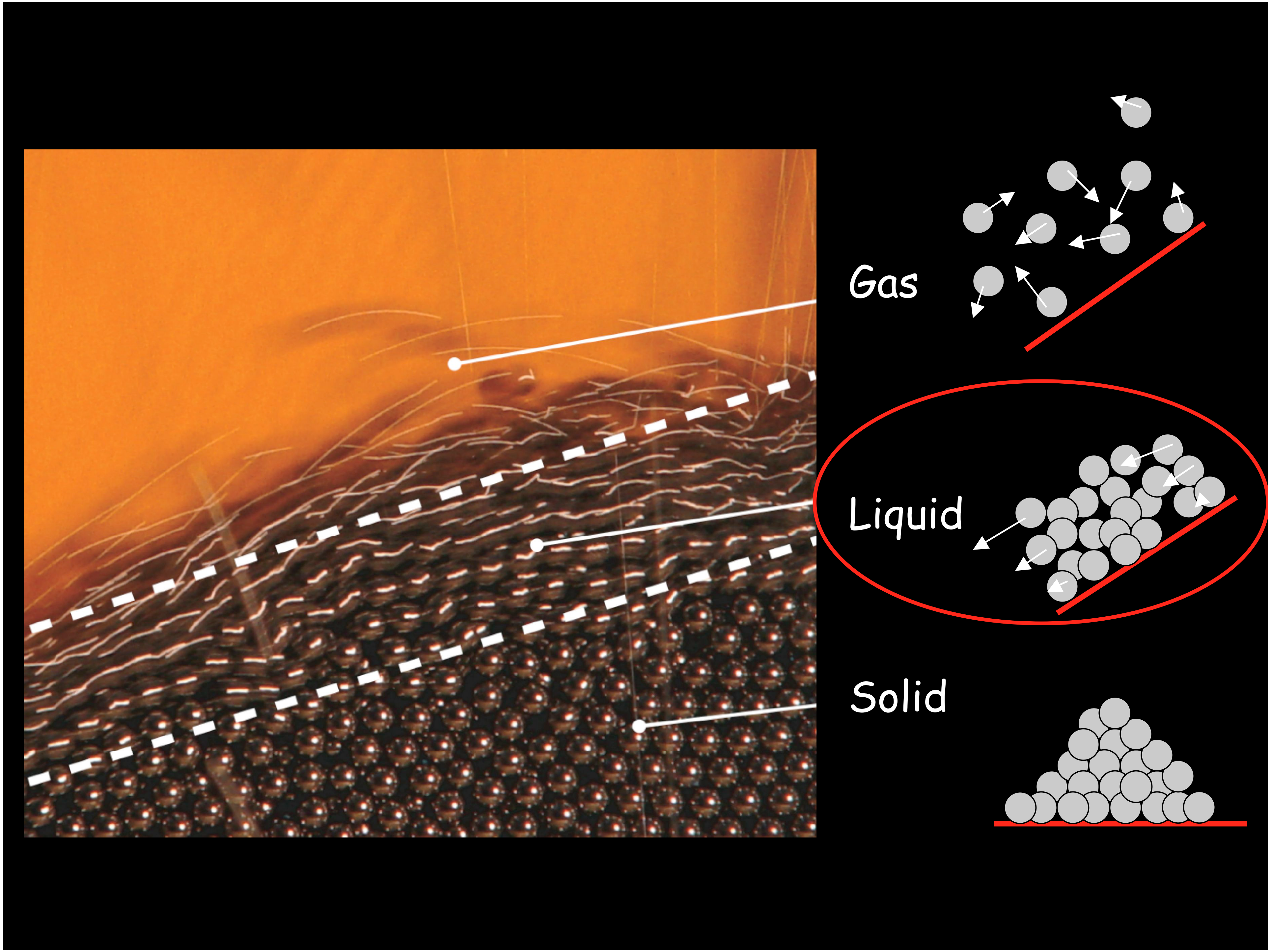
Binary collisions+inelastic collisions

⇒ constitutive equations coupling
Density, velocity and granular temperature

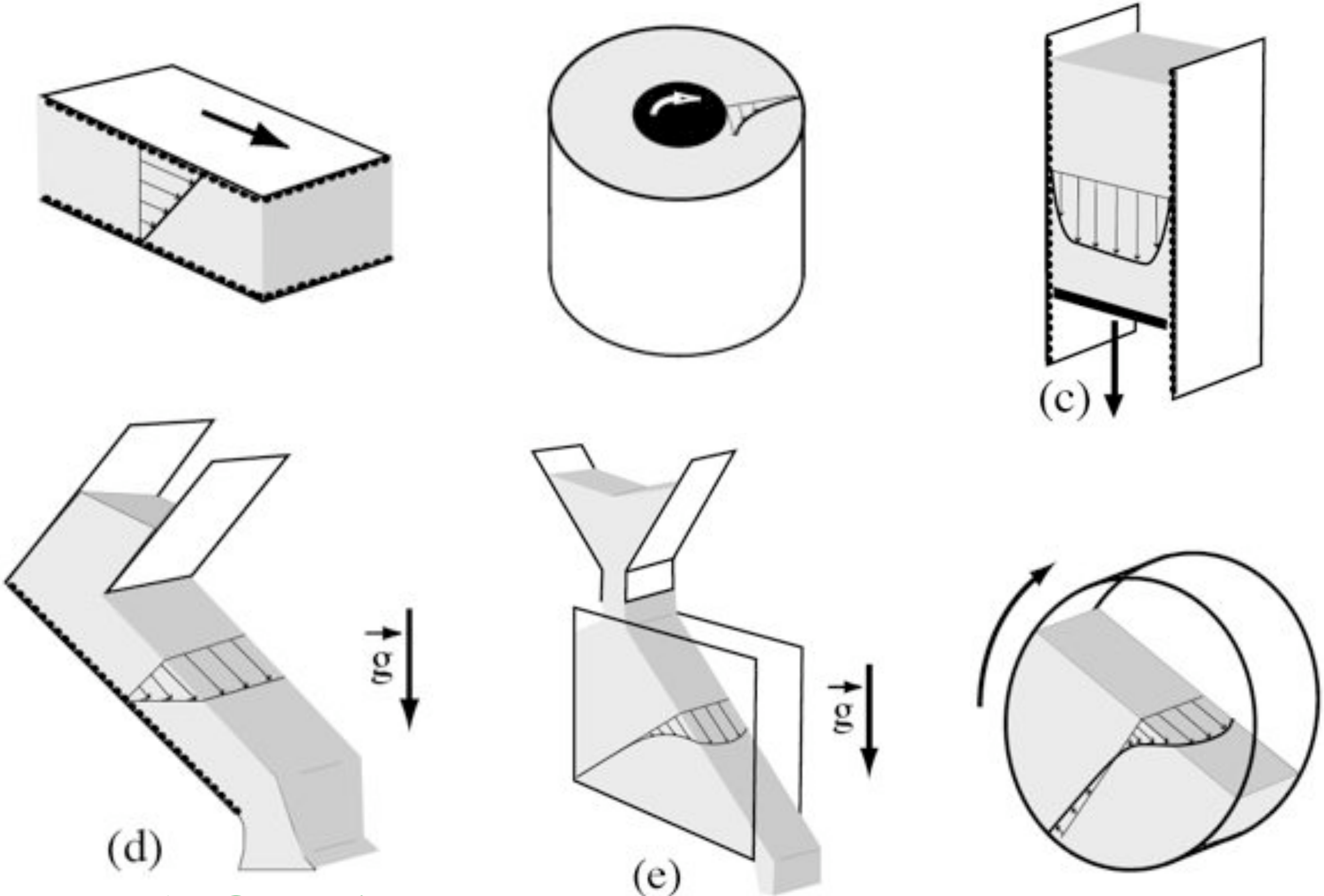


But if not enough energy is injected:

⇒ finite duration contact,
⇒ multiple contact,

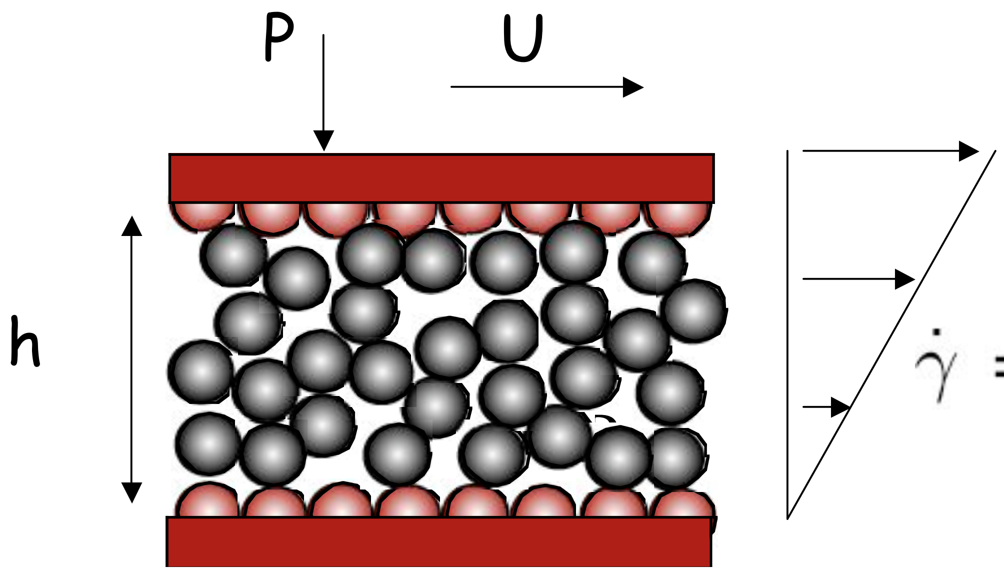

Gas
Liquid
Solid

## Different flow configurations studied both experimentally and numerically




GDR Midi, Eur. Phys. J 04

# plane shear under controlled normal stress




$\dot{\gamma} = U/h$

Lois et al 2005
Da Cruz et al, PRE 05
GdR Midi, Eur. Phys. J 04

One imposes P and $\dot{\gamma}$

Shear stress $\tau$?
Volume fraction $\phi$?

A single dimensionless number
(inertial number)

$$I = \frac{\dot{\gamma} d}{\sqrt{P/\rho_s}}$$

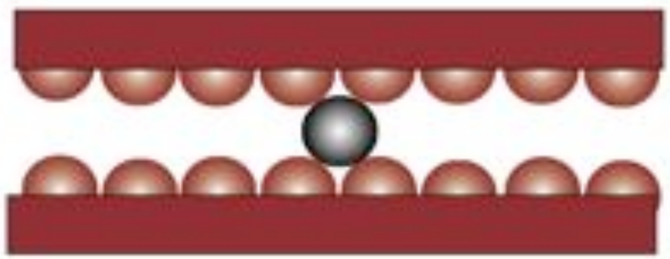
(Savage 84,
Ancey et al 99)

Inertial number $$I = \frac{\dot{\gamma} d}{\sqrt{P/\rho_s}}$$

* ratio between 2 times :

$1/\dot{\gamma}$ : time scale of the mean shear

$d/\sqrt{P/\rho_s}$ : microscopic time for rearrangement

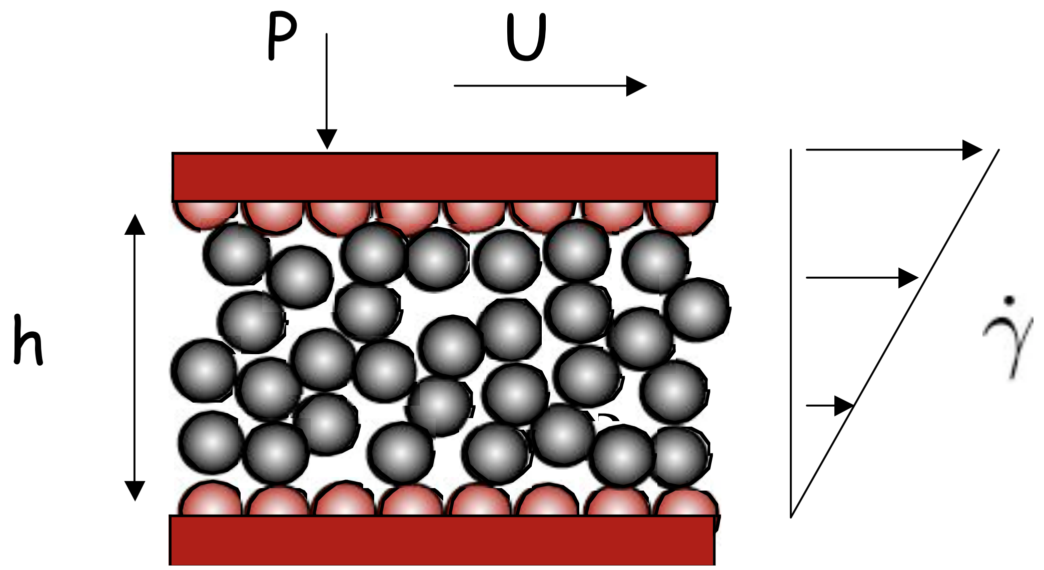


« quasi-static » « liquid » « gas »

0 1

$$I = \frac{\dot{\gamma}\, d}{\sqrt{P/\rho}}$$

Da Cruz et al, PRE 05
GdR Midi Eur. Phys. J 04

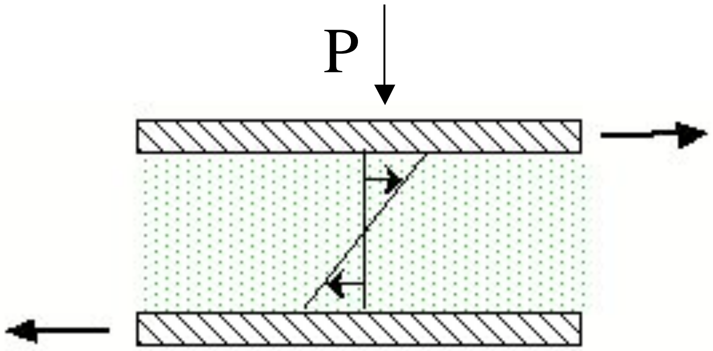



One imposes P and $\dot{\gamma}$

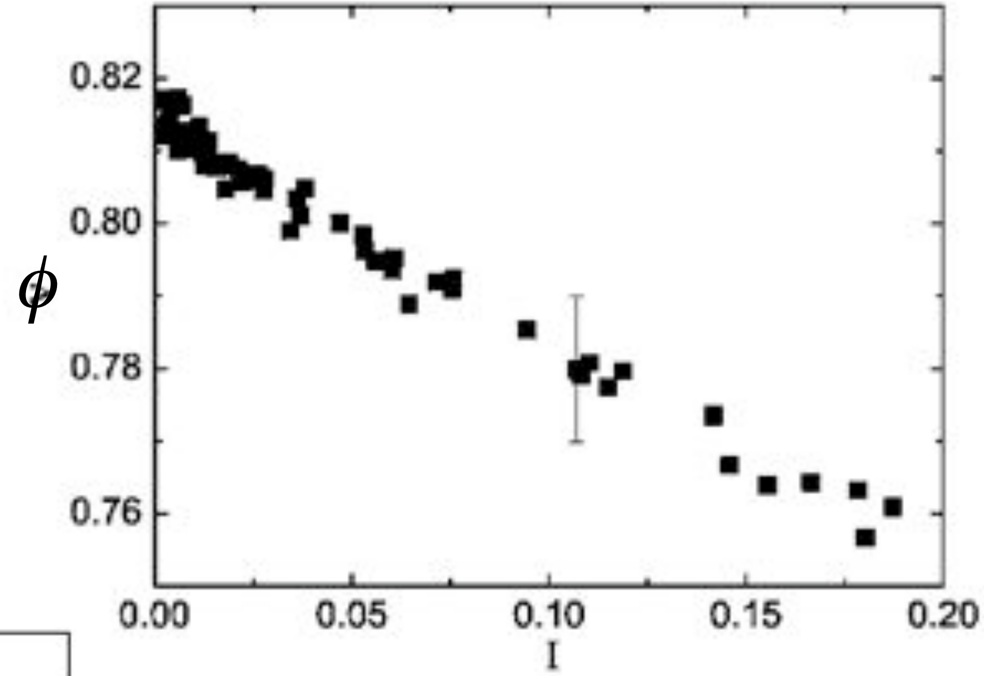
Shear stress $\tau$?
Volume fraction $\phi$?

$$\tau = \mu(I) P$$

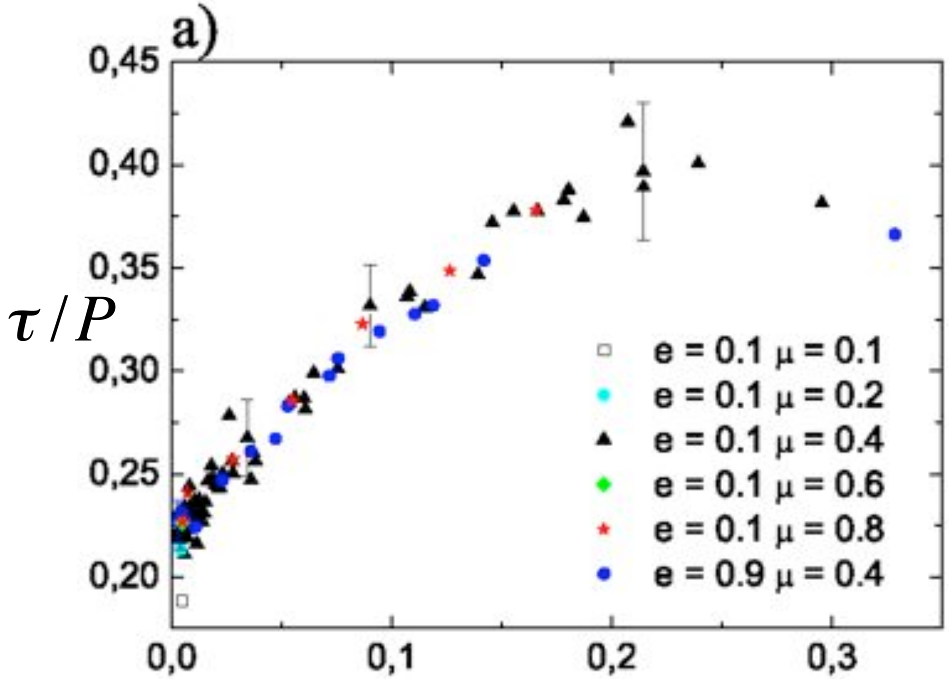
$$\Phi = \Phi(I)$$




$$\tau = \mu(I) P$$

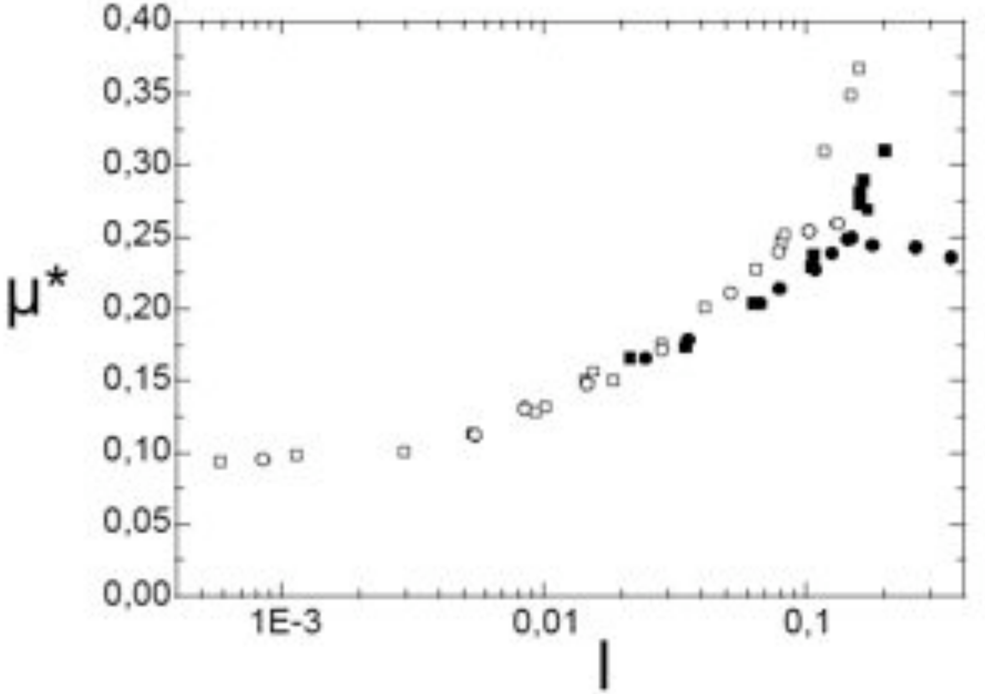



$$\phi = \phi(I)$$



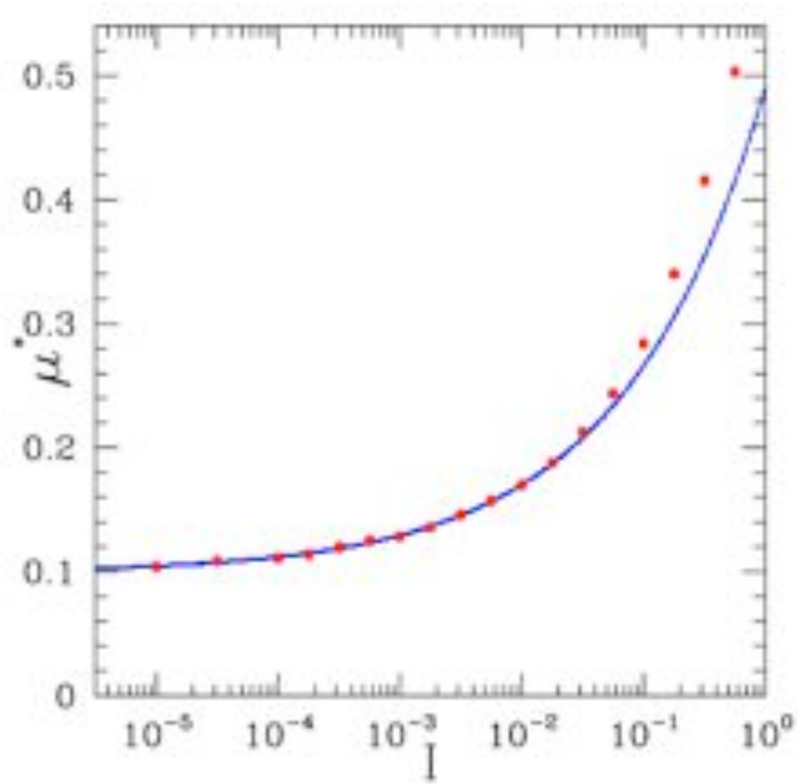

$$I = \frac{\dot{\gamma}\, d}{\sqrt{P/\rho}}$$

# remark: No velocity weakening

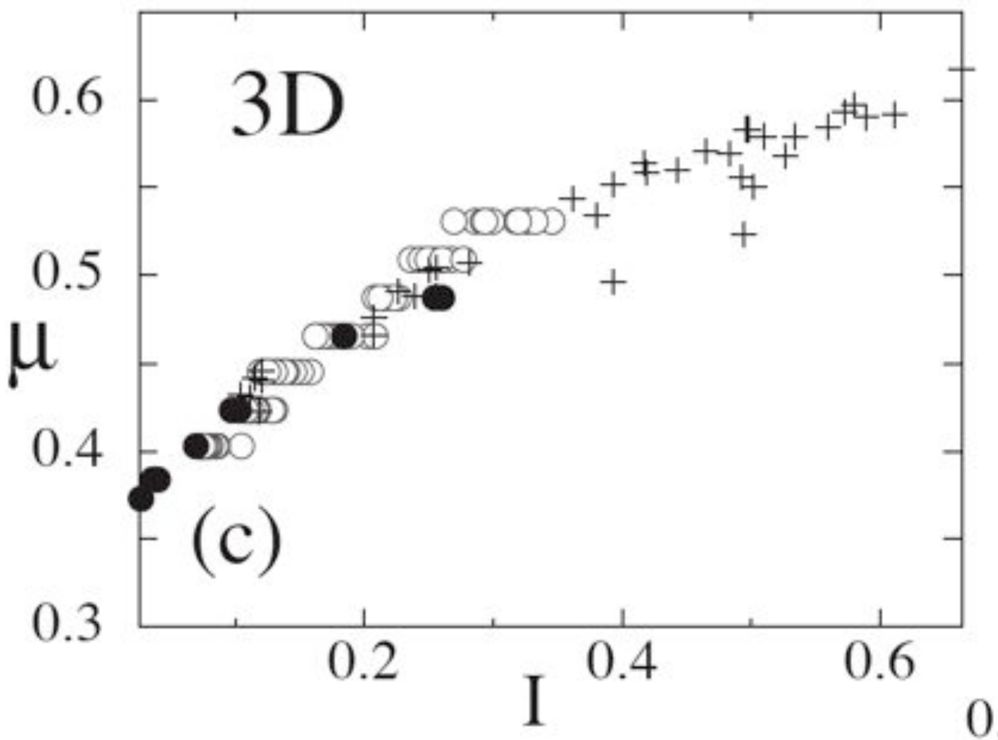



Dacruz et al PRE 05

Peyneau & Roux PRE 08



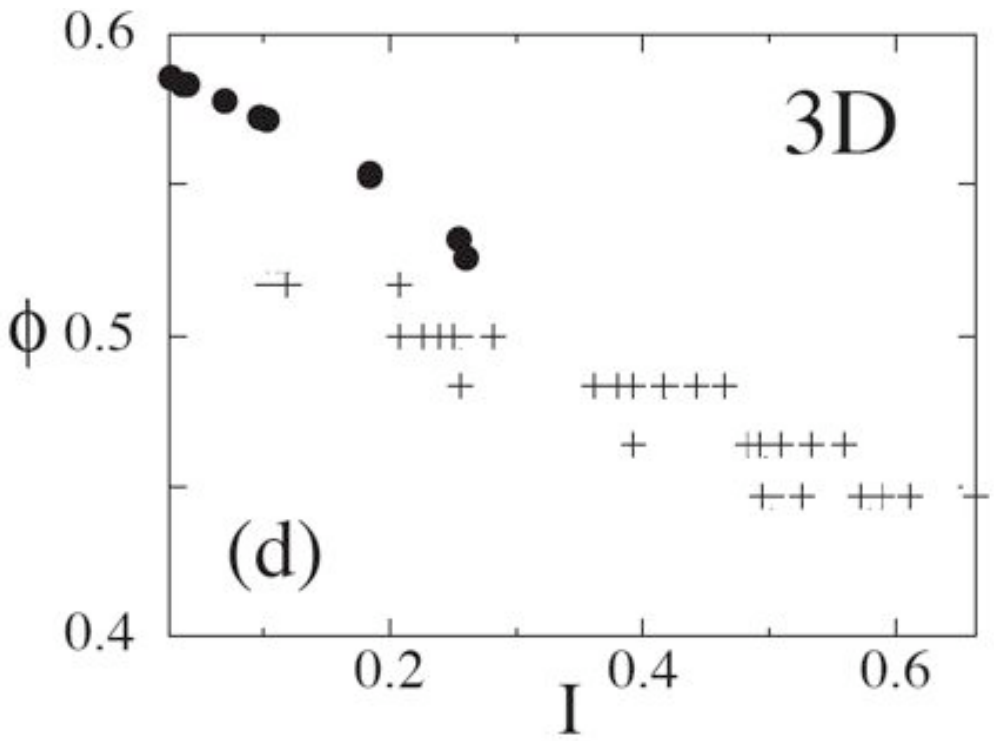
Forterre, Pouliquen ARFM 08




For spheres

Data from

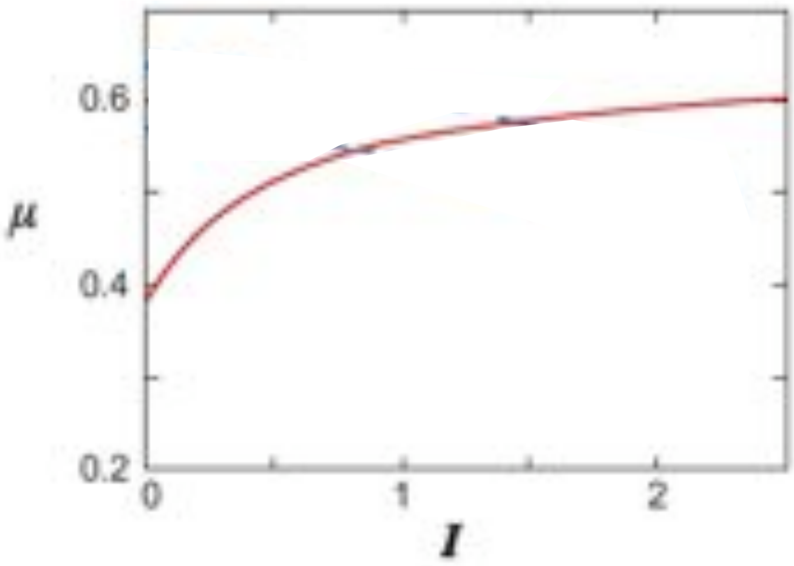
Inclined plane exp. (Pouliquen 99)
Inclined plane simulations
(Baran et al 2006)
Annular shear cell exp.
(Sayed, Savage JFM, 84)

