Rheology of granular flows: Role of the interstitial fluid

Olivier Pouliquen,
IUSTI, CNRS, Aix-Marseille University
Marseille, France

Colorado 2003, USGS
Motivations:
debris flows, landslides, avalanches, silo

Not Fault!!!
Particles of different sizes + liquid (non newtonian) + complex topography + unsteady flows …

In this talk:

1) rheology of dry granular flows ?...

2) What happen when coupling with the interstitial fluid matters ...?
Dry granular material: Collection of grains

- No cohesion
- No brownian motion
- No fluid interaction

... Only contact interactions

But not so easy...
Dry granular flows

Different flow regimes

Solid

Liquid

Gas
Dry granular flows

Different flow regimes

Solid

Liquid

Gas
Quasi-static deformations:
Soil mechanics and plasticity

Focus on initial deformation

What happens:
⇒ at large deformations?
⇒ for fast deformations?
Dry granular flows

Different flow regimes

Solid

Liquid

Gas
Kinetic theory for rapid granular gases

Binary collisions + inelastic collisions

⇒ constitutive equations coupling
Density, velocity and granular temperature

But if not enough energy is injected:

⇒ finite duration contact,
⇒ multiple contact,
Dry granular flows

Different flow regimes:
- Solid
- Liquid
- Gas
Different flow configurations studied both experimentally and numerically

One imposes $P$ and $\dot{\gamma}$

Shear stress $\tau$?
Volume fraction $\phi$?

A single dimensionless number
(inertial number)

$$I = \frac{\dot{\gamma}d}{\sqrt{P/\rho_s}}$$

(Savage 84, Ancey et al 99)
Inertial number

\[ I = \frac{\dot{\gamma}d}{\sqrt{P/\rho_s}} \]

* ratio between 2 times :

\[ \frac{1}{\dot{\gamma}} \text{ : time scale of the mean shear} \]

\[ \frac{d}{\sqrt{P/\rho_s}} \text{ : microscopic time for rearrangement} \]
« quasi-static »  « liquid »  « gas »

\[ I = \frac{\dot{\gamma} d}{\sqrt{P/\rho}} \]
One imposes $P$ and $\dot{\gamma}$.

Shear stress $\tau$?
Volume fraction $\phi$?

$$\tau = \mu(I)P$$

$$\Phi = \Phi(I)$$
\[ \tau = \mu(I)P \]

\[ I = \frac{\dot{\gamma} d}{\sqrt{P / \rho}} \]

\[ \phi = \phi(I) \]
remark: No velocity weakening

Dacruz et al. PRE 05

Peyneau & Roux PRE 08
Data from

Inclined plane exp. (Pouliquen 99)
Inclined plane simulations  
(Baran et al 2006)
Annular shear cell exp. 
(Sayed, Savage JFM, 84)
An empirical friction law:

\[ \tau = \mu(I)P \]

\[ \Phi = \Phi(I) \]

\[ \mu(I) = \mu_s + \frac{\mu_2 - \mu_s}{I_0/I + 1} \]

\[ \phi(I) = \phi_{max} - AI \]
Constant pressure

\[ I = \frac{\dot{\gamma} d}{\sqrt{P/\rho_s}} \]

\[ \tau = \mu(I)P \]

\[ \Phi = \Phi(I) \]

\[ \Phi(I) = f_1^{-1}(1/I