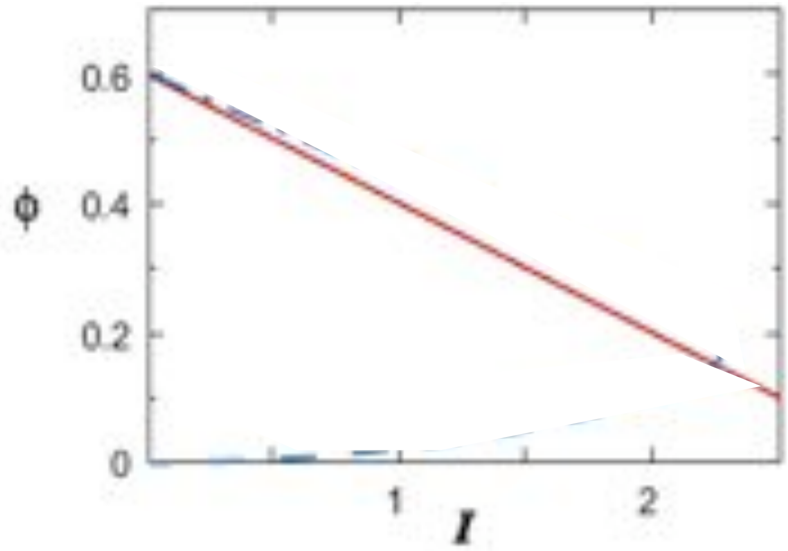


## An empirical friction law:

$$\tau = \mu(I)P$$

$$\Phi = \Phi(I)$$



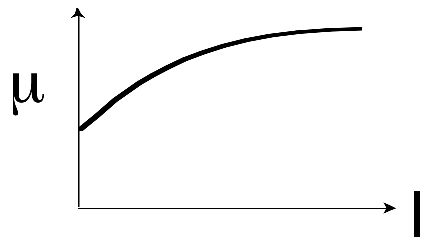

$$\mu(I) = \mu_s + \frac{\mu_2 - \mu_s}{I_0/I + 1}$$

$$\phi(I) = \phi_{max} - AI$$

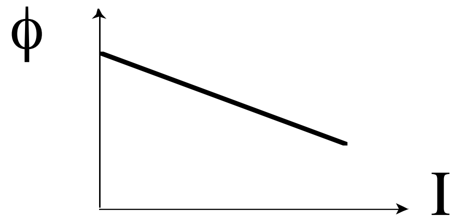
## Constant pressure

$$I = \frac{\dot{\gamma} d}{\sqrt{P/\rho_s}}$$

$$\tau = \mu(I) P$$




$$\Phi = \Phi(I)$$




$$\Phi(I) = f_1^{-1}(1/I^2) \qquad \mu(I) = I^2 f_2(f_1^{-1}(1/I^2))$$

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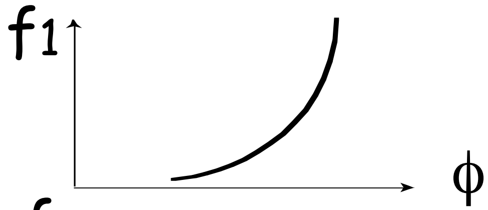
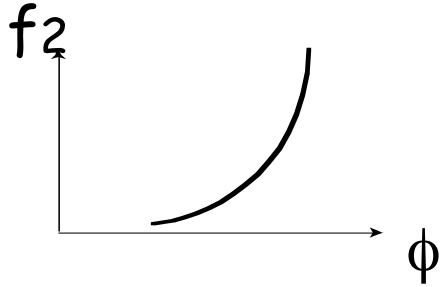
## And shear at constant volume fraction ??

Bagnold Proc. R. Soc 54
Lois et al PRE 07
Lemaitre PRE 05
Da Cruz et al PRE 05

$$\tau = f_1(\Phi) \rho_s d^2 \dot{\gamma}^2$$

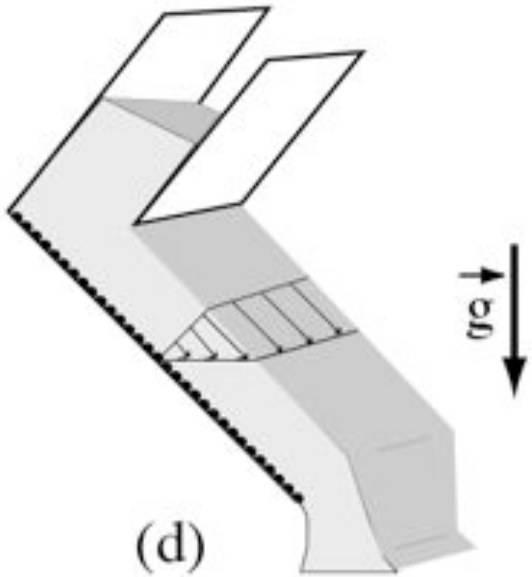
$$P = f_2(\Phi) \rho_s d^2 \dot{\gamma}^2$$



$$\tau = \mu(I)P$$
$$\Phi = \Phi(I)$$

allows to describe (not perfectly) velocity profiles on inclined plane,



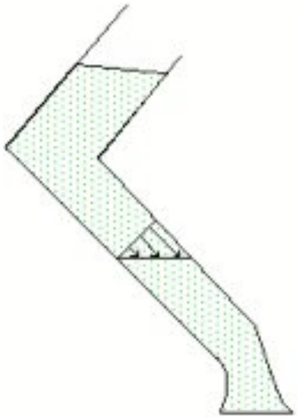
(d)

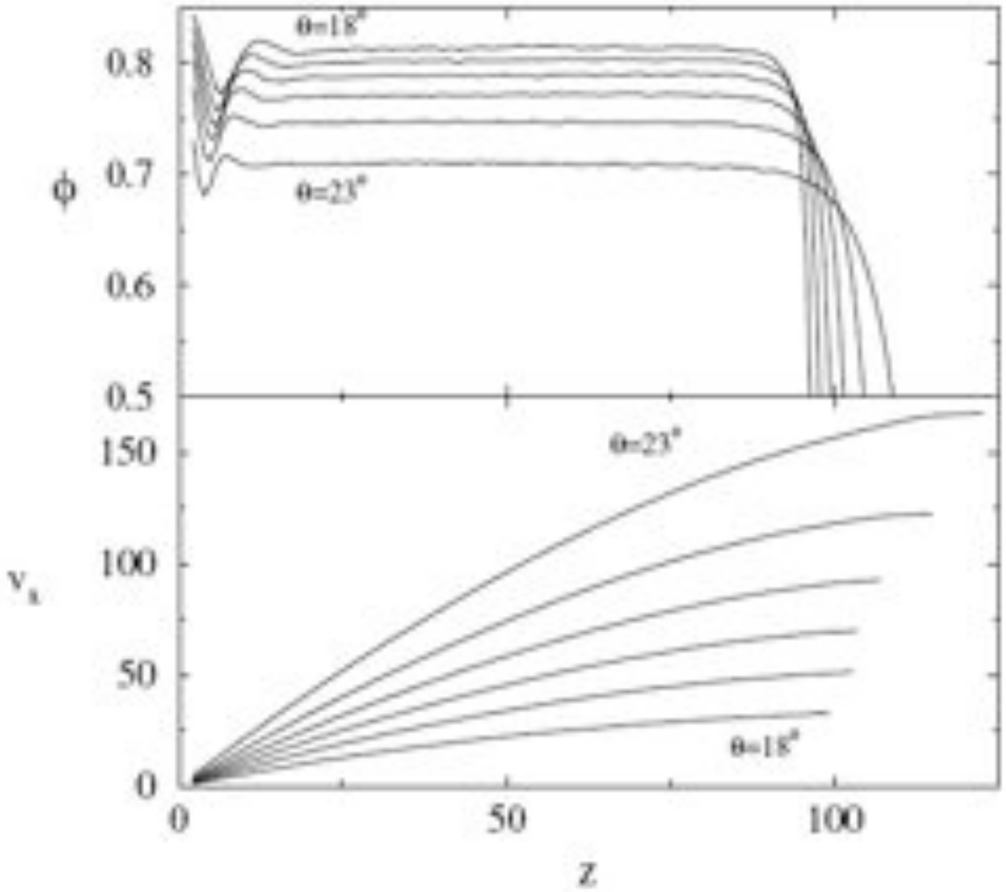
Let's go further...

# Predicted velocity and volume fraction profiles

Rheology $\mu(I)$ predicts - $V \propto h^{1.5} - (h-z)^{1.5}$

d $\Phi$=cte





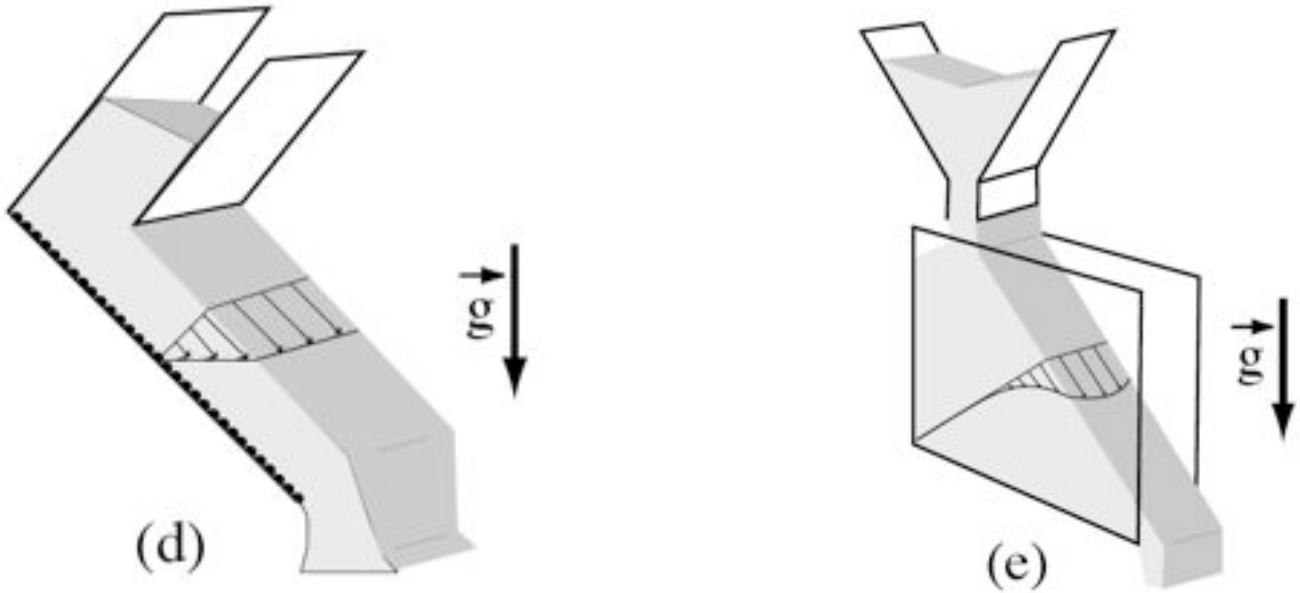

Gdr Midi et al, 2004,
Da cruz et al 2002,
Silbert et al 2001

-Pb with thin flows and close to free surface

Gdr Midi et al, 2004,
Da cruz et al 2002,
Rajchenbach 2003

$$\tau = \mu(I)P$$
$$\Phi = \Phi(I)$$

allows to describe (not perfectly) velocity profiles on inclined plane, on pile,...




Let's go further...

# 3D generalisation: a visco-plastic model (Jop et al Nature 06)

assumptions : 1) P isotropic

2) $\dot{\gamma}_{ij}$ and $\tau_{ij}$ are colinear

(Savage 83, Goddard 86, Schaeffer 87,...)

$$\tau_{ij} = \frac{\mu(I)P}{||\dot{\gamma}||}\dot{\gamma}_{ij}$$

Effective viscosity

$$\dot{\gamma}_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}$$

$$||\dot{\gamma}|| = \sqrt{1/2\dot{\gamma}_{ij}\dot{\gamma}_{ji}}$$

# 3D generalisation of the friction law : granular flows as a viscoplastic fluid

(Jop et al Nature 06)

assumptions:

1) P isotropic
2) $\dot{\gamma}_{ij}$ and $\tau_{ij}$ are co-linear

(Savage 83, Goddard 86, Schaeffer 87,...)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} = 0,$$

$$\rho_s \phi \left( \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial z} \right) = \rho_s \phi g \sin\theta - \frac{\partial P}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z},$$

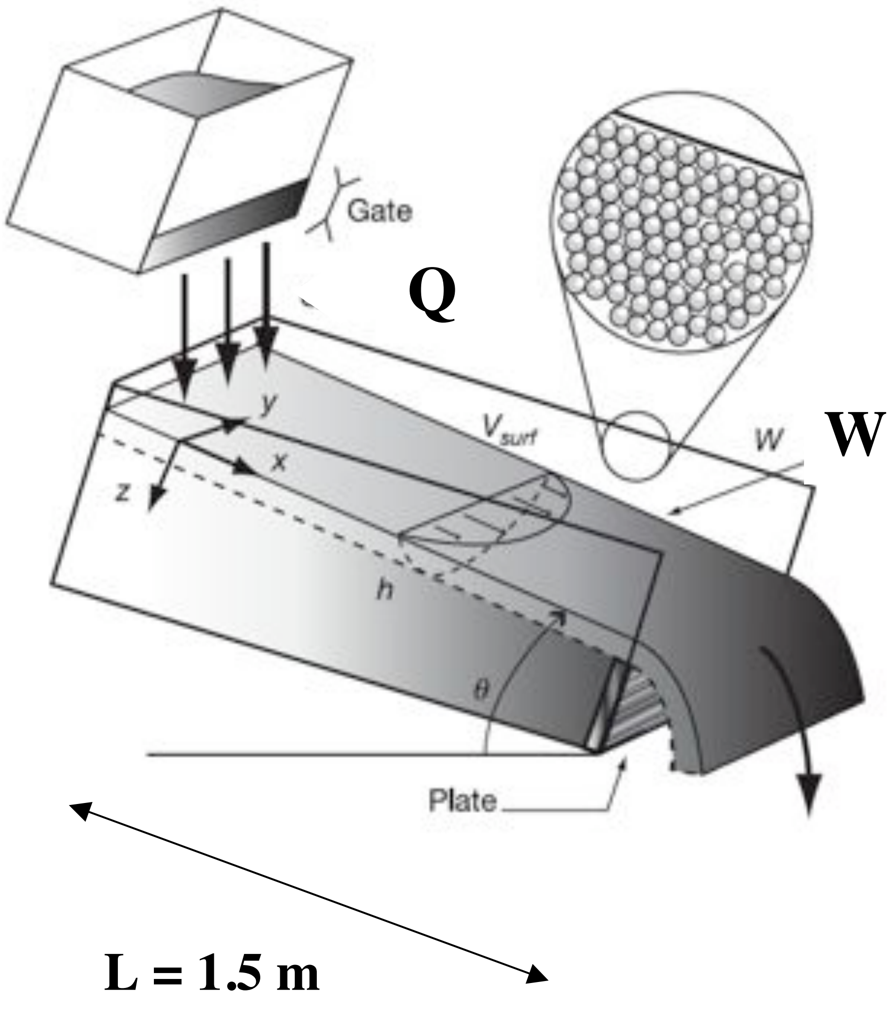
$$\rho_s \phi \left( \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial z} \right) = -\rho_s \phi g \cos\theta - \frac{\partial P}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z},$$

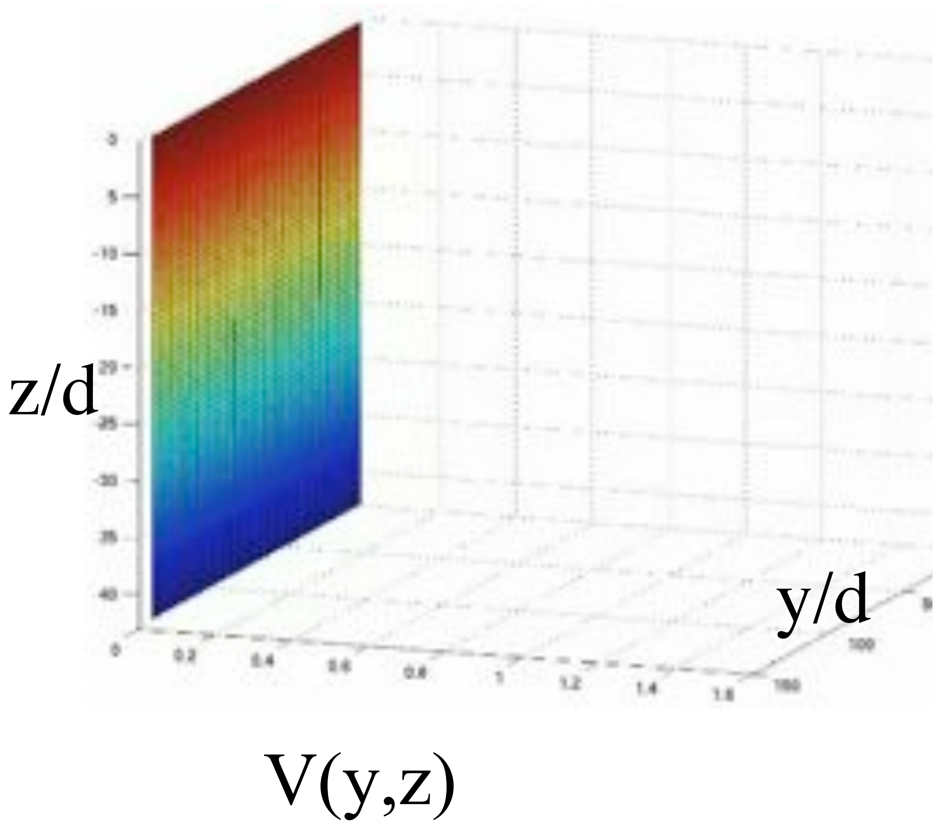
$$\tau_{ij} = \frac{\mu(I)P}{\dot{\gamma}} \dot{\gamma}_{ij} \qquad I = \frac{\dot{\gamma} d}{\sqrt{P/\rho_s}}$$

Pressure dependent viscosity

# flows on a heap : a full 3D problem

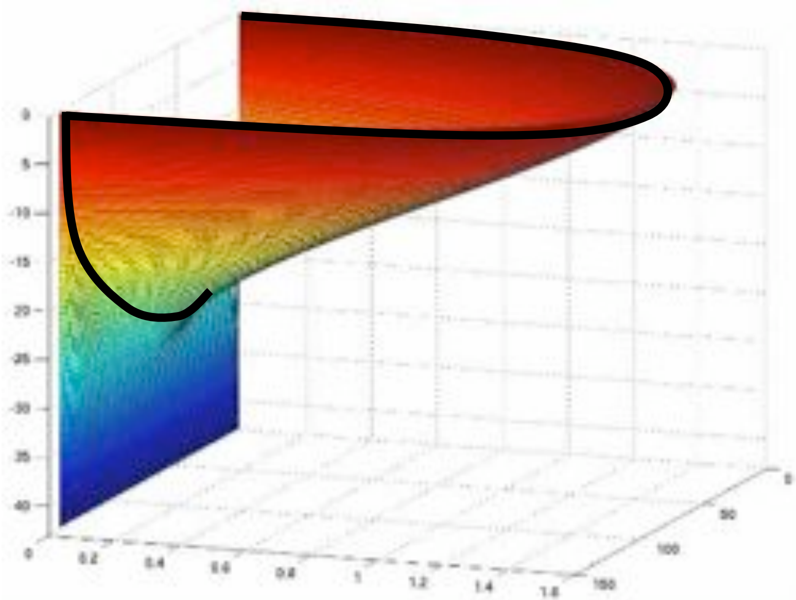
(P. Jop et al Nature 06)

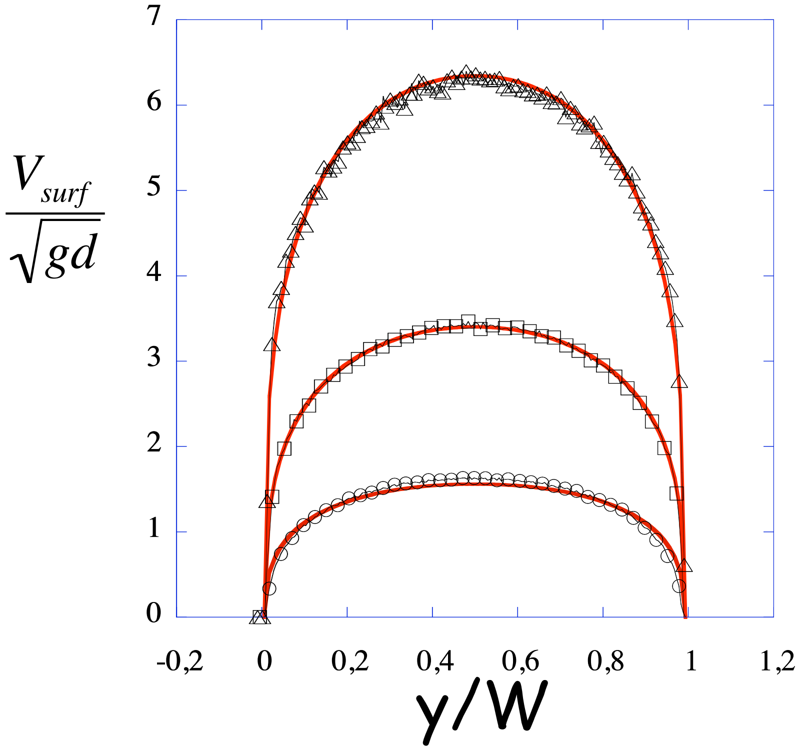


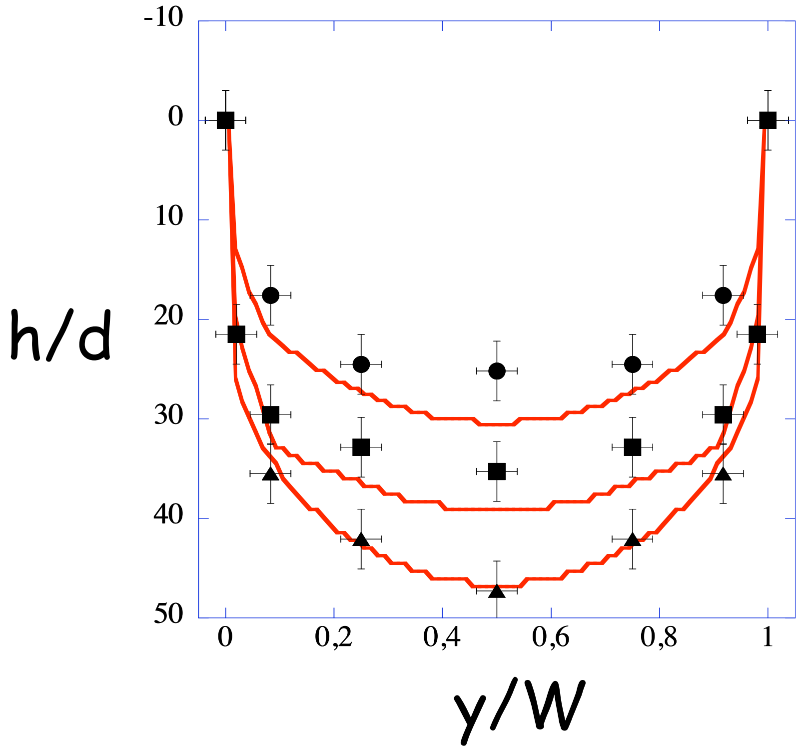



# Flow between rough lateral walls:

Jop et al , Nature 2006

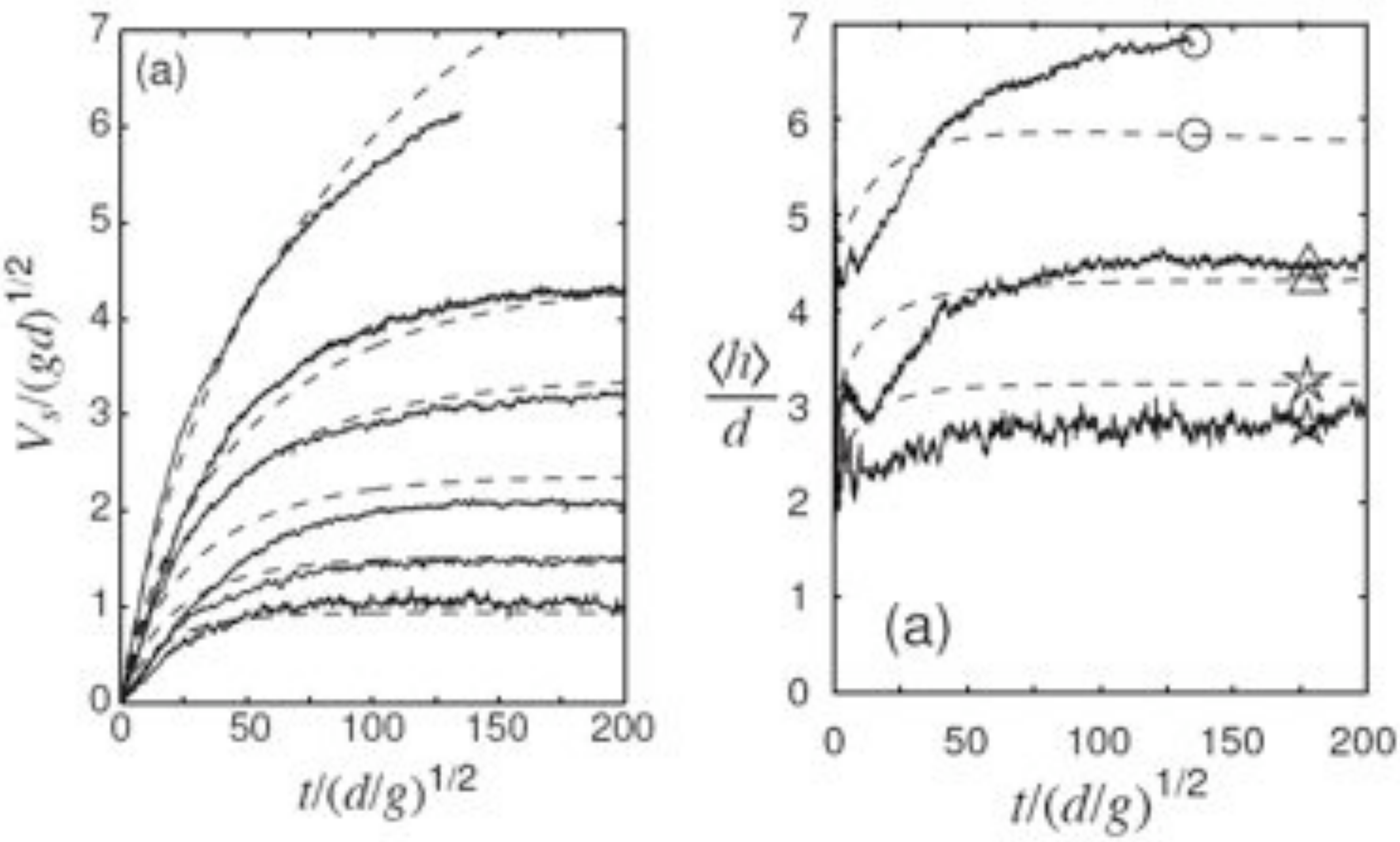




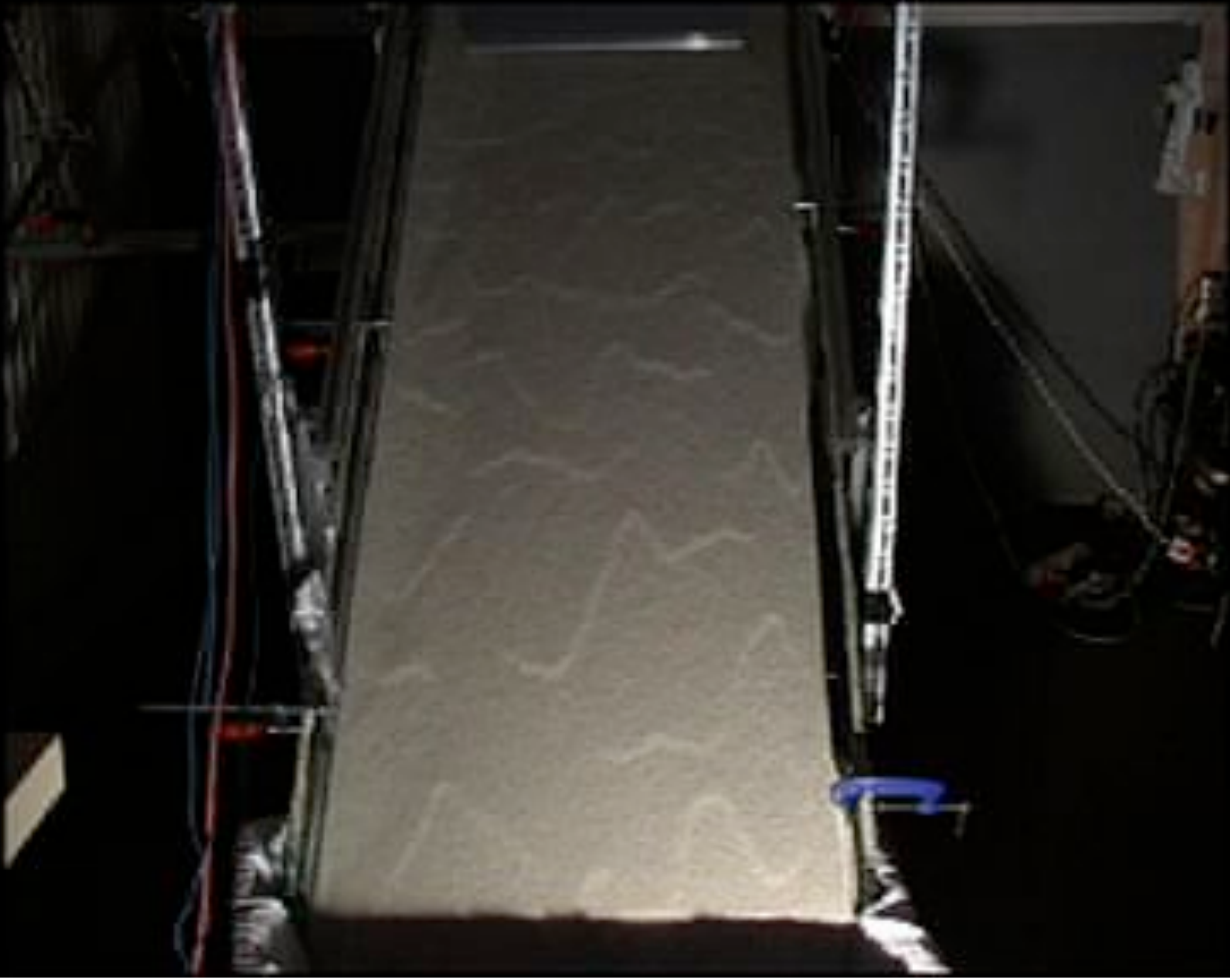
Jop et al,
Phys. Fluids 2007

Initiation of the flow?

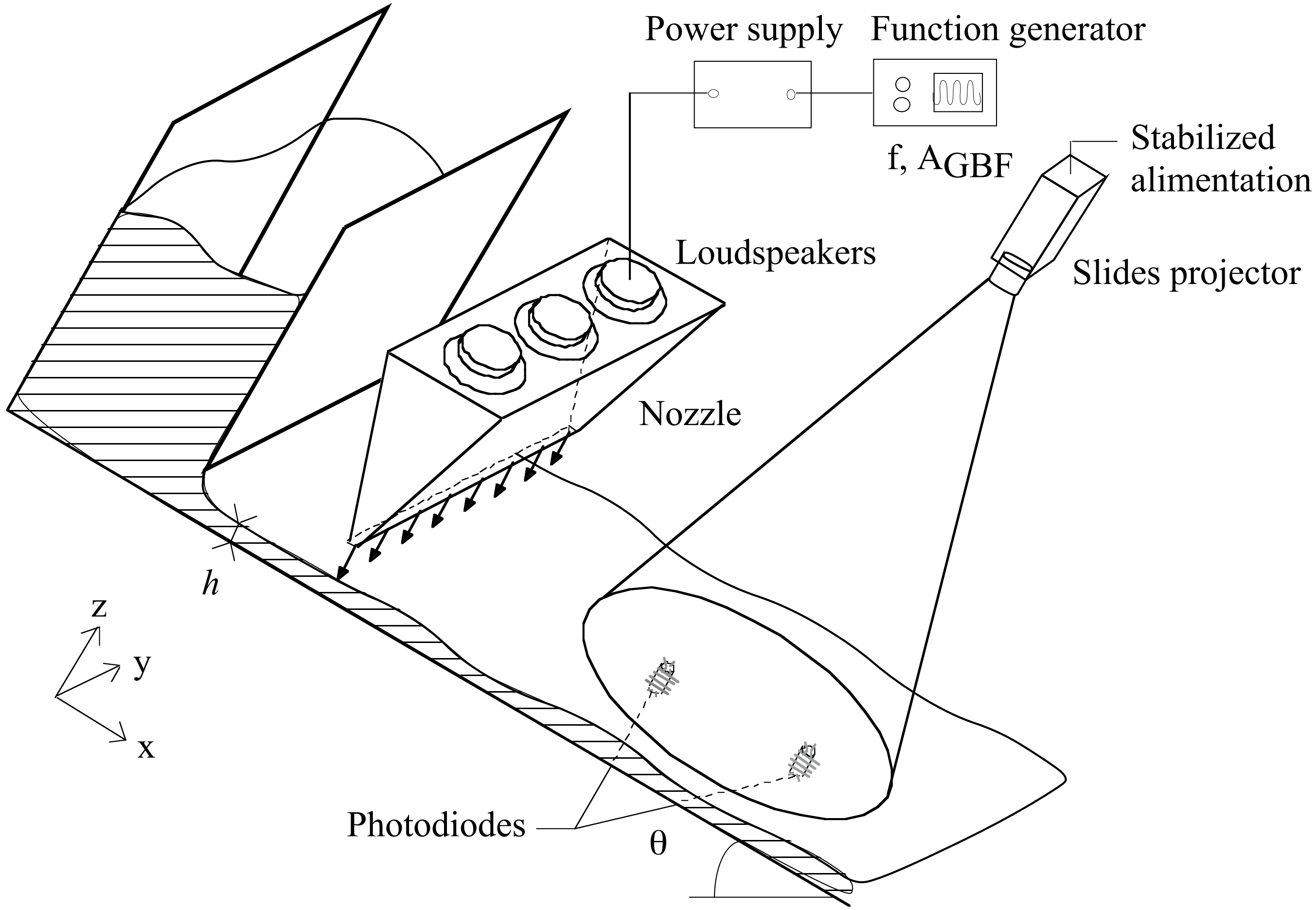


# Long wave instability in granular flows

(Y. Forterre, JFM 06 )



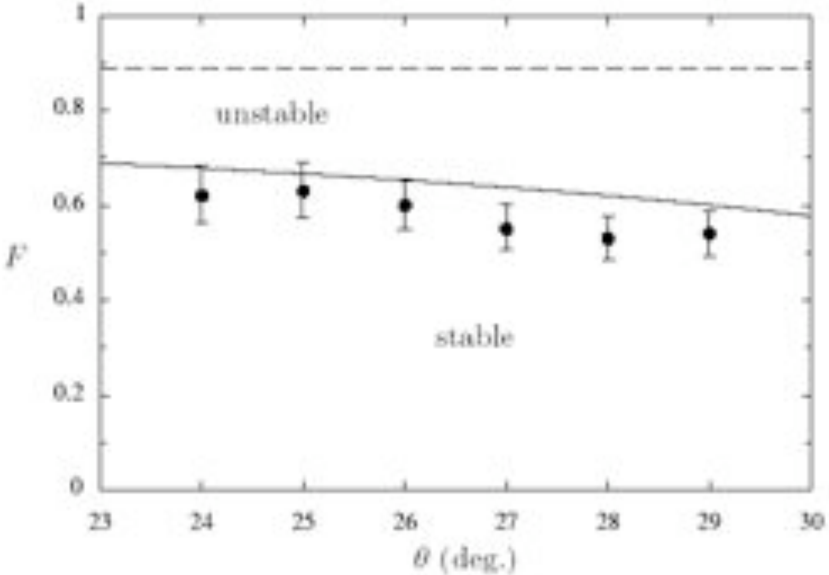
# Experimental Setup : forcing of the instability



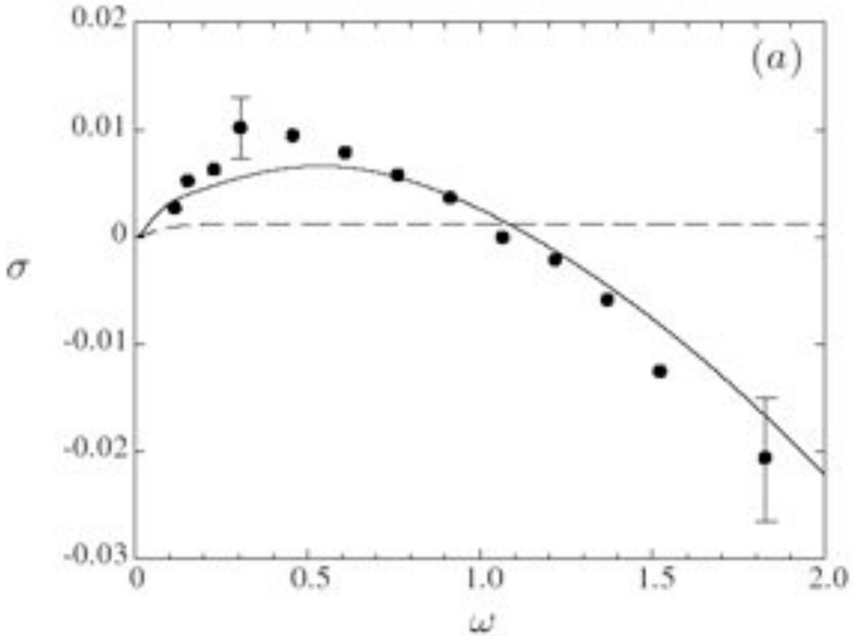

Forterre and Pouliquen JFM 02

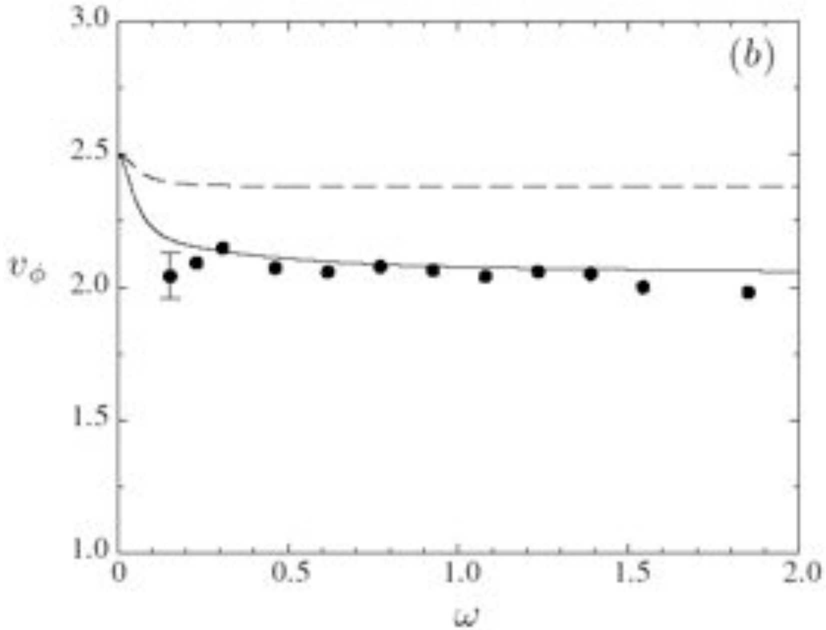
# Instability threshold

Forterre, JFM 06

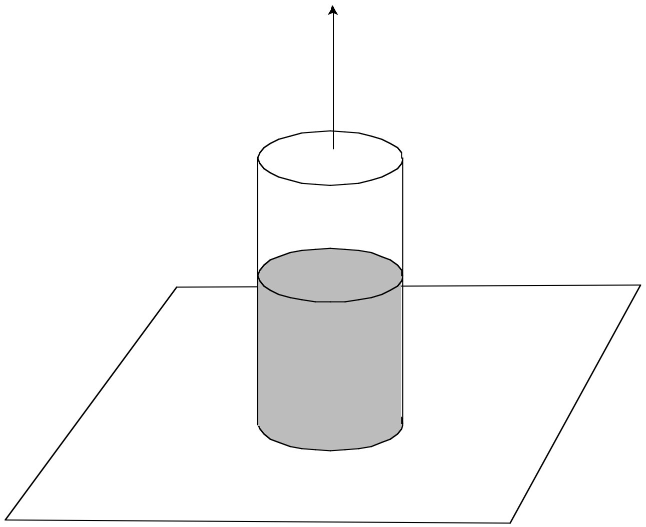



# Dispersion relation

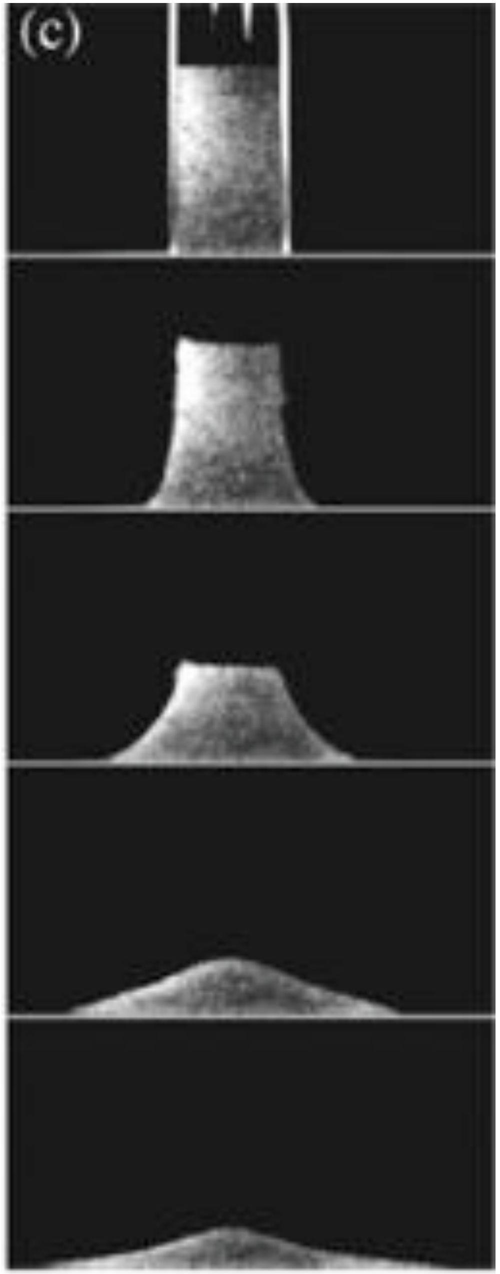


# Granular slumping
(Lacaze and Kerswell 08)

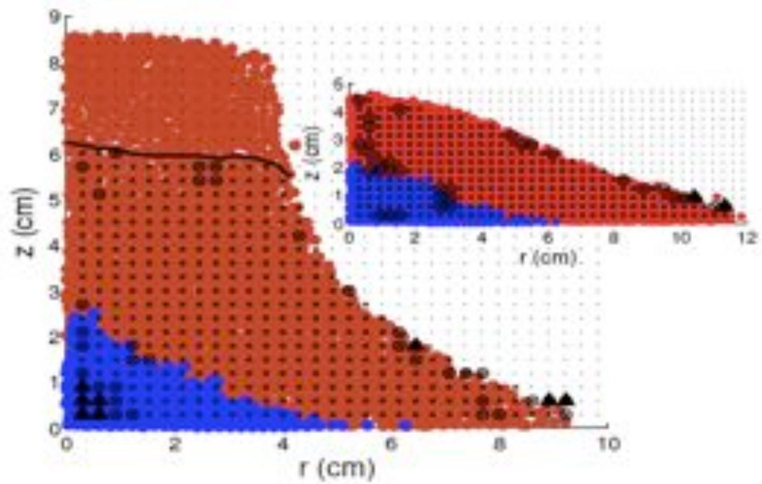


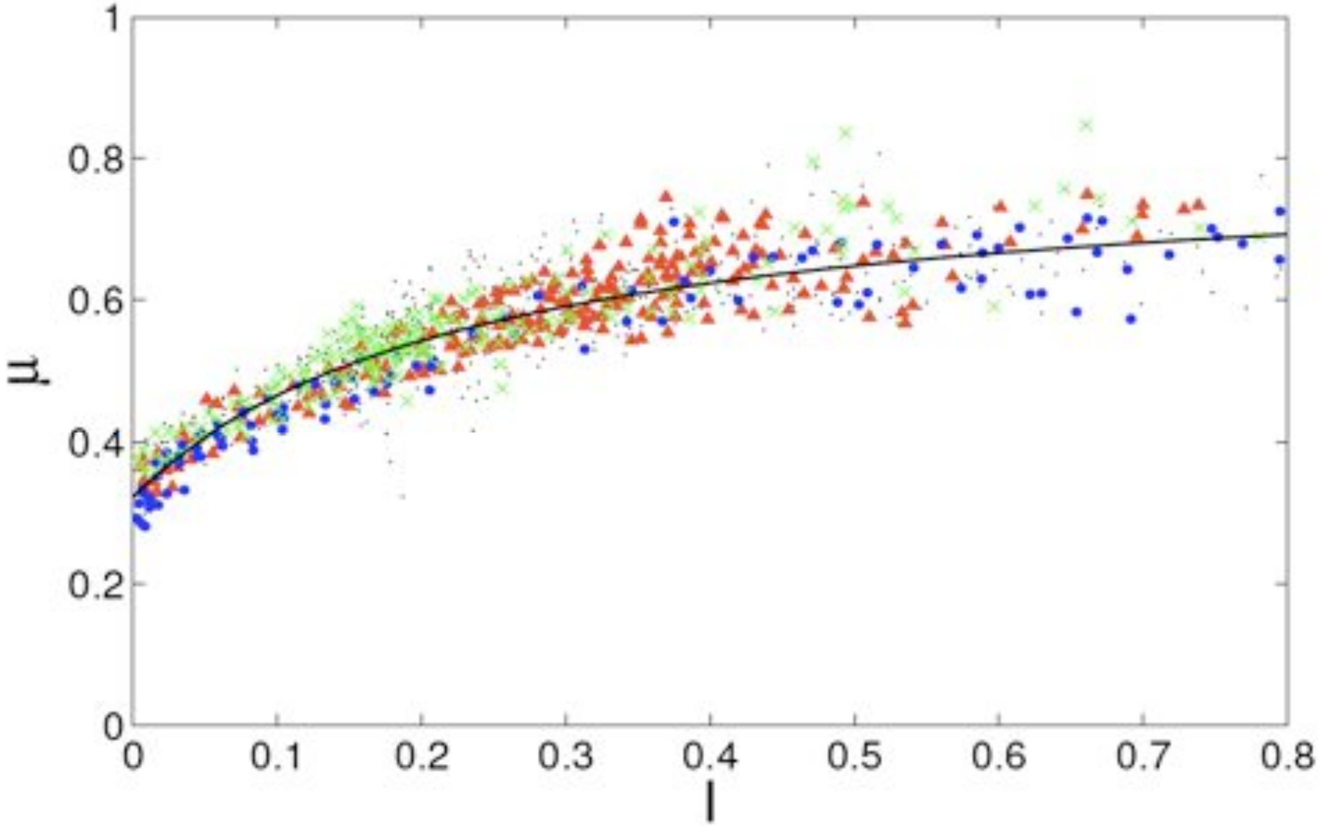
Lajeunesse et al Phys. Fluids 2004,2005
Lube et al JFM 2004,
Larrieu et al JFM 2006,
Staron & Hinch JFM 2005,
Lacaze et al Phys. Fluids 2008
...




Lajeunesse et al 05

Lacaze and Kerswell
(preprint 08)



Relative Success of the visco-plastic description.

A starting point to adress other configurations...
(simulating the pressure dependent visco-plastic rheology is non trivial...)

But there are problems when approaching the solid...

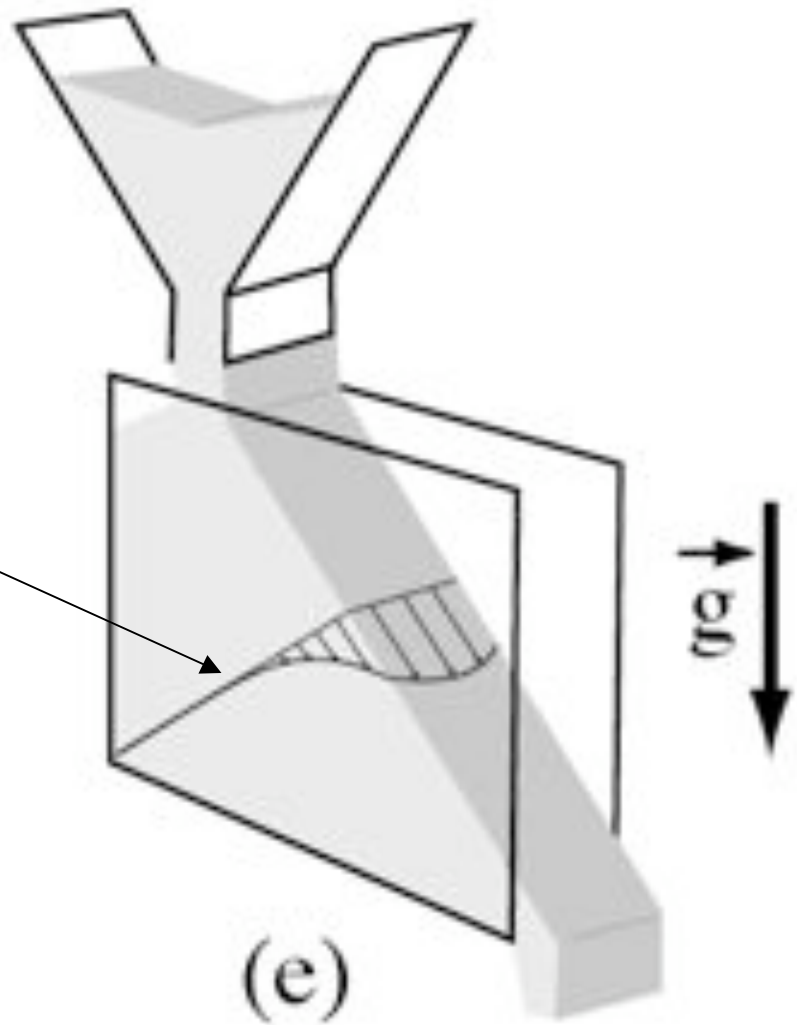
# Limits of the viscoplastic approach:

1) Quasistatic flows (shear band, finite size effect….)
A need for non local approach…

2) Transient flows when preparation plays a crucial role

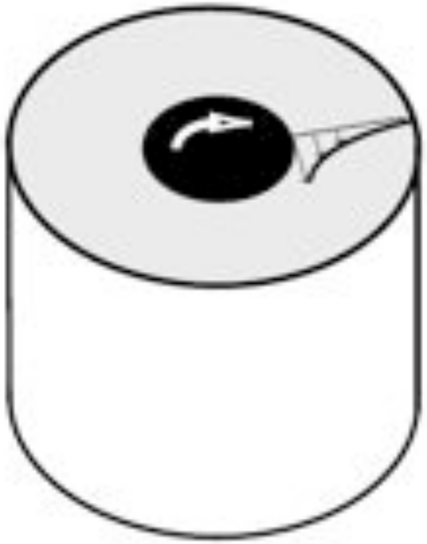
# Velocity profile

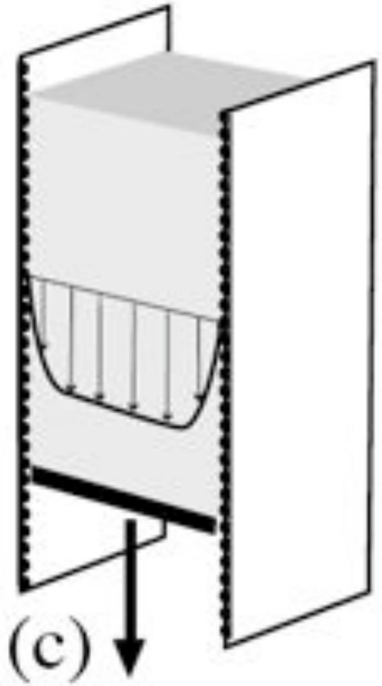
Exponential tail
Not predicted..

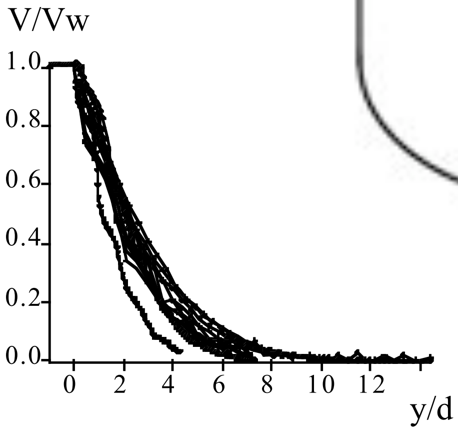


# Shear bands in quasi-static flow

(Forterre & Pouliquen ARFM 08, Jop PRE 08)





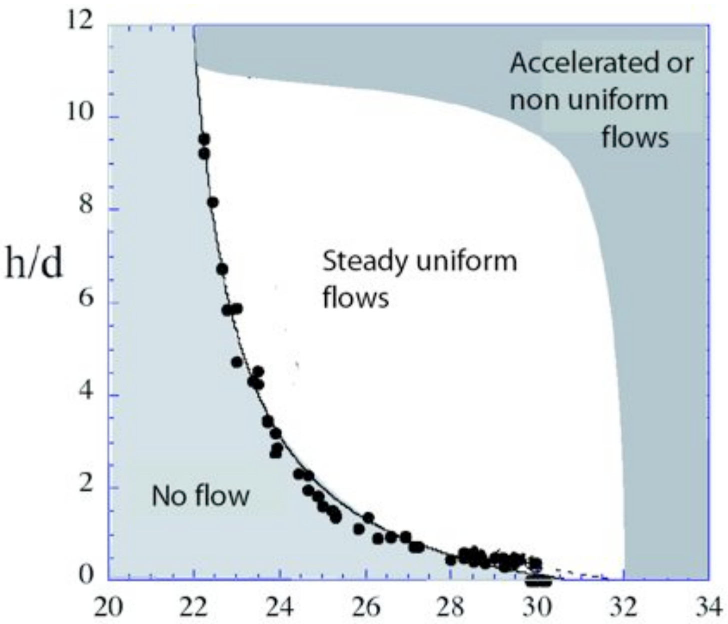

Howell et al PRL 99
Mueth et al Nature 00
Bocquet et al PRE 02
...

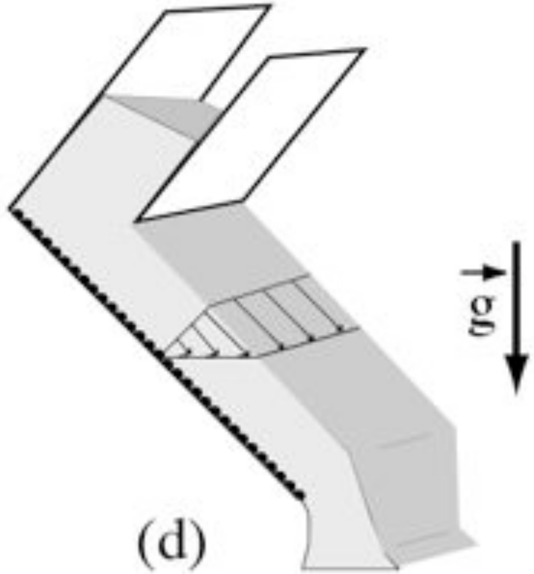
Not captured by the viscoplastic approach

# Flow threshold

## hysteresis

## Finite size effects

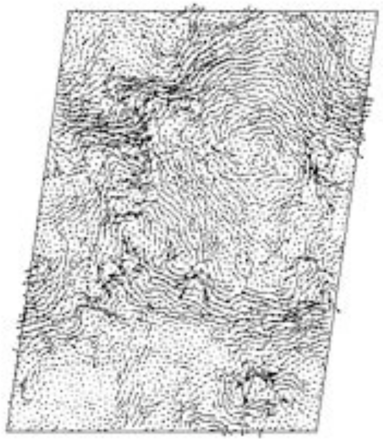



**Not captured by the viscoplastic approach**

Limit of a local rheology ?

# to go further?



Radjai and Roux PRL 02

## Role of the fluctuations ?

Aranson and Tsimring PRE,01,

Louge Phys. Fluids 03,

Josserand et al 06

Lemaitre 02
Bazant 07
Nott 08
Behringer 08...

## Role of the correlations ?

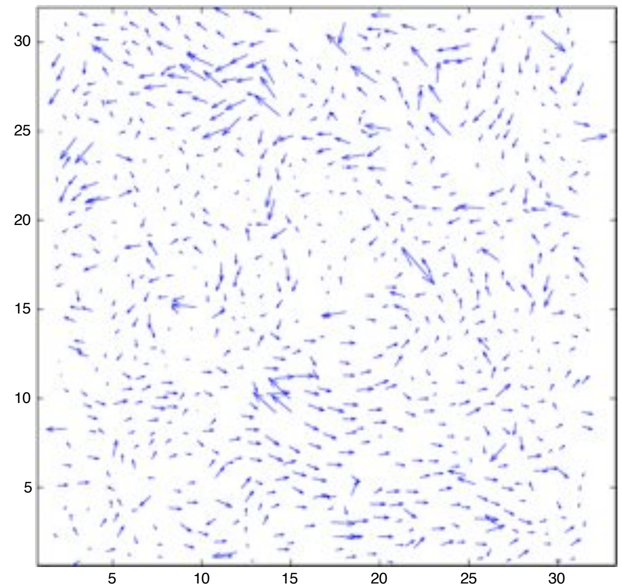
Pouliquen et al 01,
Ertas and Halsey 03,

Mills et al 08

Jenkins and Chevoir 01,

Jenkins Phys. Fluids 06,

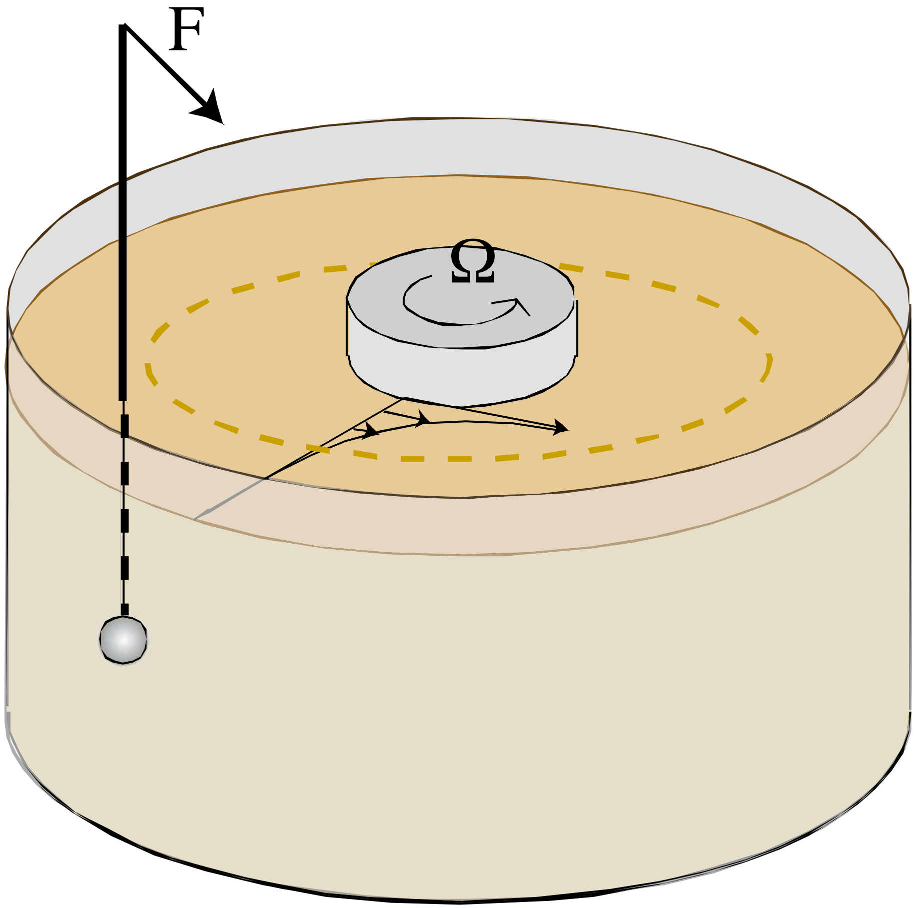
...

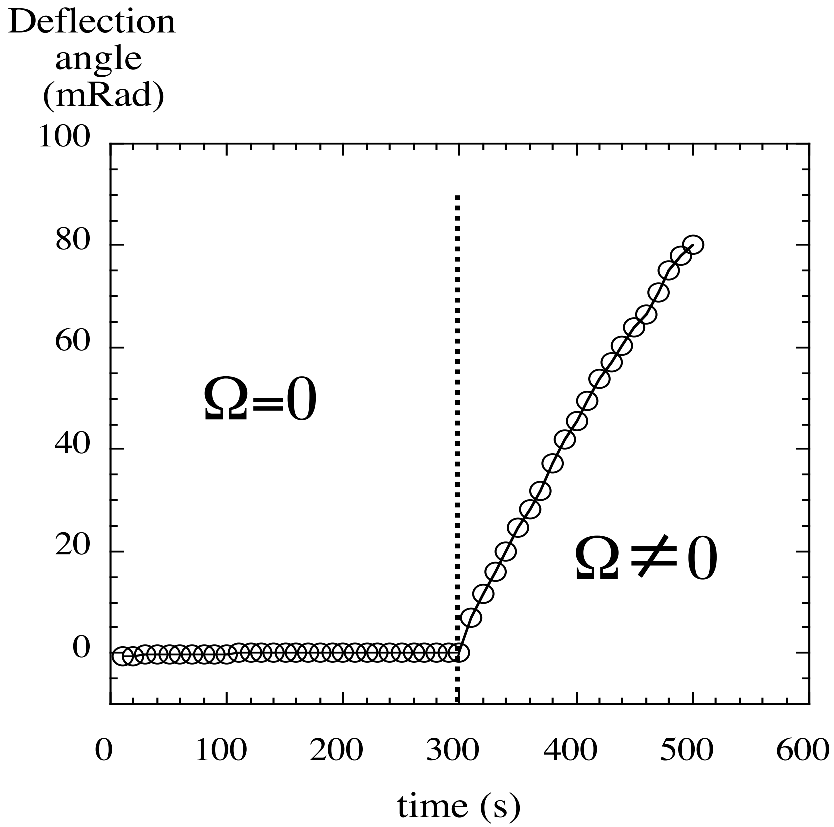



Pouliquen PRL 04

## Link with plasticity of other amorphous and glassy systems

# Evidence for non local effects: Microrheology experiments

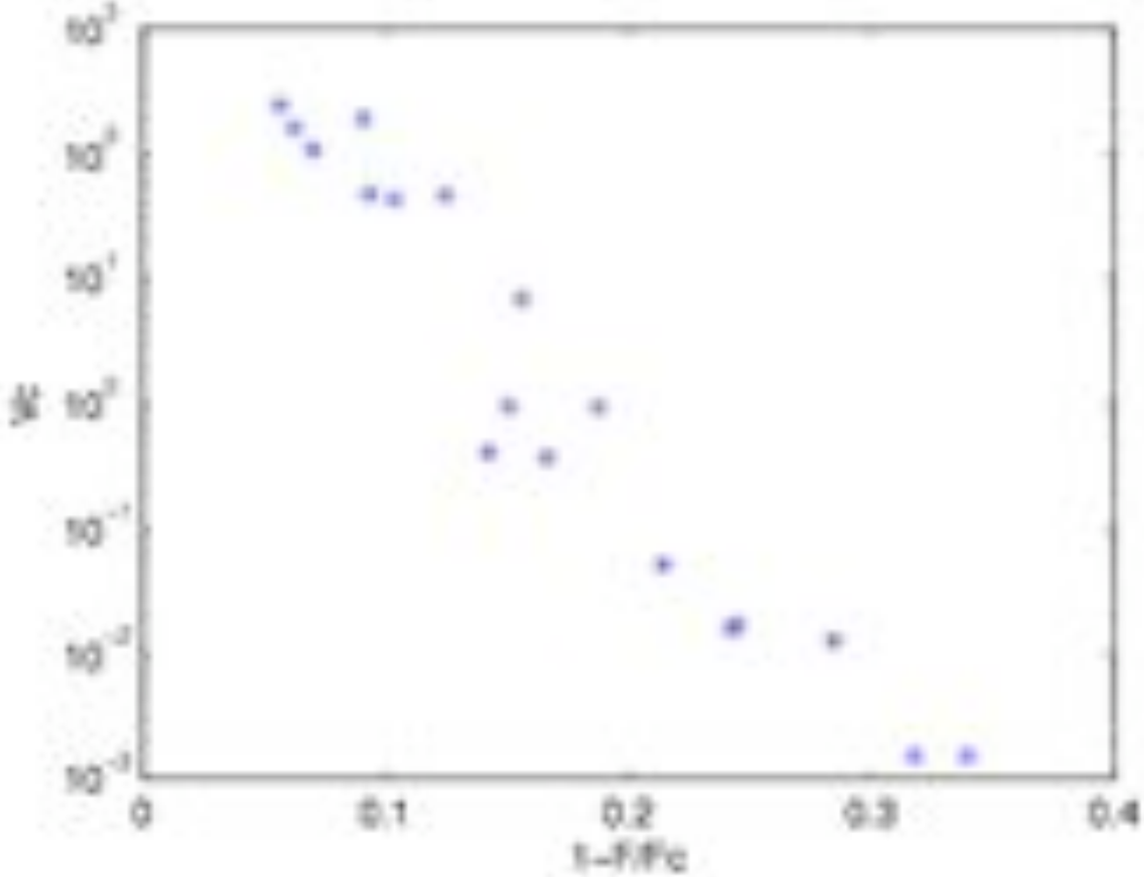


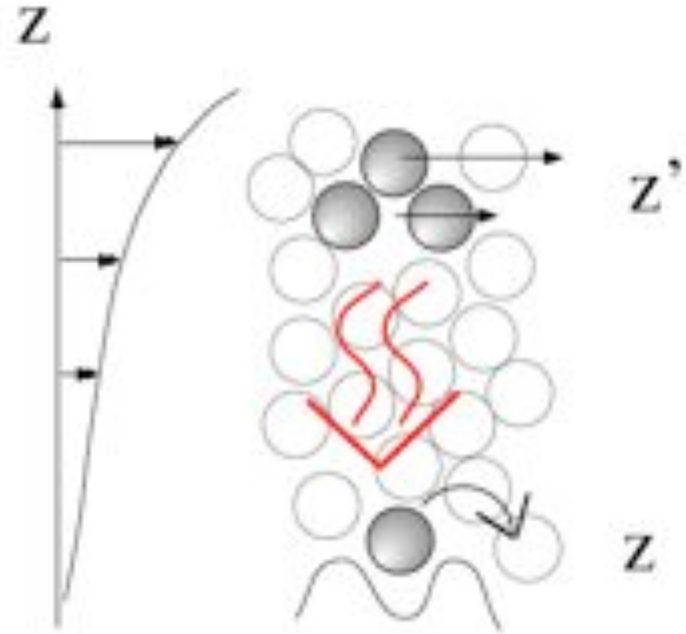

M. Van Hecke 2008

Pouliquen, Forterre, Nott

$$v_{creep} \propto \exp(\frac{F - F_c}{F_c})$$

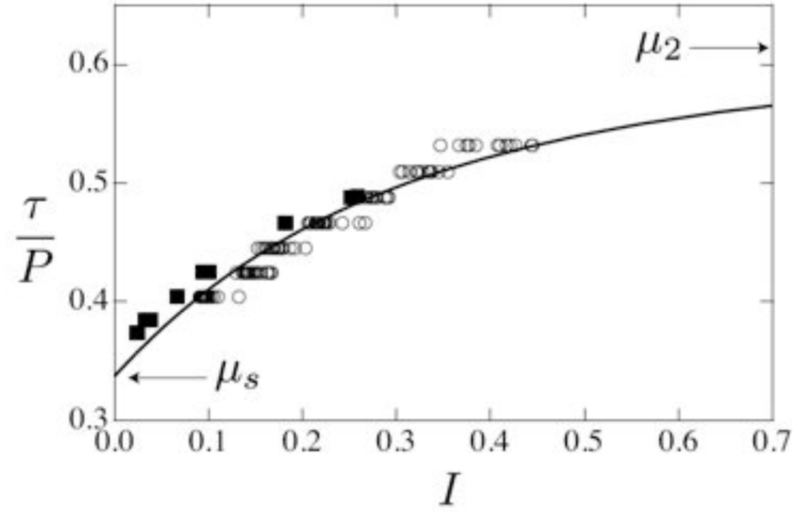


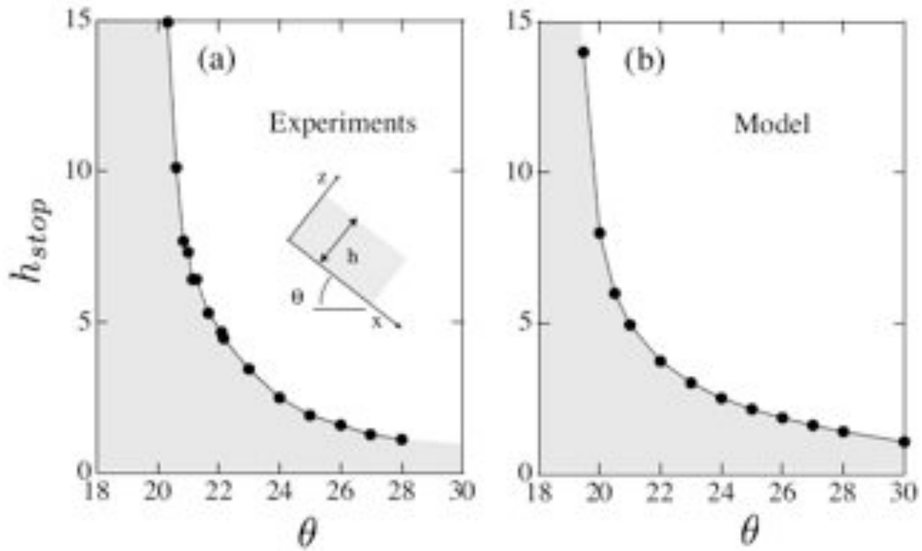
# Self activated process




$$\dot{\gamma}(z) = \sum_{z'} f_{z' \to z} \left( p^{+}_{z' \to z} - p^{-}_{z' \to z} \right),$$

$${}_{z'->z} = |\dot{\gamma}(z')|$$




Pouliquen & Forterre, Phil. Trans, 2009

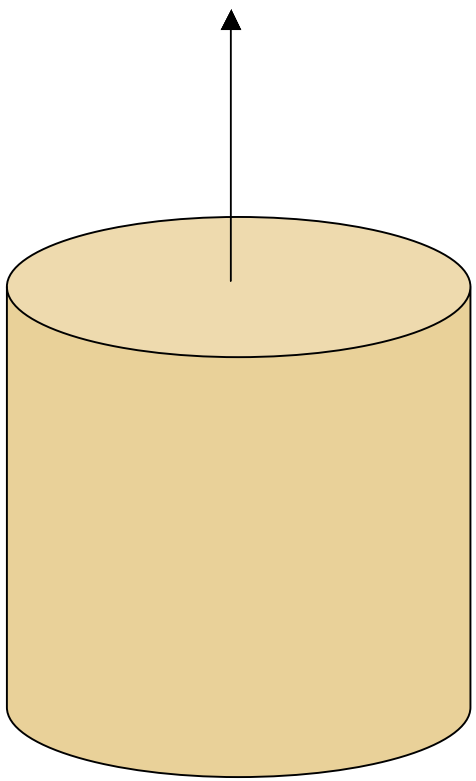
# Limits of the viscoplastic approach:

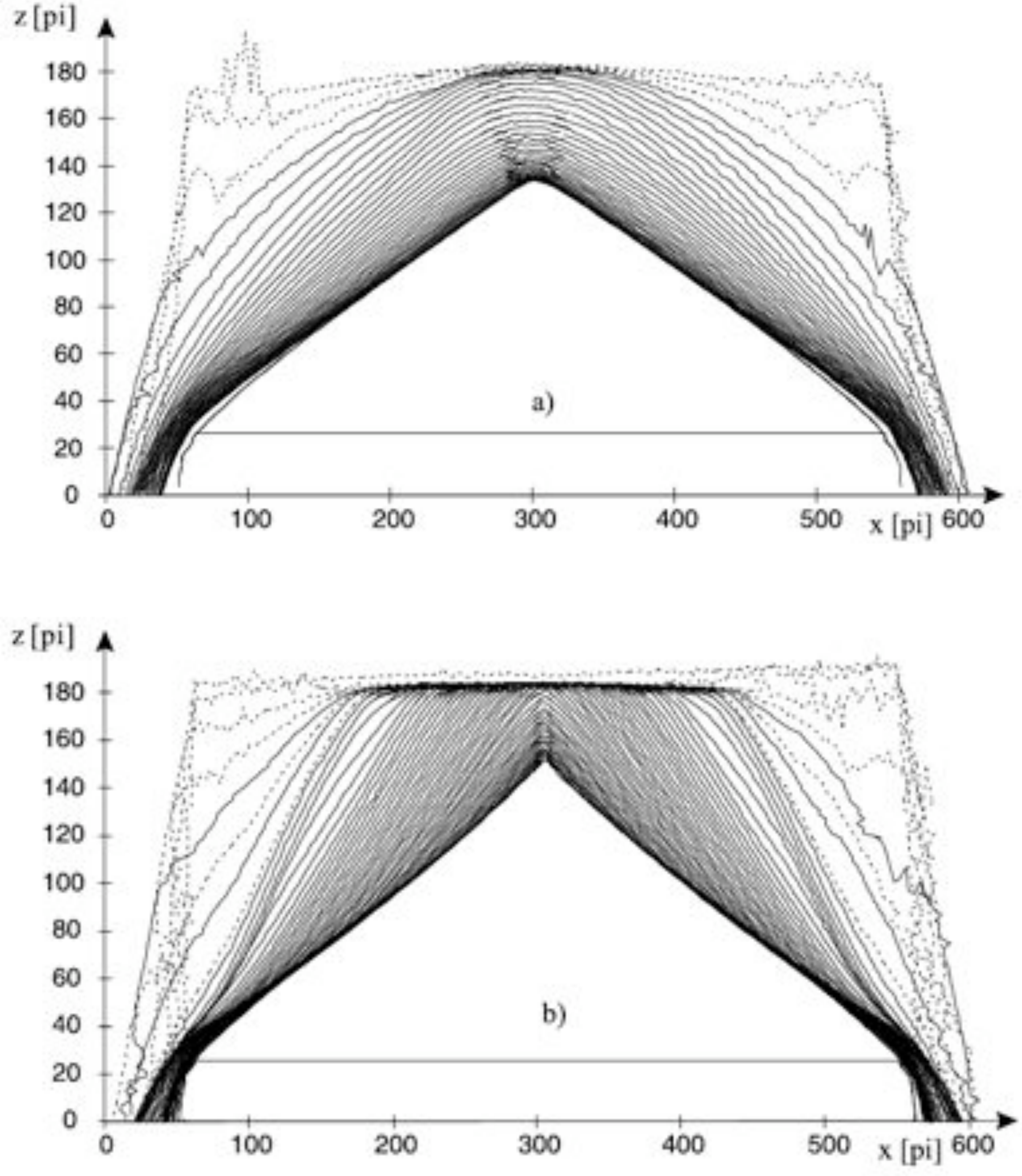
1) Quasistatic flows (shear band, finite size effect....)
A need for non local approach...

2) Transient flows when preparation plays a crucial role

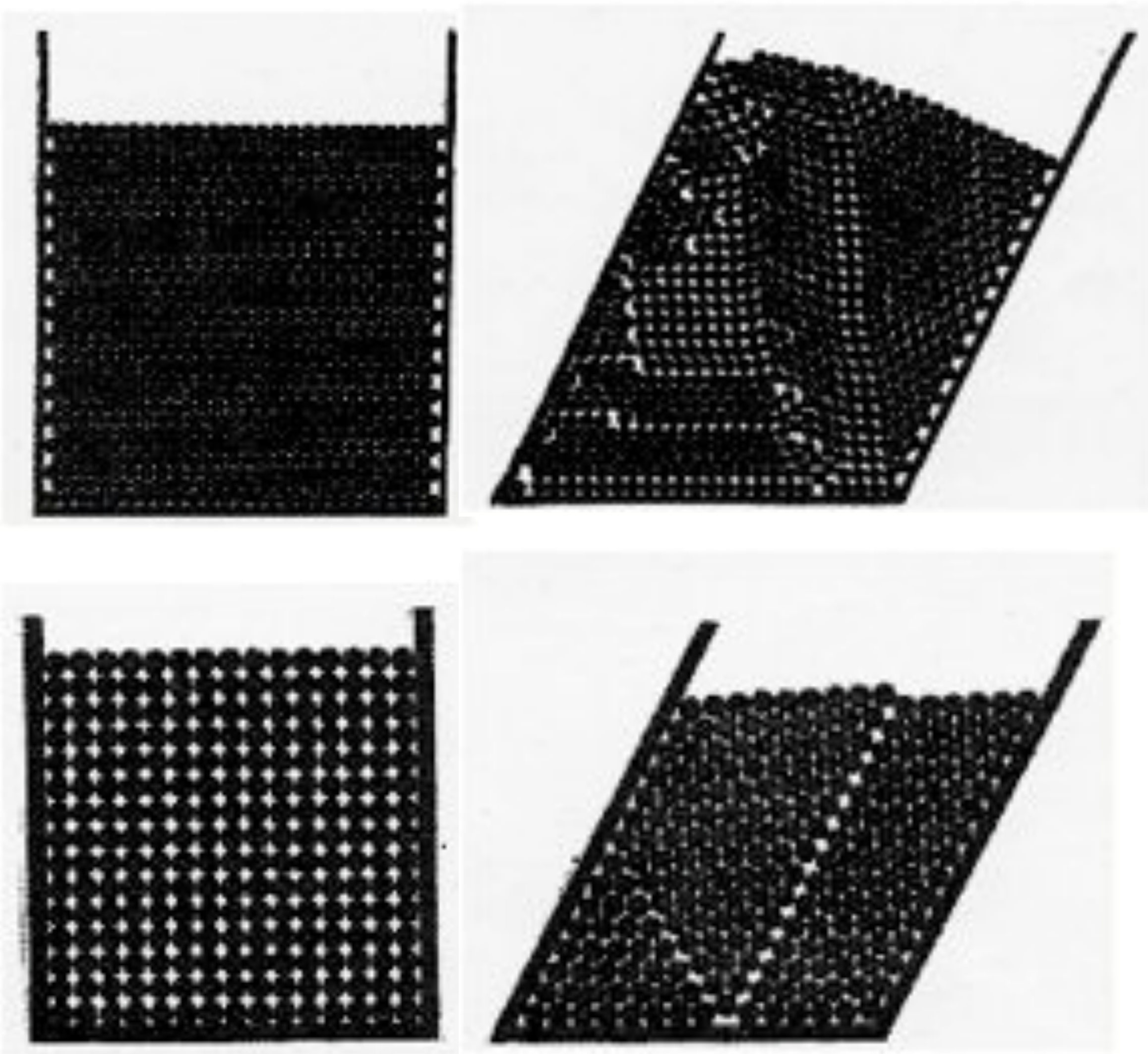
Influence of the initial Volume fraction on the Collapse of a pile.

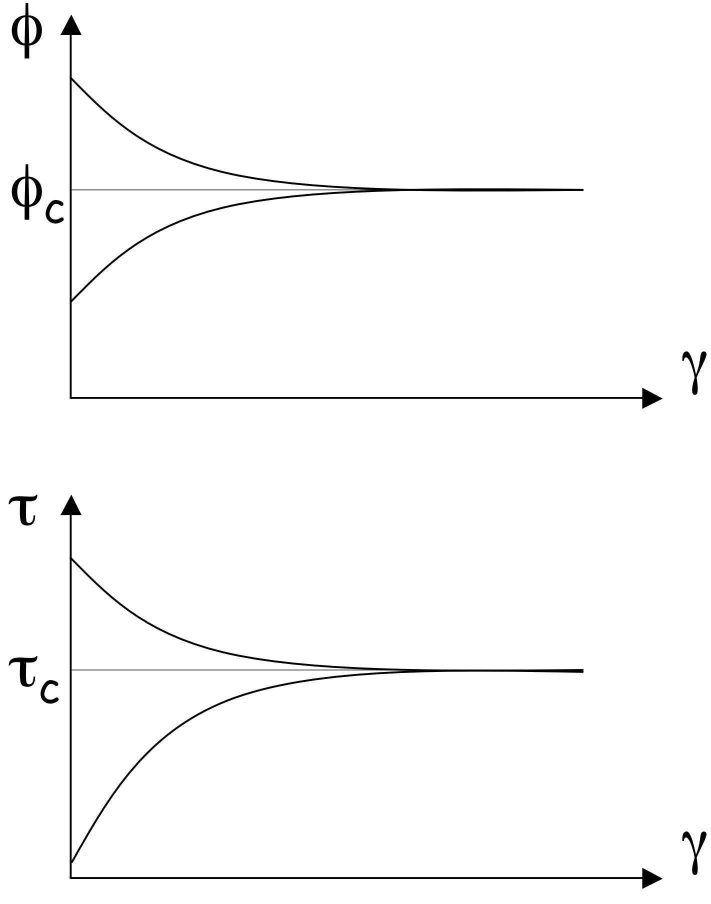
Daerr & Douady 99





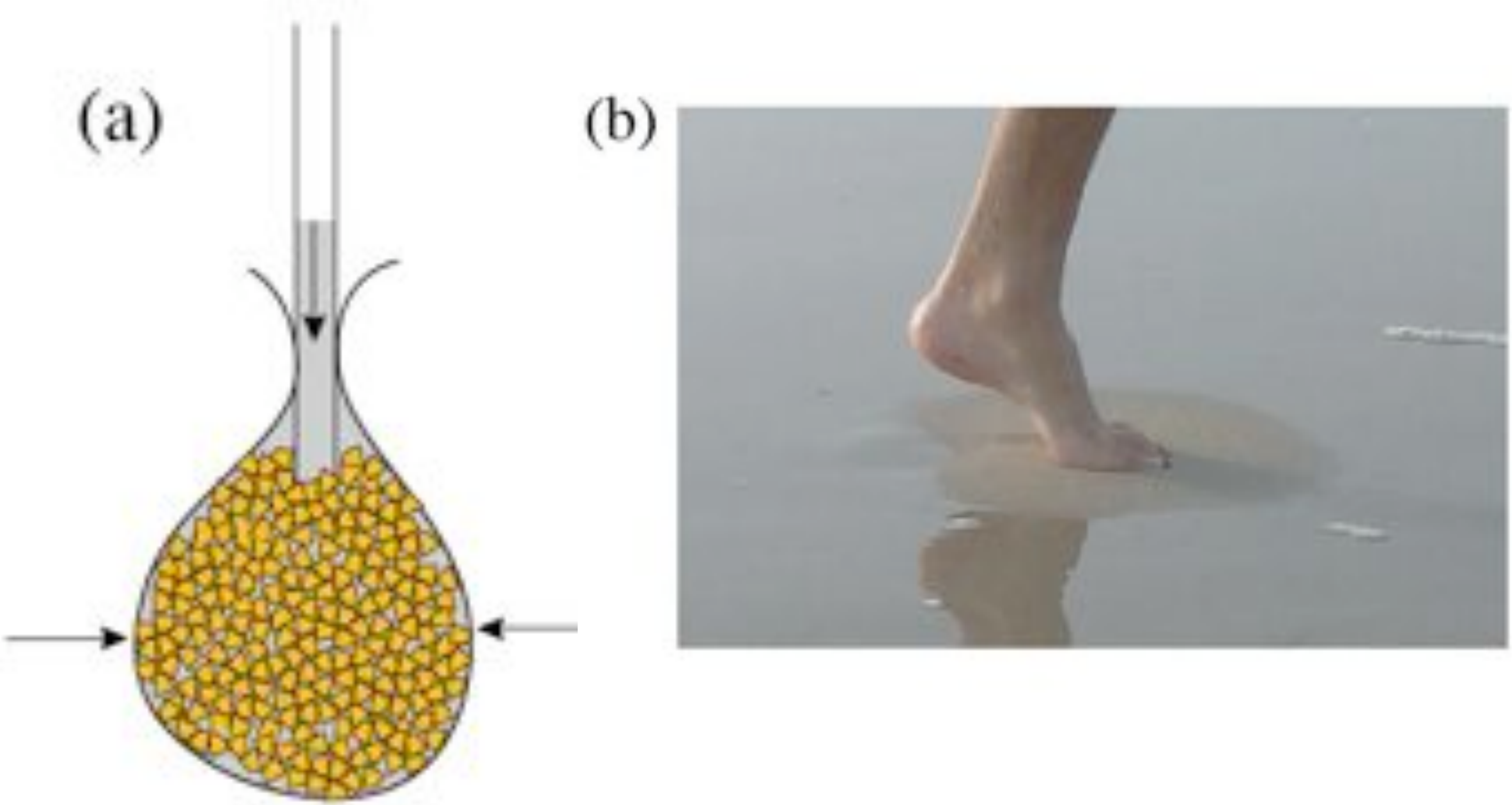
# Quasi-static case : critical state theory



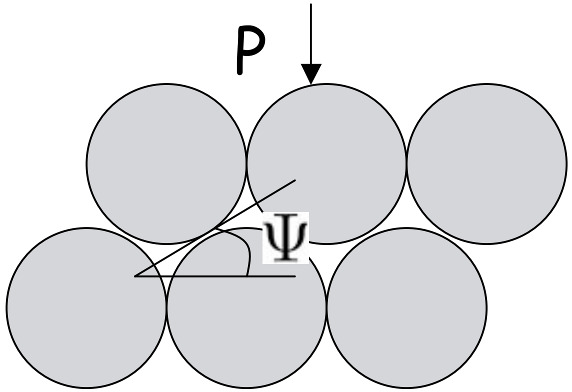



Coupling
Friction- dilatancy

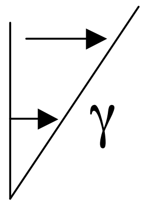
# Reynolds Dilatancy



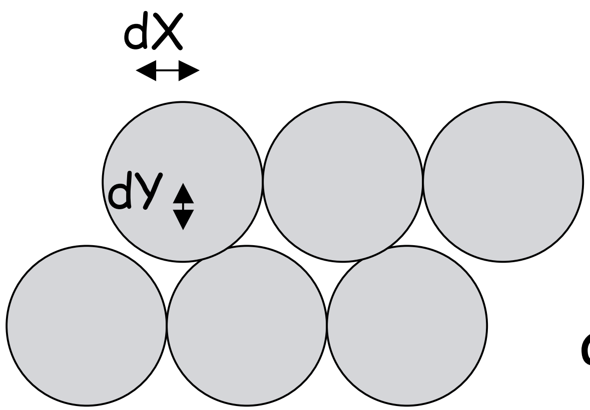
# Simple critical state theory




$\Psi$ Dilatancy angle

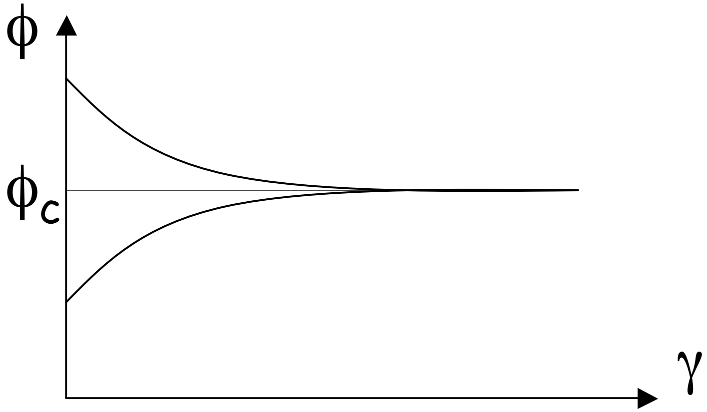



$$\frac{dY}{dX} = -\frac{1}{\Phi}\frac{d\Phi}{d\gamma} = \tan\Psi$$




$$\tau = (\mu + \tan\Psi)P$$

assumption: $\exists$critical volume fraction $\phi_c$




$$\tan\Psi = K(\Phi - \Phi_c)$$

(Radjai and Roux 98)

Visco plastic theory :

shear rate dependence
but no dilatancy

critical state theory :

dilatancy but
no shear rate
dependence

Shear rate dependent critical state theory

## Shear rate dependent critical state theory :

$$\boxed{\tau = (\mu(I) + \tan\Psi)P}$$

$$\boxed{\tan\Psi = K(\Phi - \Phi_{eq}(I))}$$

$$\boxed{\frac{1}{\Phi}\frac{d\Phi}{dt} = \dot{\gamma}\tan\Psi}$$

$$I = \frac{\dot{\gamma}\, d}{\sqrt{P/\rho}}$$

3D generalisation :

$$\dot{\tilde{\gamma}}_{ij} = \dot{\gamma}_{ij} - \frac{1}{3}\delta_{ij}\dot{\gamma}_{kk}$$

$$I = \frac{\|\dot{\tilde{\gamma}}\| d}{\sqrt{P/\rho_s}}$$

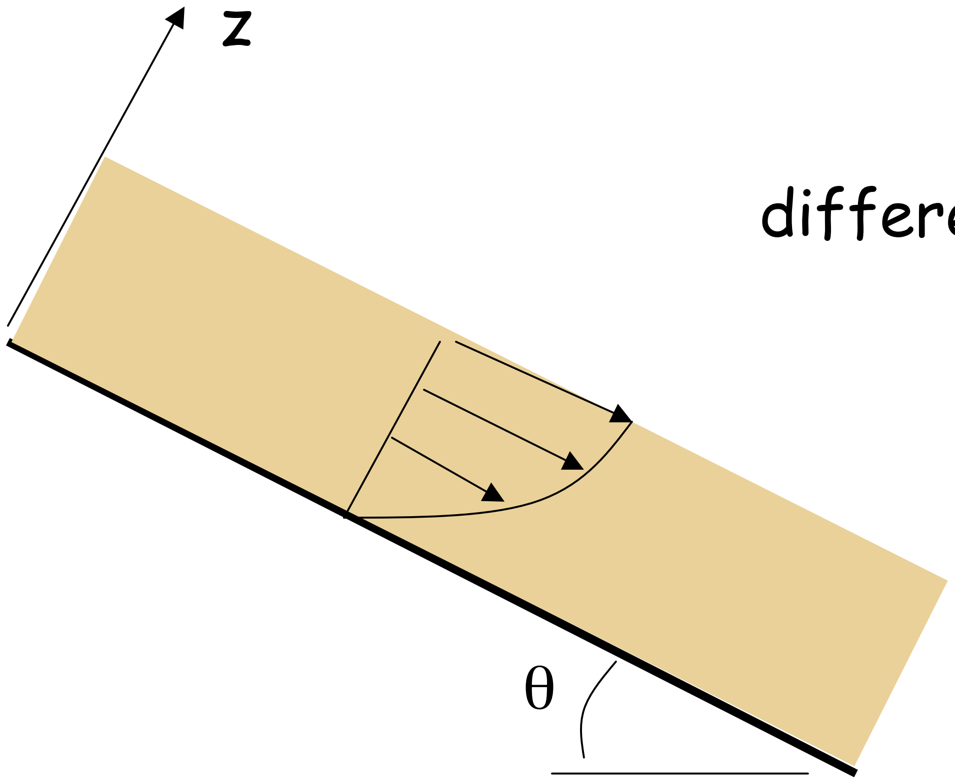
$$\tan\psi = K(\Phi - \Phi_c(I))$$

$$\tau_{ij} = (\mu(I) + \tan\Psi)\frac{\dot{\tilde{\gamma}}_{ij}}{\|\tilde{\gamma}\|}$$

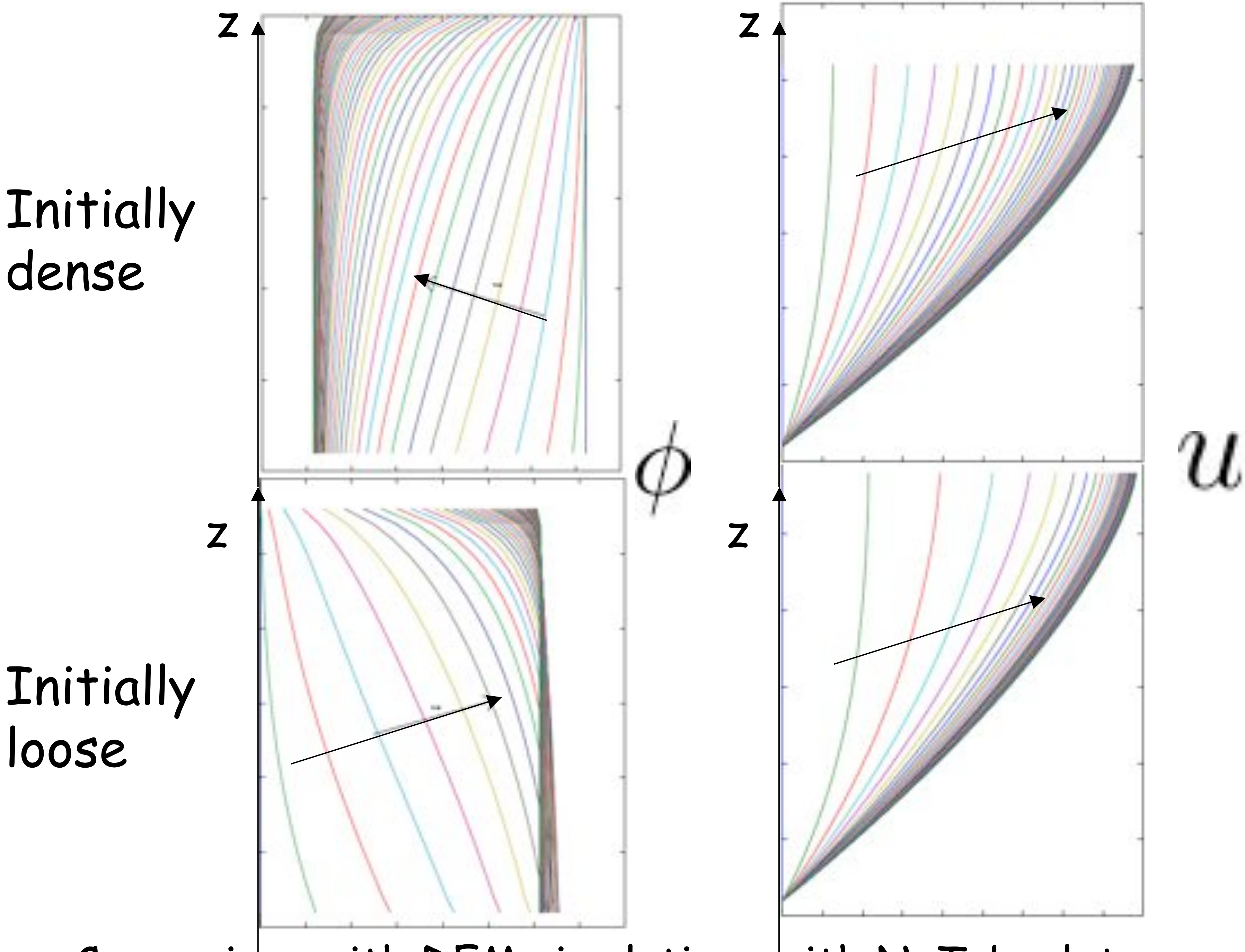
$$\frac{\partial u_i}{\partial x_i} = \tan\Psi\|\dot{\tilde{\gamma}}\|$$

# Application to a dry flow:

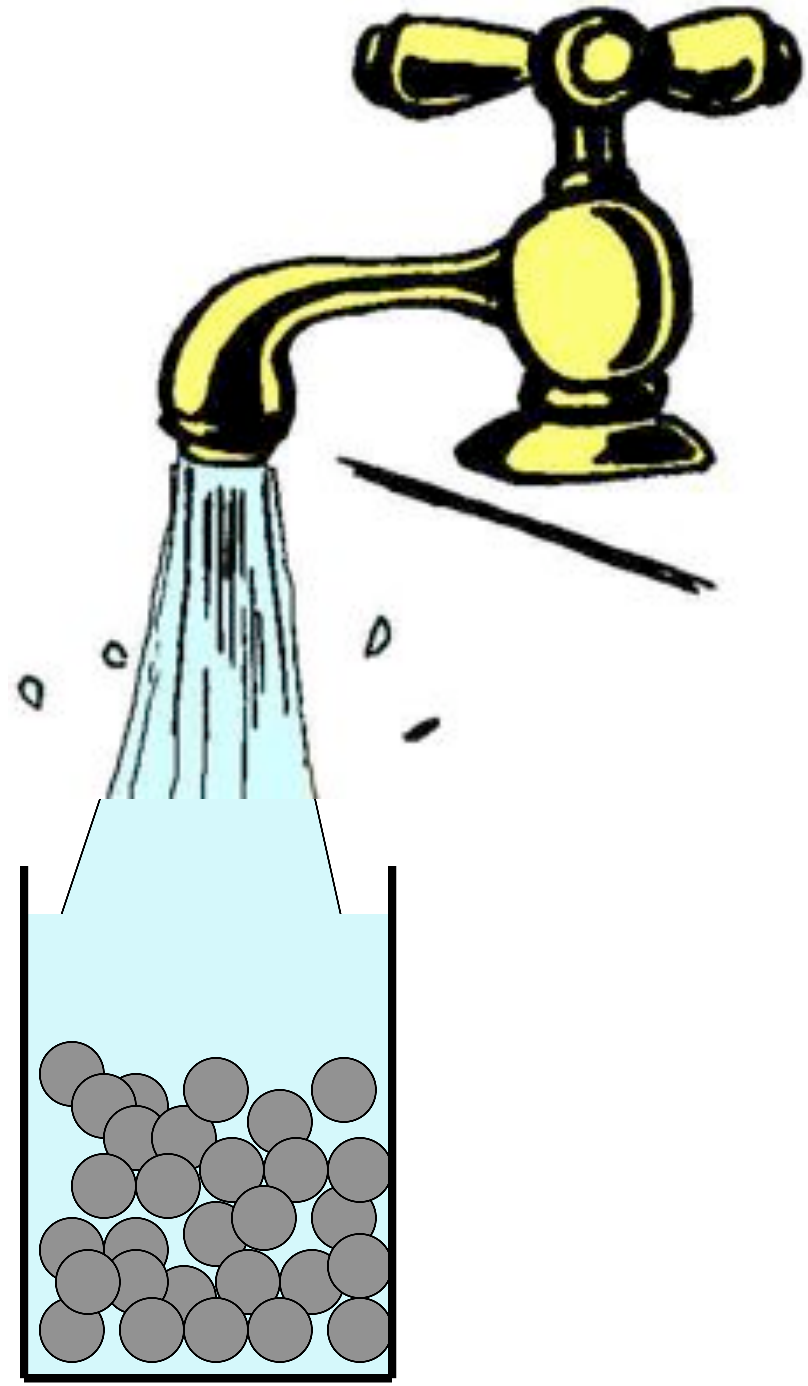
Initiation of flow on an inclined plane:



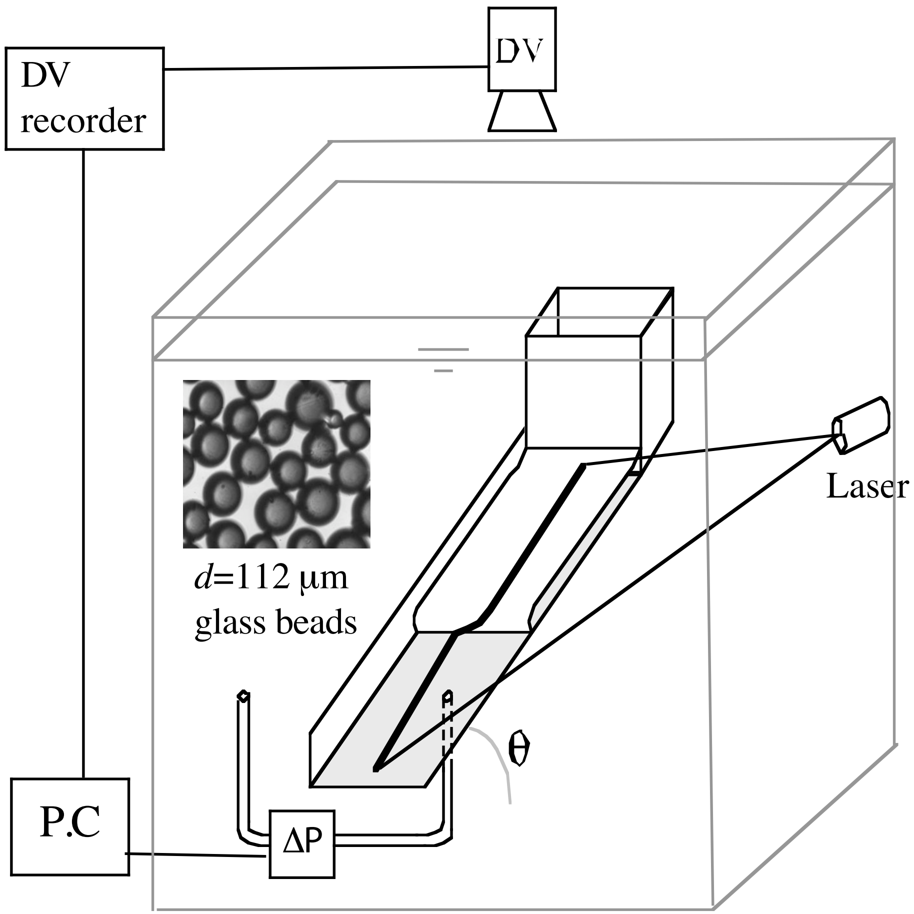

different $\Phi_i$

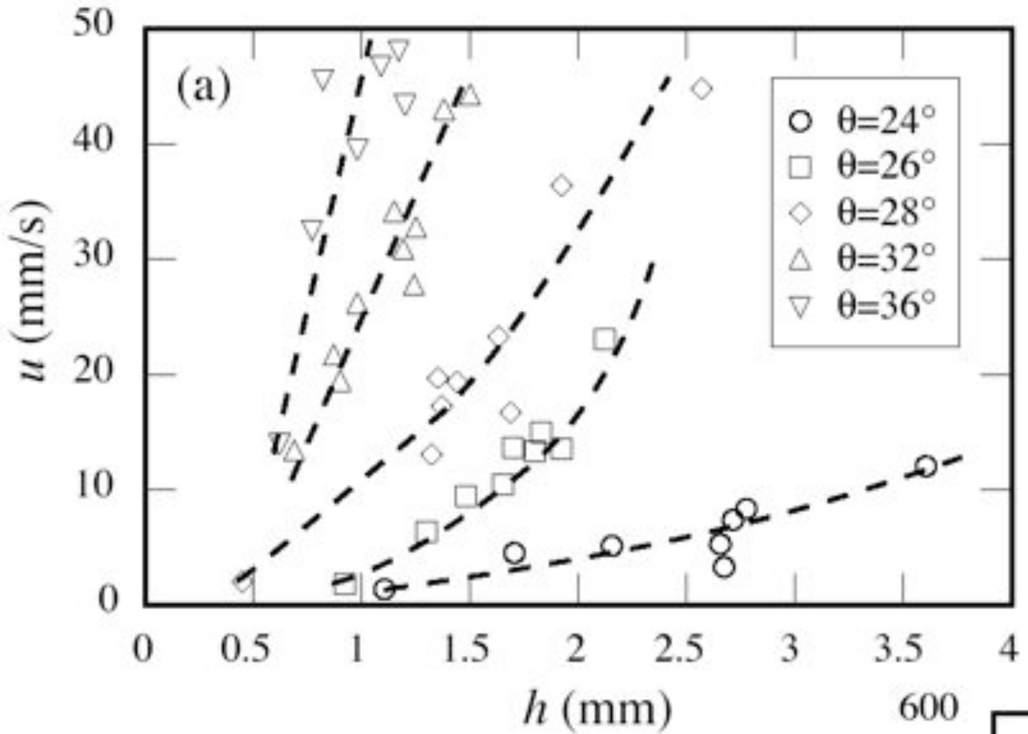



Comparison with DEM simulations with N. Taberlet…

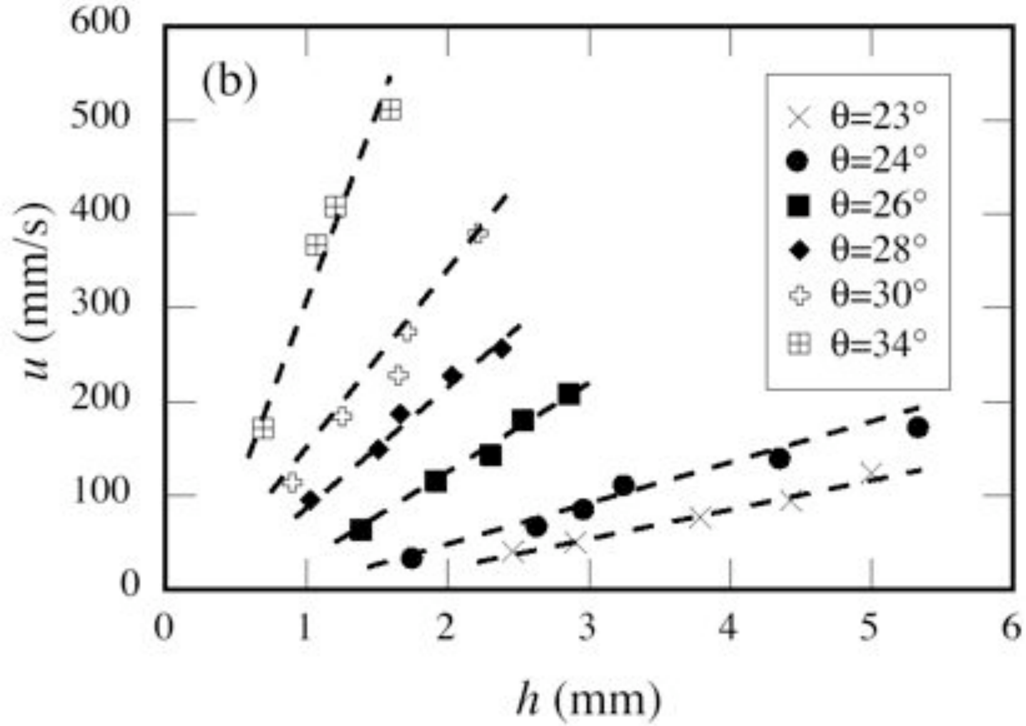


# Changing time scales…
# by putting the granular material in water
(Cassar et al, Phys. Fluids 06)






Immersed

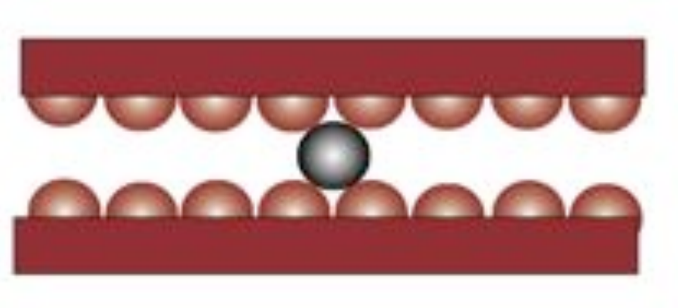



Dry

A naive idea :

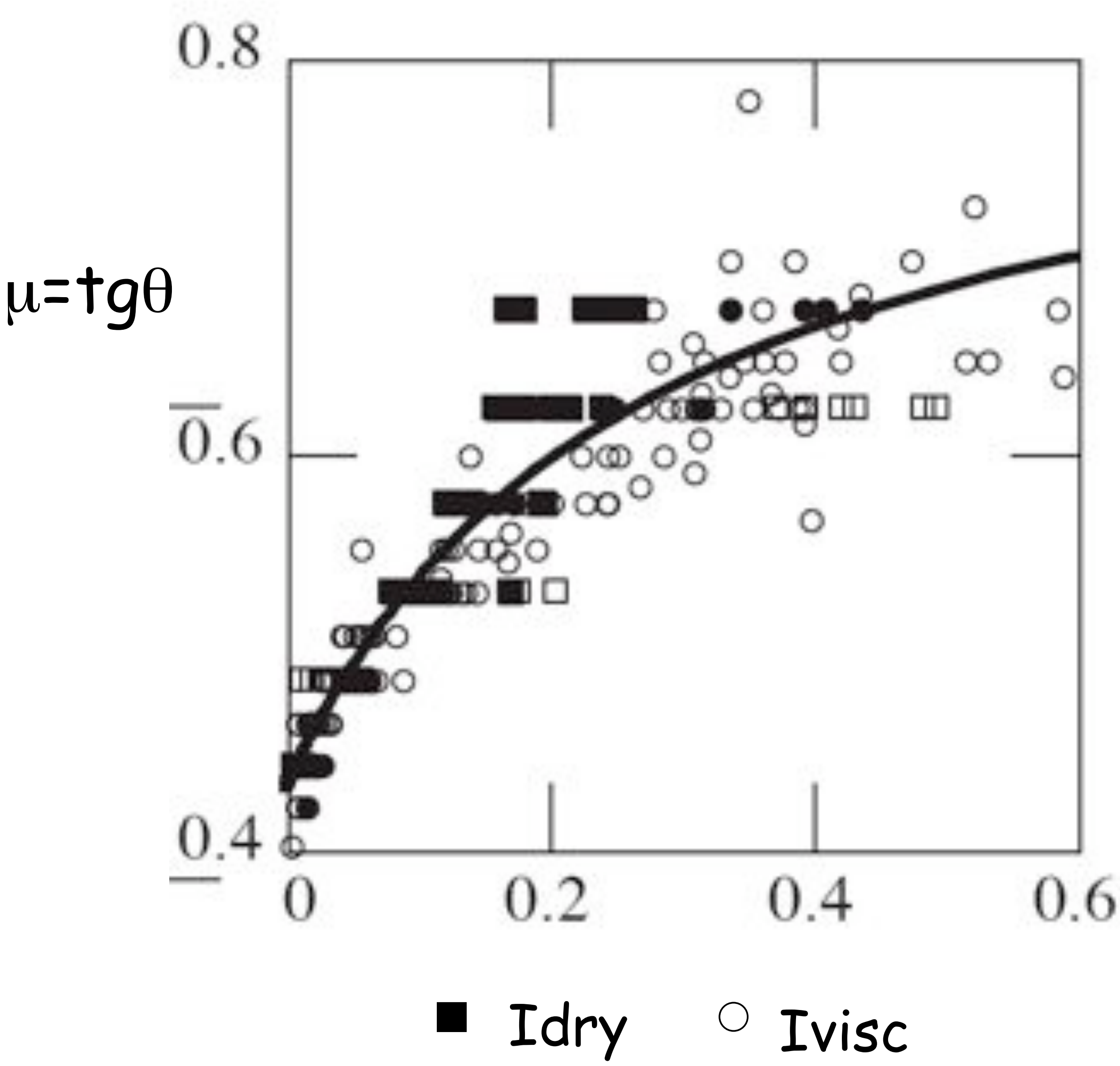
fluid only plays a role by changing the time scale of rearrangements

$$\tau = P\,\mu(I) \qquad \text{with} \qquad I = \dot{\gamma}\, t_{micro}$$



viscous : $$t_{micro} = \frac{\eta_f}{P}$$

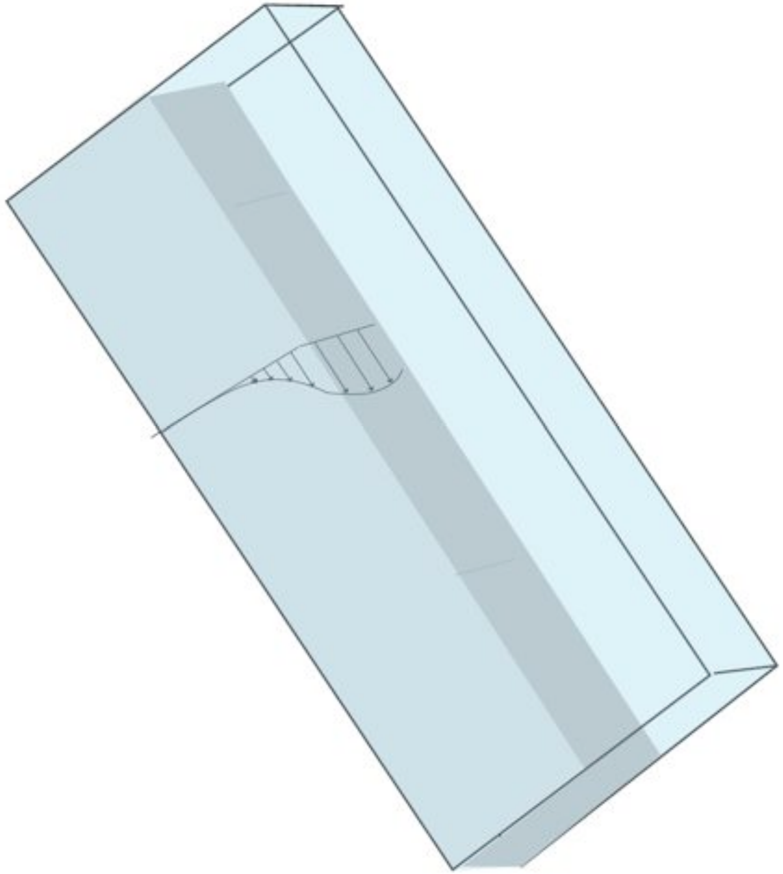
dry : $$t_{micro} = \frac{d}{\sqrt{P/\rho_s}}$$

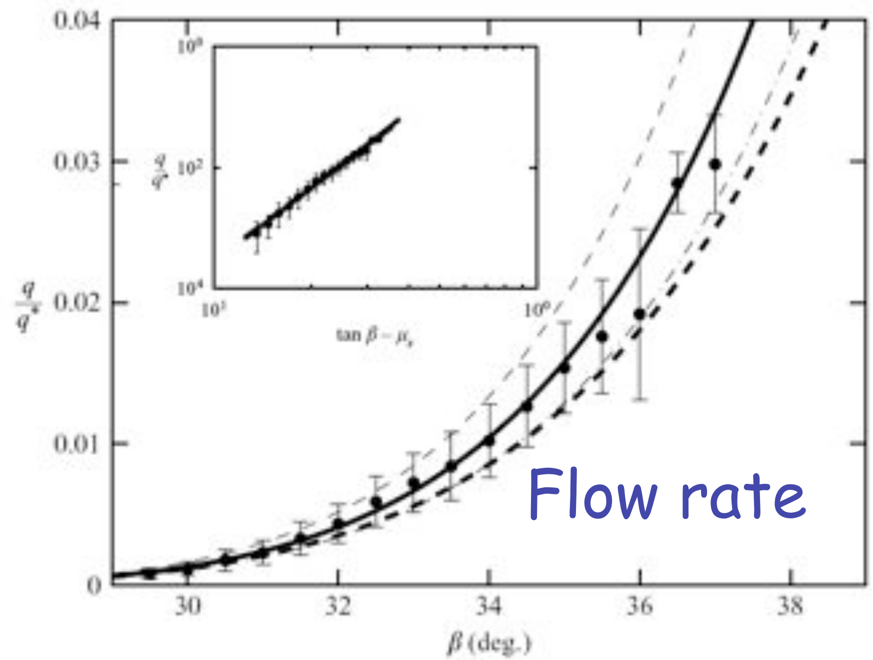


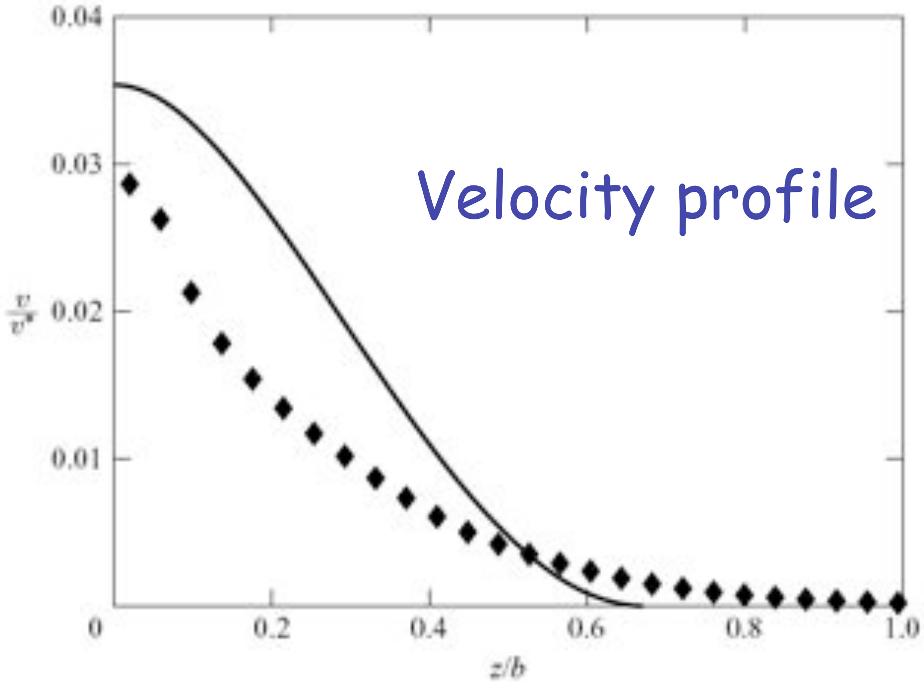

Cassar et al. Phys. Fluids 05

# Submarine flows on heap

Doppler et al, JFM 07





And dilatancy ???

And Pore pressure ??

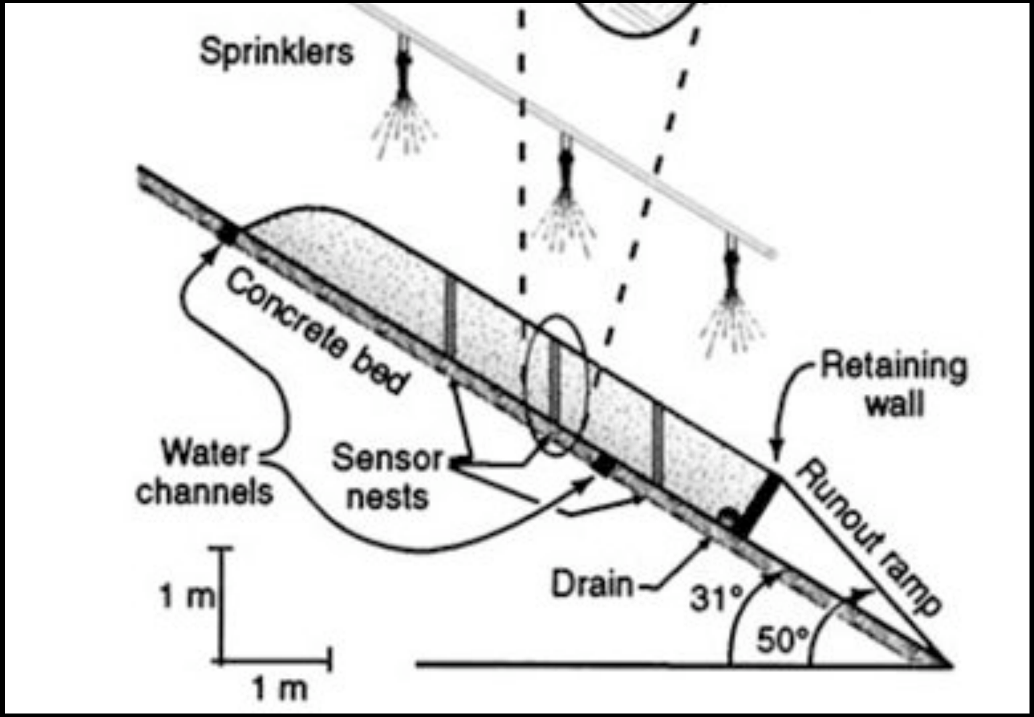
Cf In Faults

Rice JGR 75, Rudnicki JGR 84, …

How to explain the variety of landslides observed in nature ?

Iverson et al , (2000) Science

Large scale experiments in the USGS facility



Dense preparation

Courtesy of Dick Iverson

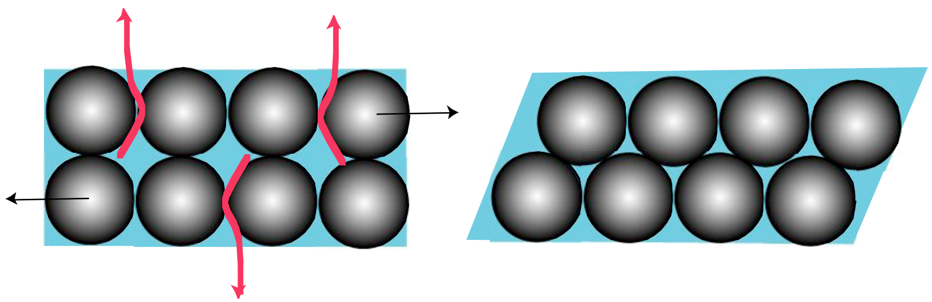
Loose preparation

Courtesy of Dick Iverson

# Pore Pressure feedback argument

(Iverson Rev. Geo. 97, JGR 05)

## Loose case



$\Phi \nearrow$

$\Rightarrow$ Fluid expelled

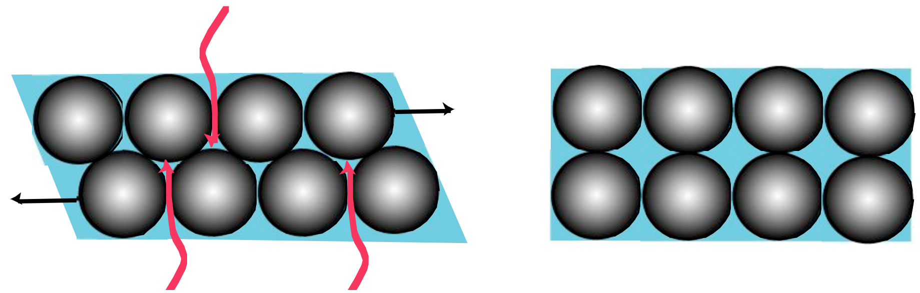
$\Rightarrow P_{fluid} \nearrow$

$\Rightarrow P_{eff} \searrow$

$\Rightarrow$ Friction $\searrow$

$\Rightarrow$ Less friction between grains

## Dense case



$\Phi \searrow$

$\Rightarrow$ Fluid sucked

$\Rightarrow P_{fluid} \searrow$

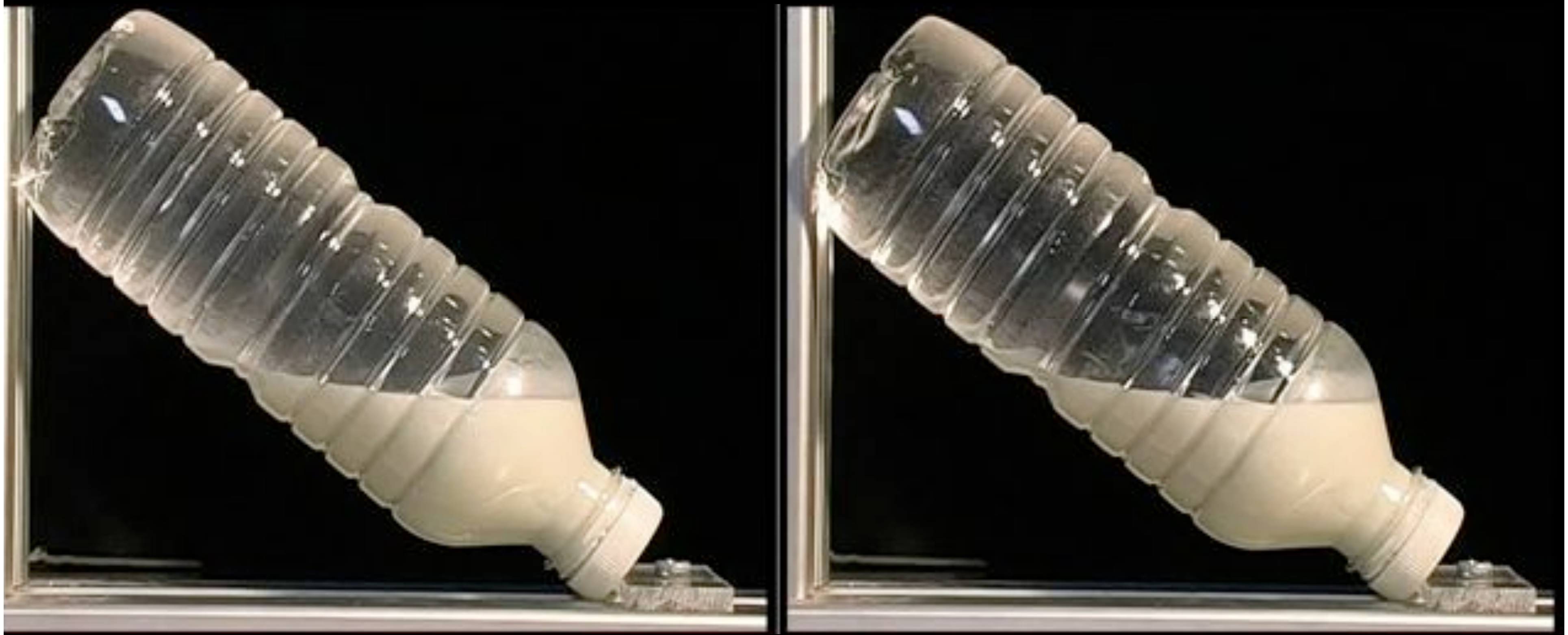
$\Rightarrow P_{eff} \nearrow$

$\Rightarrow$ Friction $\nearrow$

$\Rightarrow$ higher friction between grains

In faults: rice JGR 75, Rudnicki JGR 84, ...

A simple experiment:



Dense sample

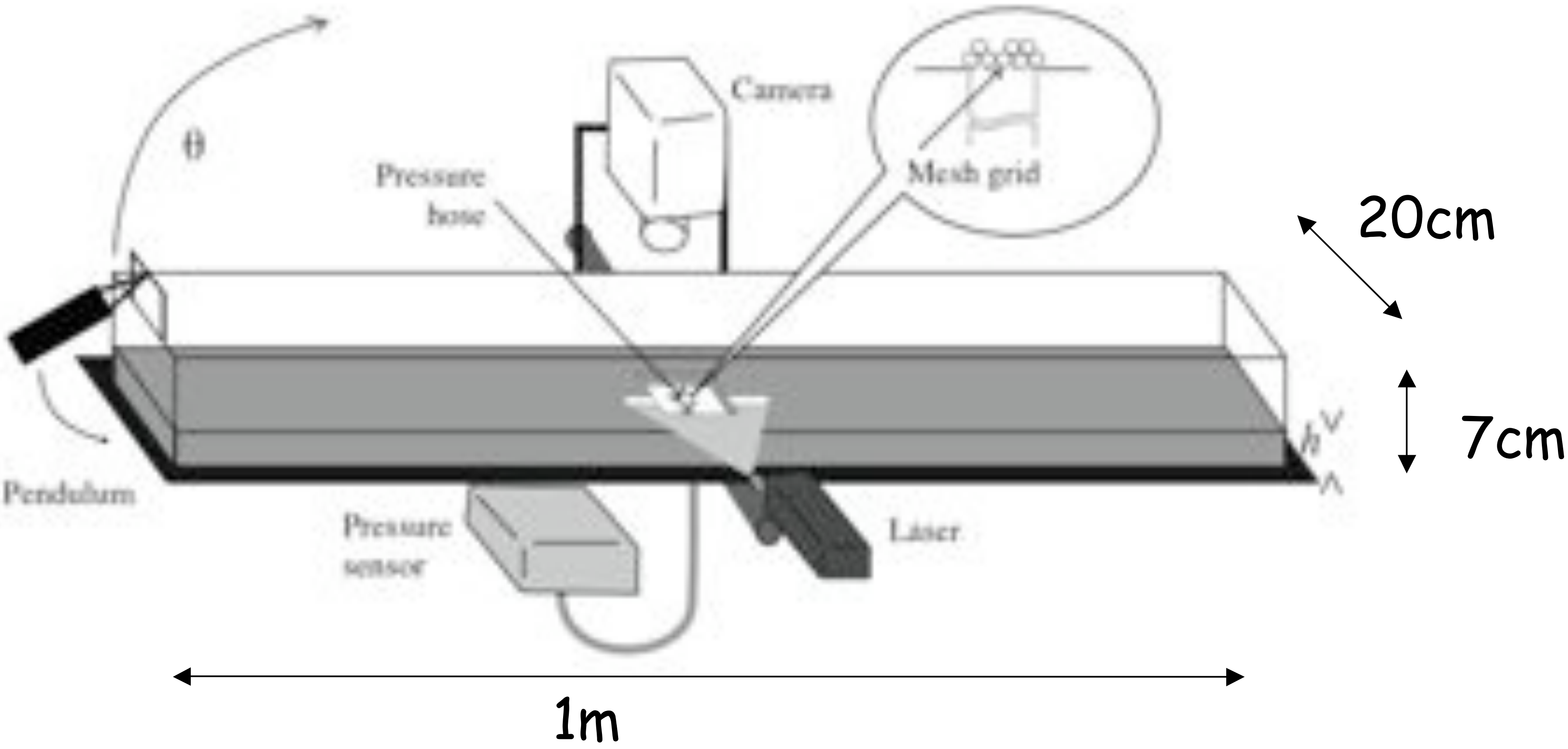
Loose sample

# Experimental setup

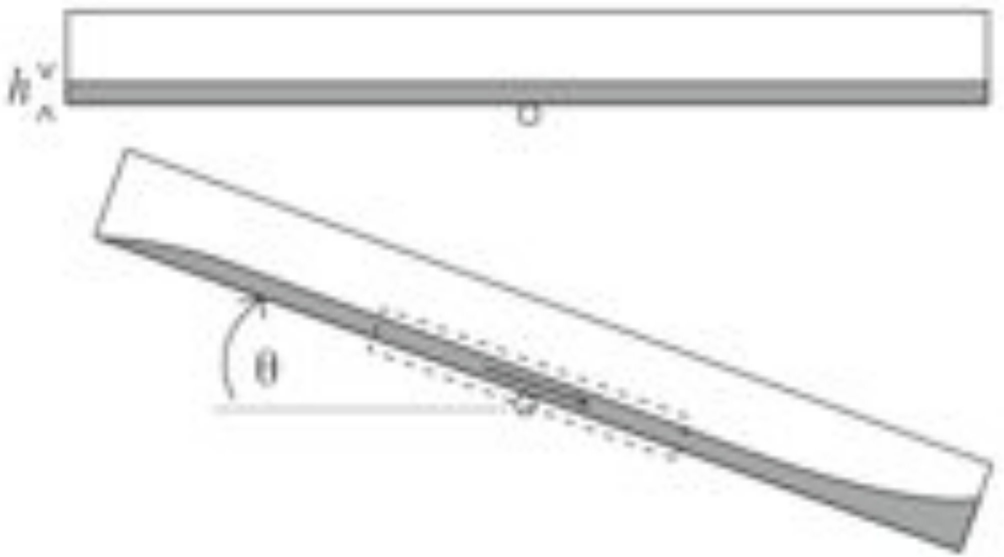
(Pailha et al 08)

Glass beads : 160μm

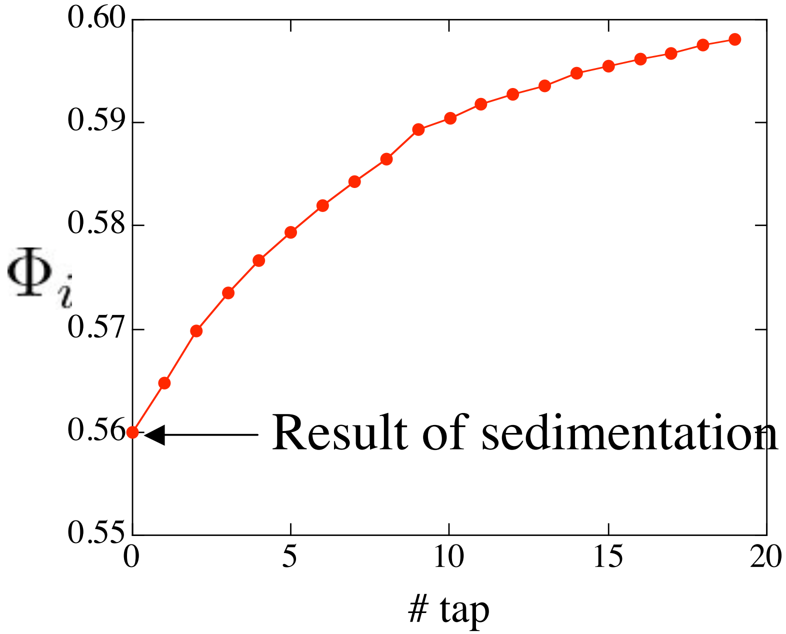
Liquid: mixture of water and Ucon oil:
$\eta$=9.8 $10^{-3}$ kg/m.s
$\eta$=96 $10^{-3}$ kg/m.s



# Experimental procedure




*Compaction by taps*

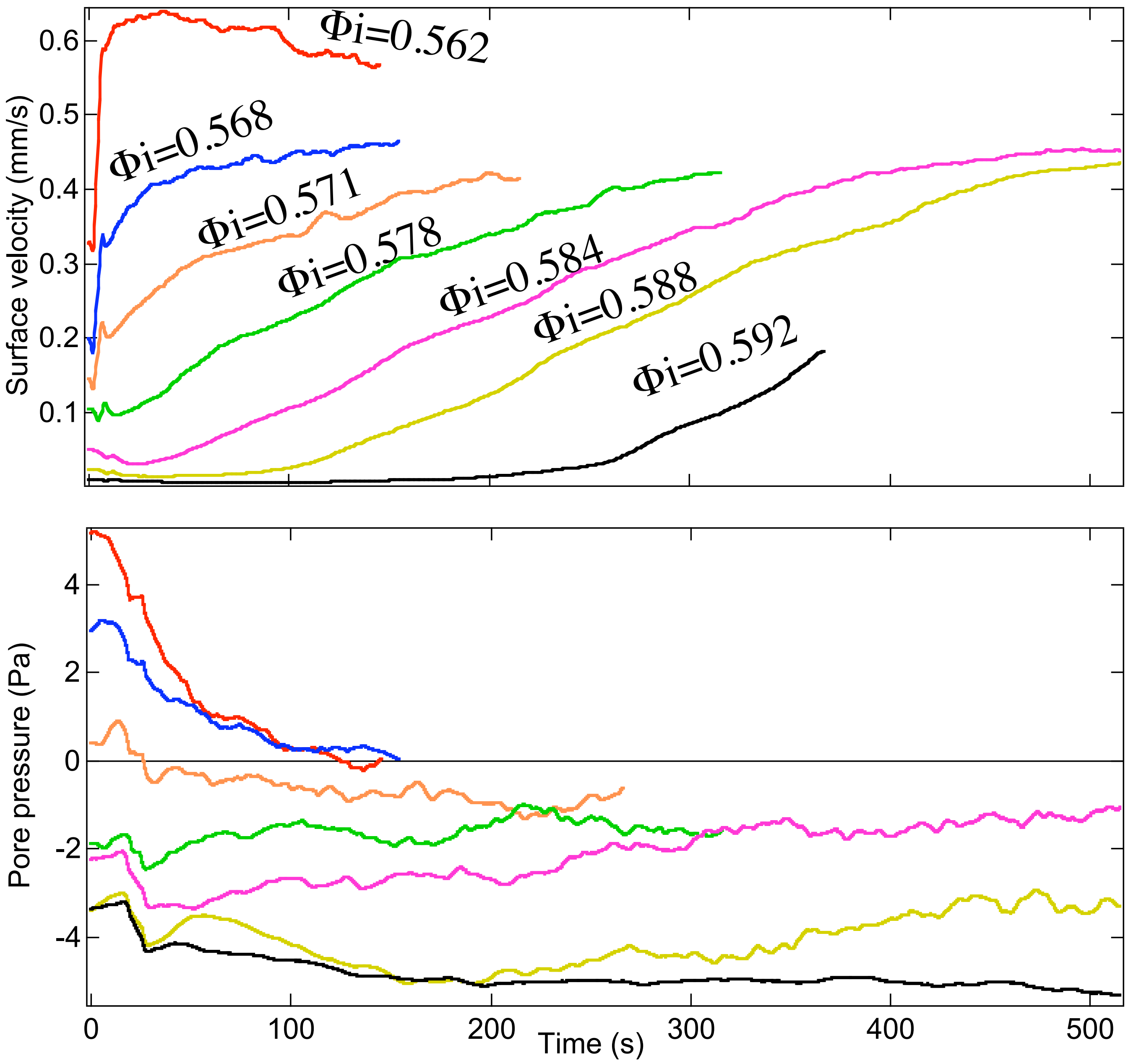


Velocity of the
Free surface

# Typical results

$\Theta=25°$
$h=5mm$
$\eta=96\ 10^{-3}$ Pa.s

Pressure under
The avalanche




(Pailha et al Pof 08)

Velocity of the
Free surface

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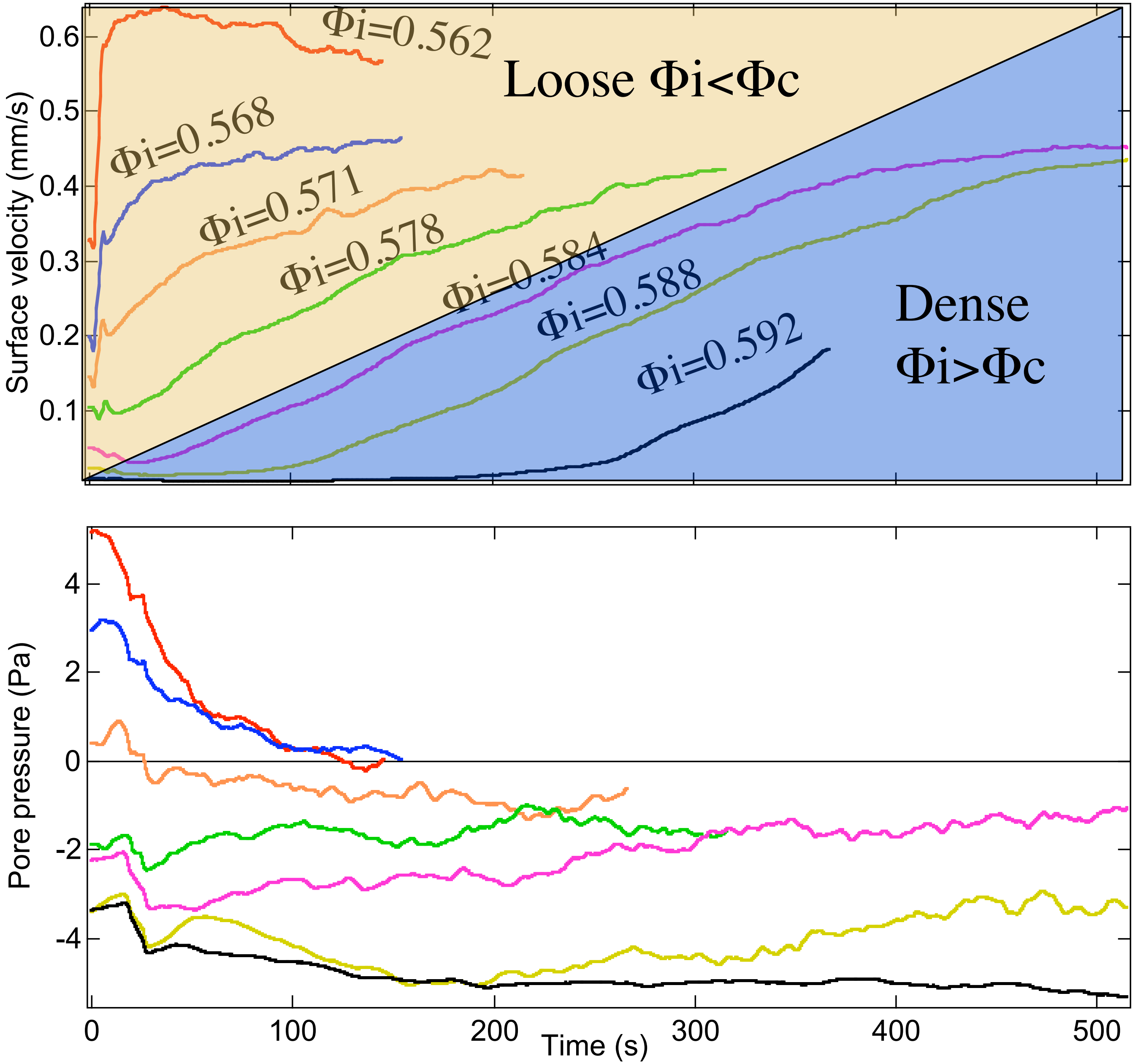
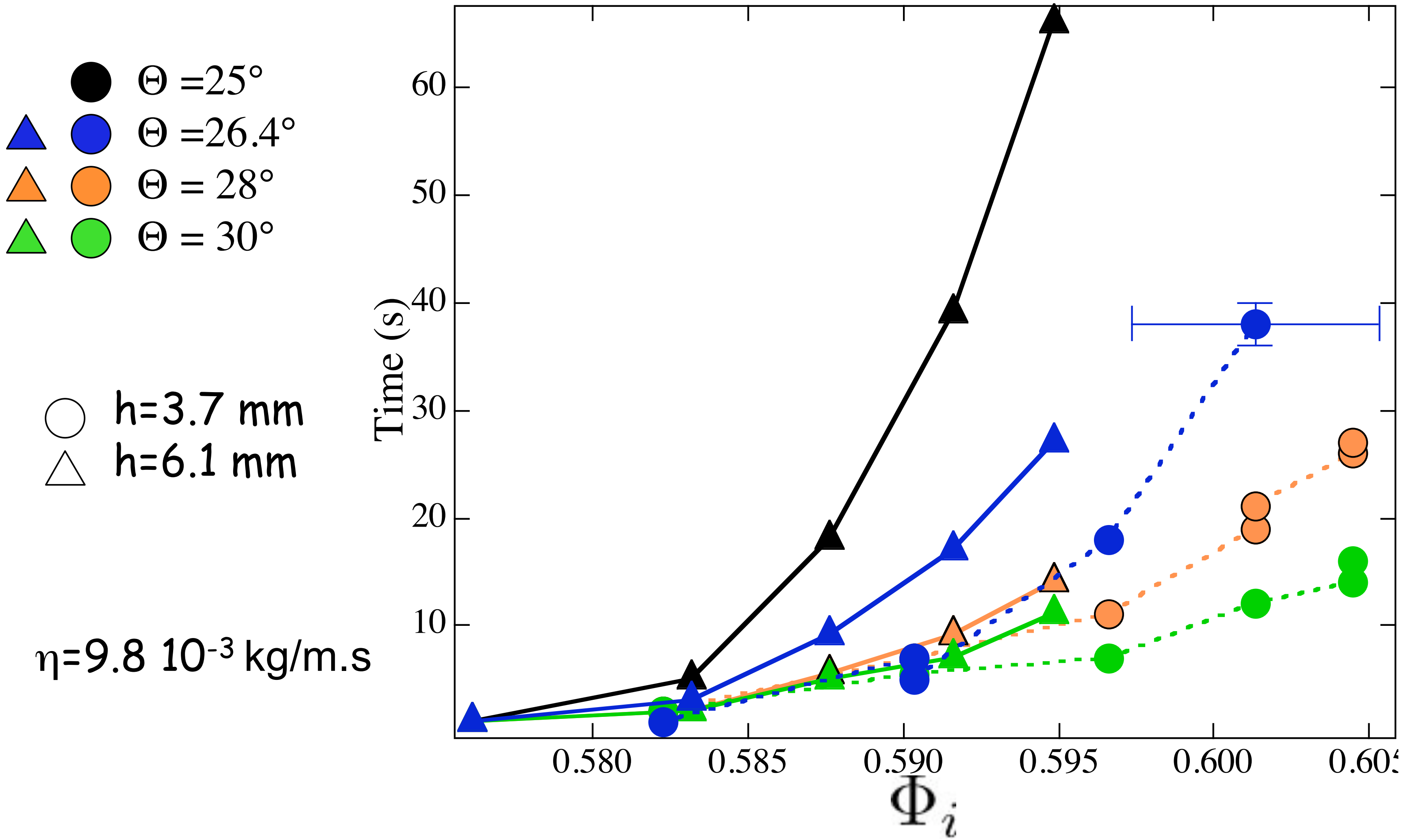


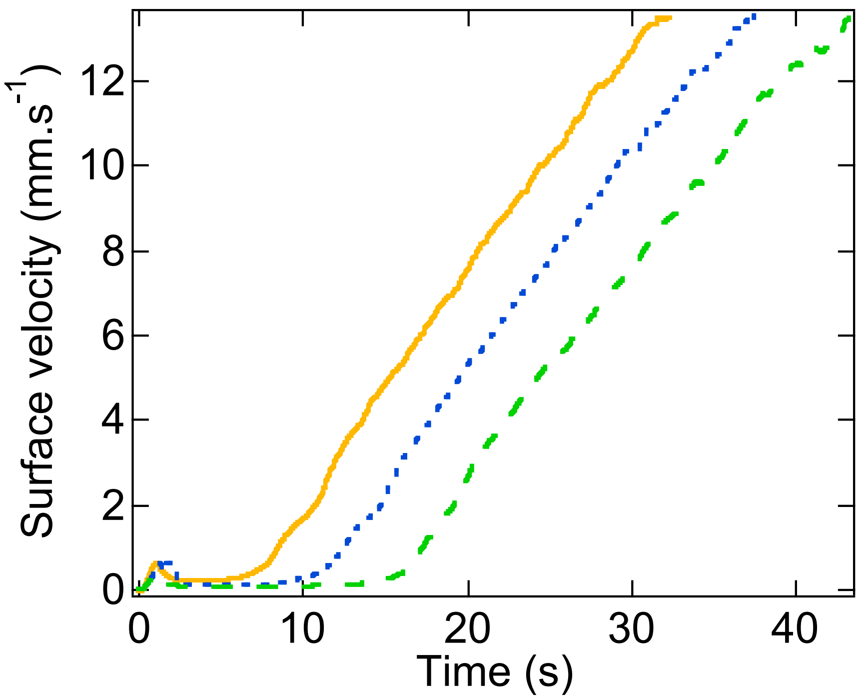
# *Triggering time*

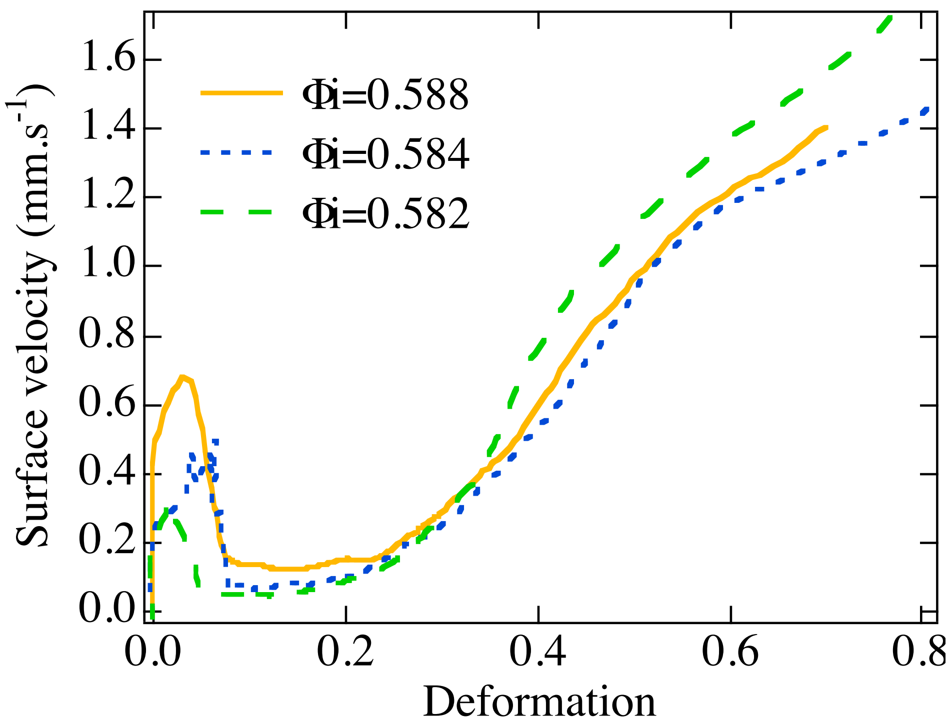
# *in the dense case*

# Triggering time

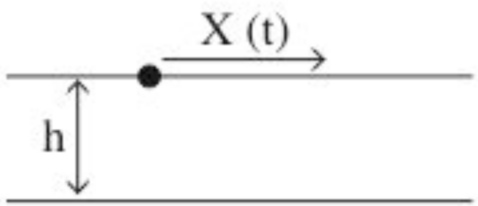


# Deformation




For $\frac{X}{h} = 0.25$




initial preparation erased

$$\frac{t_{trig} u}{h} = 0.25$$

# Two phase flow model

1 Coupling with the liquid:
Two phase equations

Fluids mechanics

2 Rheology of the
granular phase

Granular matter

3 Dilatancy

Soil mechanics

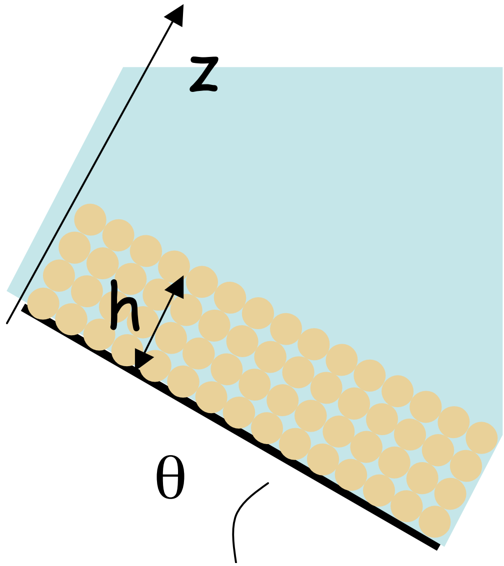
## submarine avalanches :

Depth averaged approach (Pitman and Le 05) :

$$\frac{d\overline{\phi}h}{dt} = 0$$

$$\rho_p \frac{d\overline{\phi}h\overline{u}^p}{dt} = (\rho_p - \rho_f)g\overline{\phi}h\sin\theta - \tau_b^p + (1-\overline{\phi})\beta(\overline{u}^f - \overline{u}^p)h$$

$$\rho_f \frac{d(1-\overline{\phi})h\overline{u}^f}{dt} = -(1-\overline{\phi})\beta(\overline{u}^f - \overline{u}^p)h$$



# Submarine granular avalanches:

Shear rate critical state theory

$$\boxed{\tau^p = (\mu(I) + \tan\Psi)p^p}$$

$$\boxed{\tan\Psi = K(\Phi - \Phi_{eq}(I))}$$

$$I = \frac{\dot{\gamma}_b \eta_f}{p^p}$$

$$\boxed{\frac{1}{\Phi}\frac{d\Phi}{dt} = \dot{\gamma}_b \tan\Psi}$$

(Cassar et al 05, Doppler et al 07 )

$$\dot{\gamma}_b = 3\frac{u^p}{h}$$

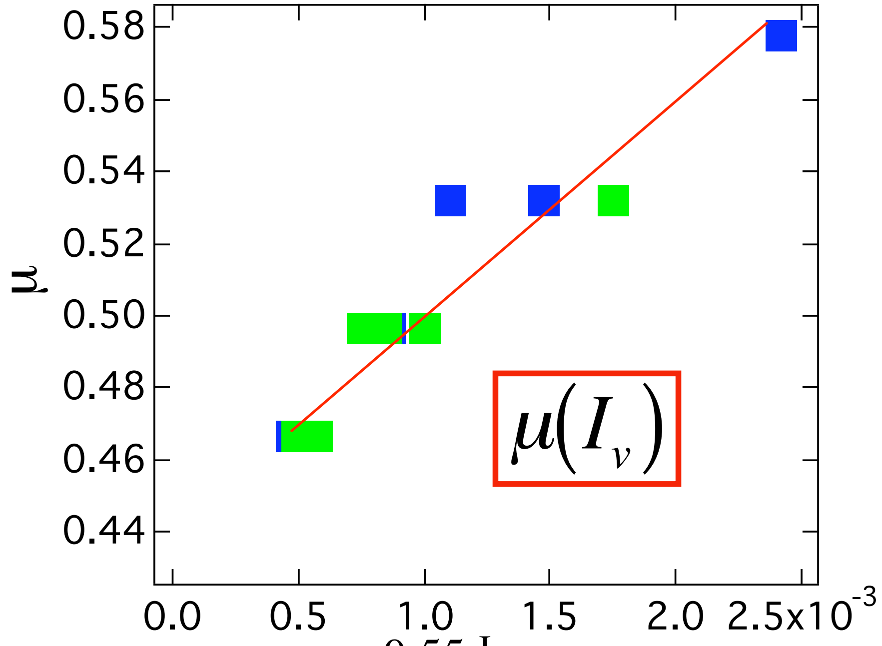
Particle -fluid coupling

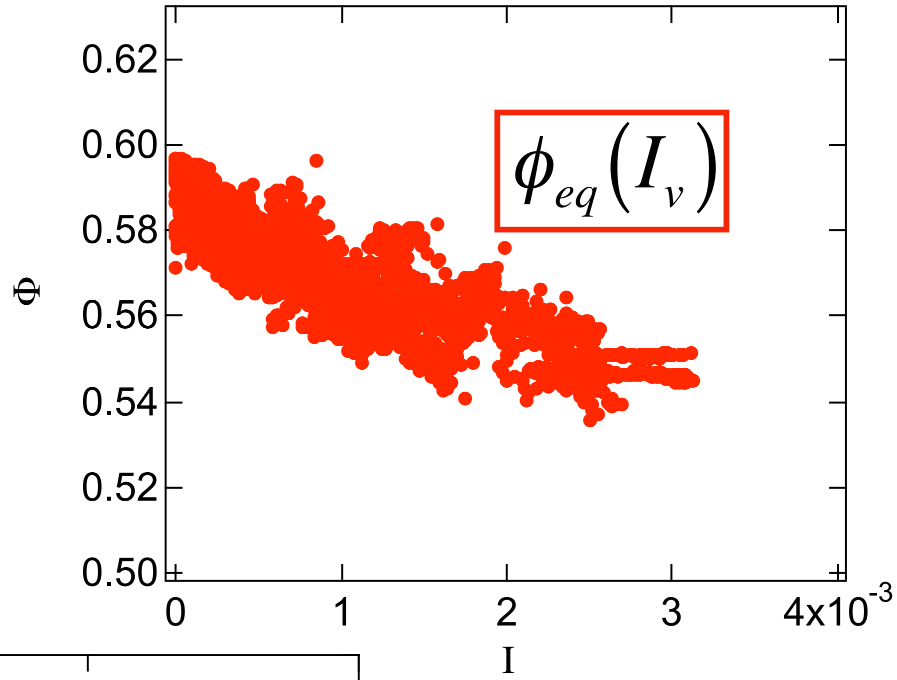
$$p^p = \Delta\Phi g h \cos\theta - K_2 \frac{\eta}{d^2} h u^p \tan\Psi$$

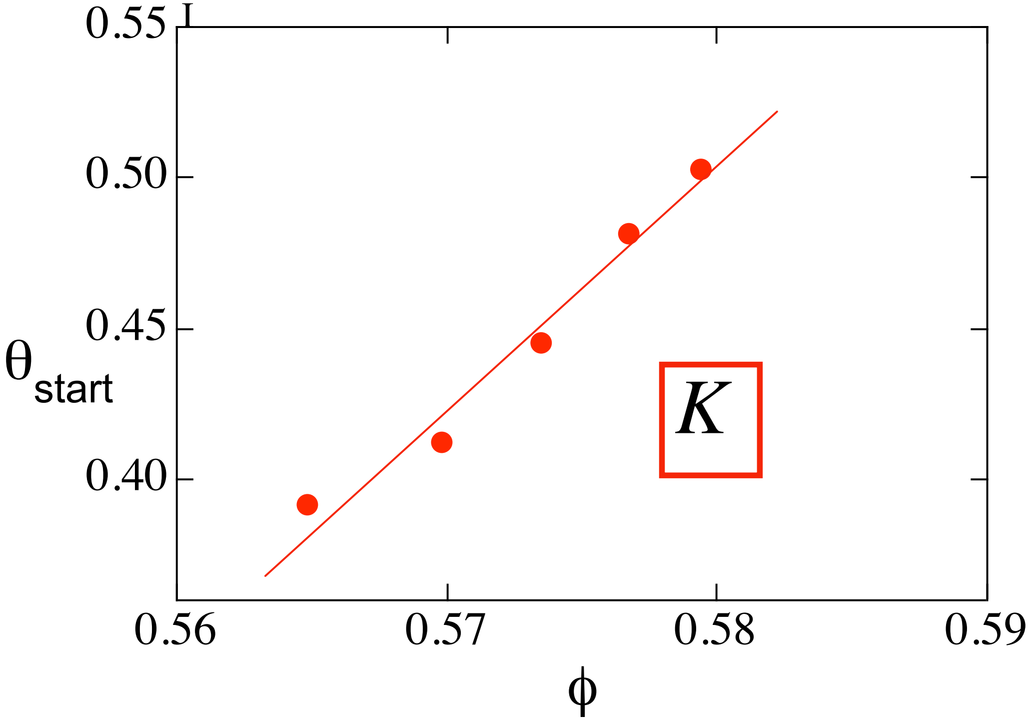
Relative weight

Viscous drag due to the Vertical displacement

## Calibration of the model looking at the steady state



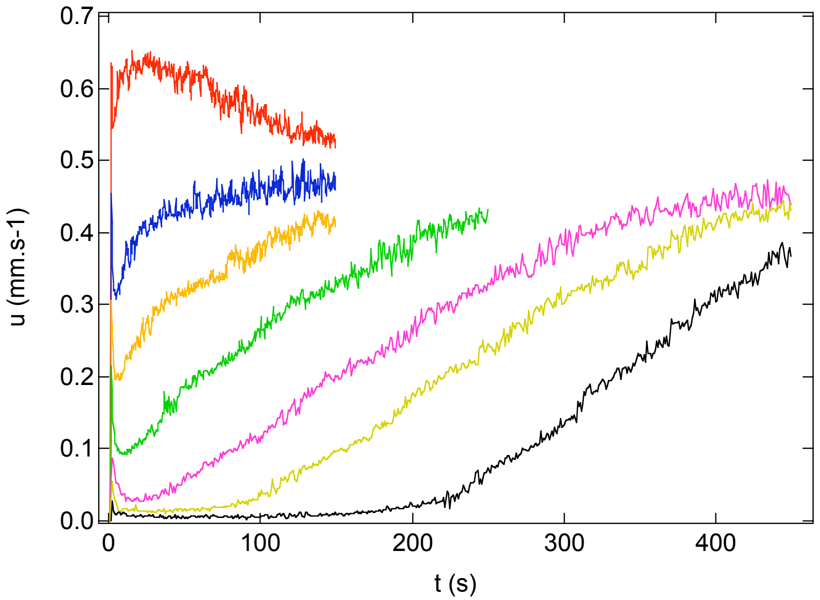



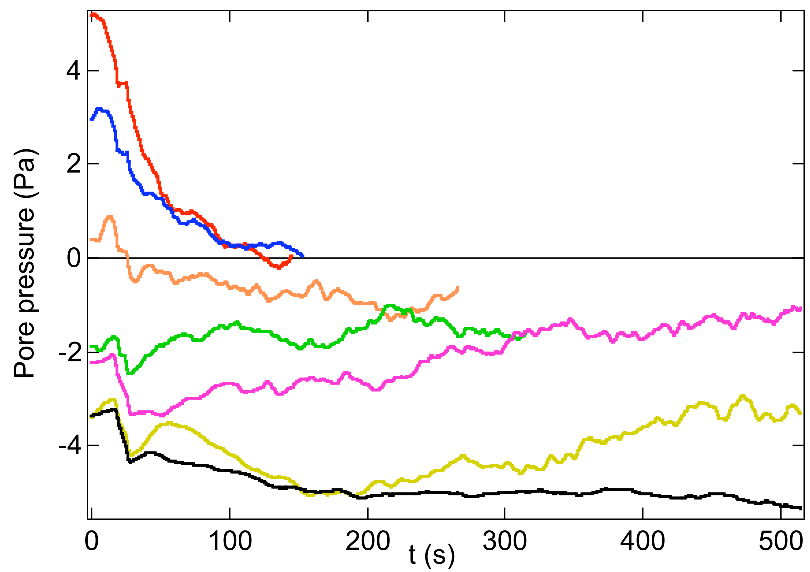

A single free
Parameter
K2

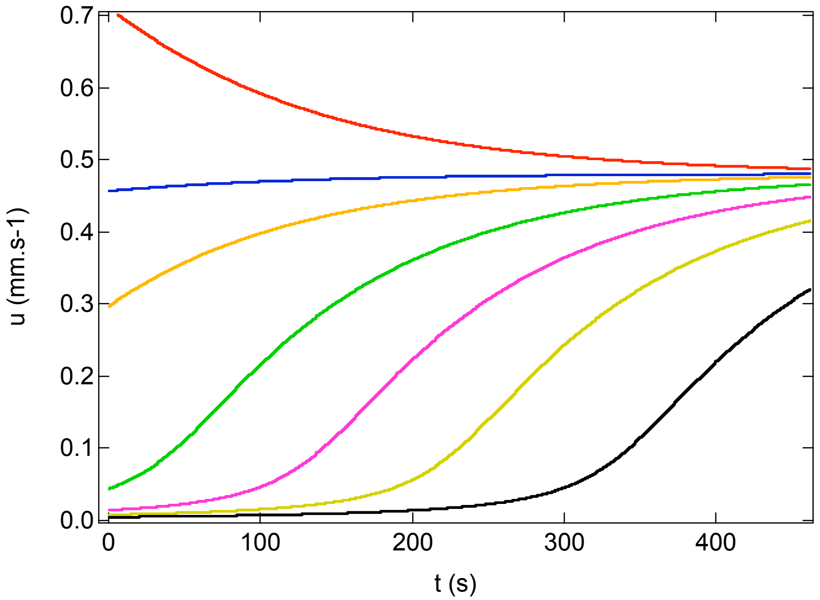
# Predictions :

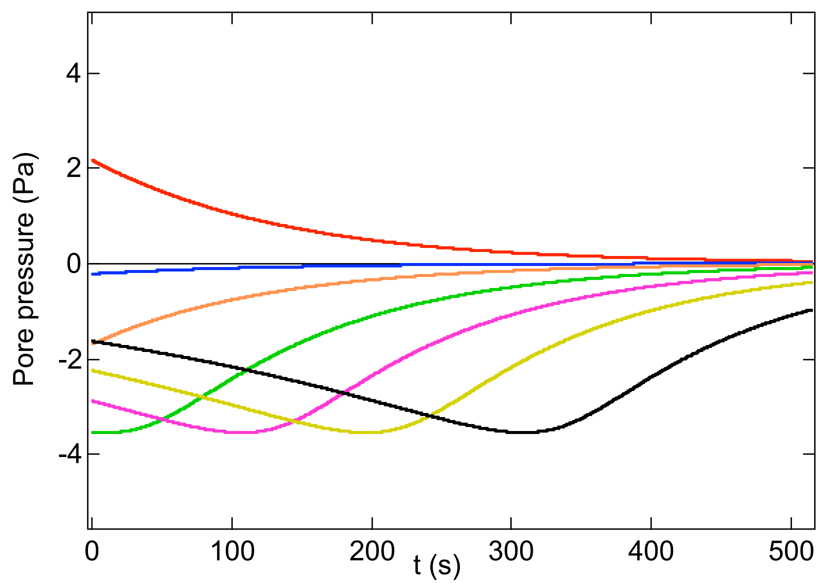
## Velocity

## Pressure



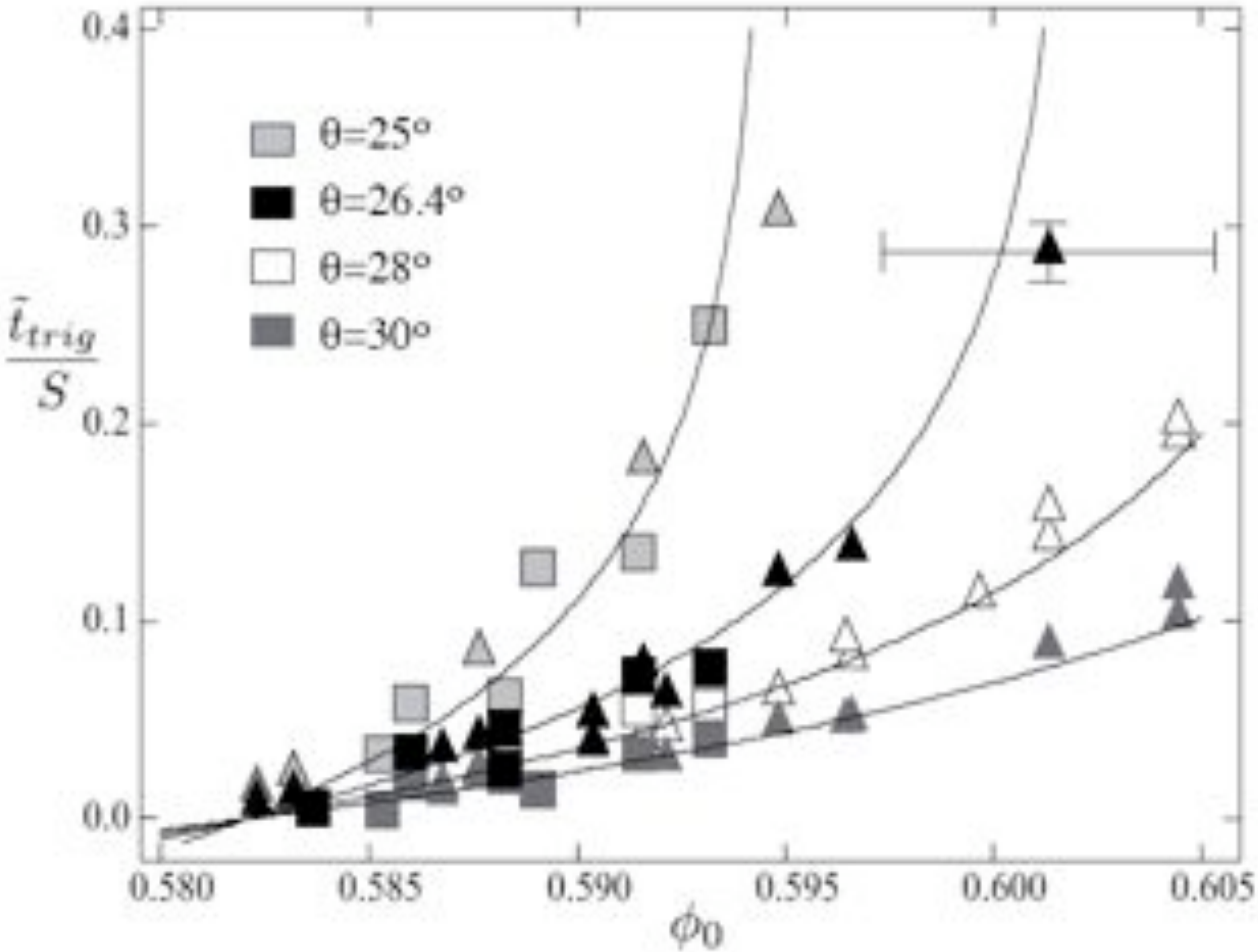



# Scaling of the triggering time

t/t0

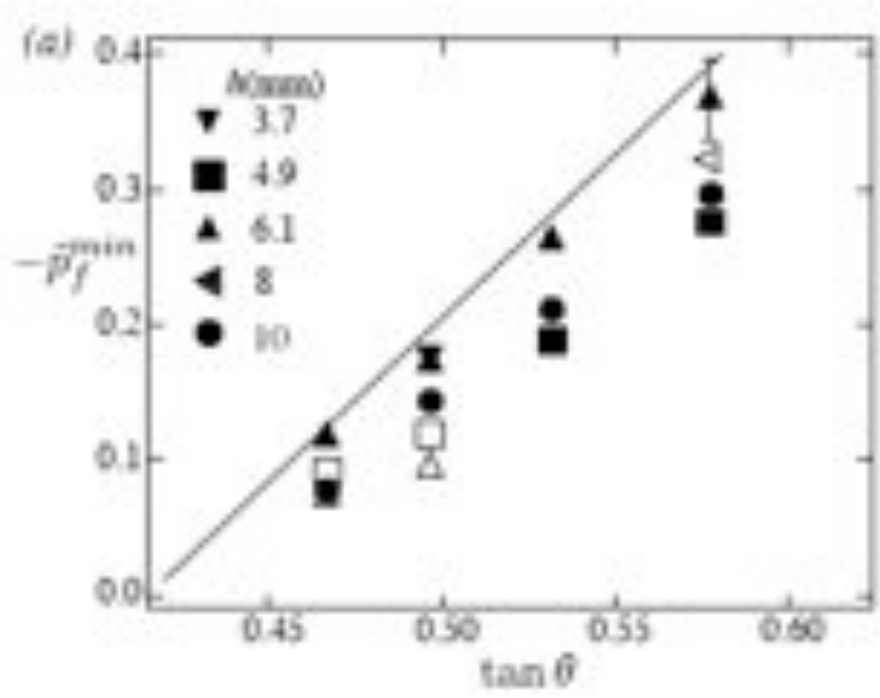


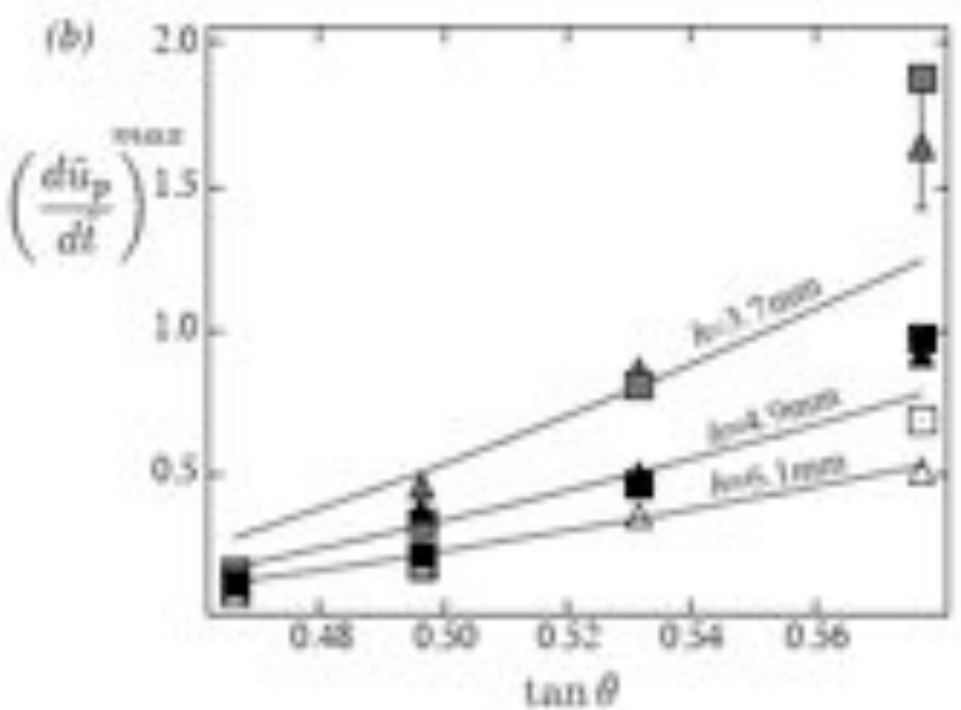

$$t_0 = \frac{0.25\eta h}{2\alpha d^2 \Delta\rho g \Phi_i \cos\theta}$$

Different h
Different viscosities

## Pore Pressure

## Maximum acceleration



# Conclusions for submarine avalanches

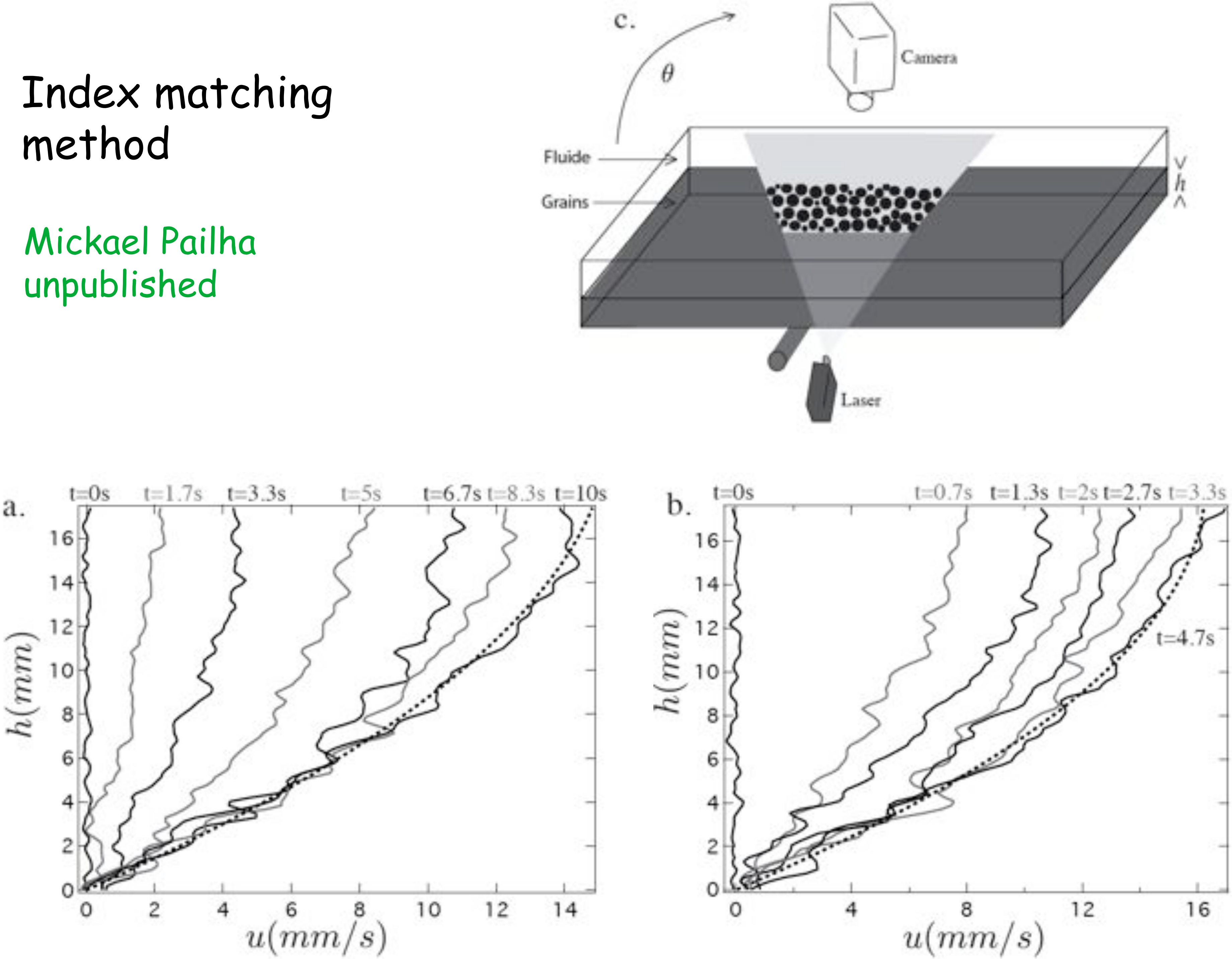
A simple critical state approach+ a viscoplastic rheology
+ two phase flow equations

⇒Semi quantitative predictions in the complex dynamics of the flow initiation of submarine avalanches

Beyond the depth averaged approach ?
Question of the numerical implementation of such models?

# Index matching method

Mickael Pailha
unpublished



# Conclusions for constitutive modeling of granular flows

Visco-plastic approach gives the order zero of viscous behavior of granular flows

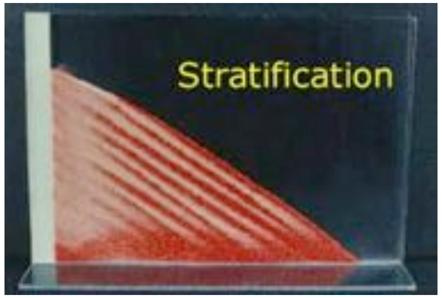
It can serve as a base for further developments:
-irregular particles?
-cohesive particles?
-polydispersed materials?
-breakable particles?
(dilatancy, underwater granular flows, cohesive flows…)

-link with the microscopic physics?
-how to capture non local effects (role of fluctuations, link with glassy systems,….) ?

# Towards more complex granular media:

Polydispersed :

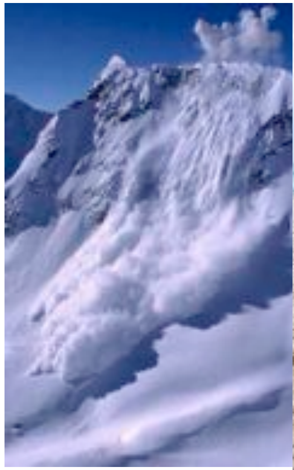
Felix et Thomas PRE 04
Rognon et al 06…




granular matter with fluid interactions

Cohesive granular matter:

Rognon et al 08
Halsey et al 06
Richefeu et al 06 …



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DE LA RECHERCHE
SCIENTIFIQUE

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UNIVERSITÉ DE PROVENCE
Aix - Marseille 1

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Cyril Cassar
Yoël Forterre
Pascale Aussillous
Maxime Nicolas
Prabhu Nott
Jeff Morris
Neil Balmforth
Bruno Andreotti
Olivier Dauchot…

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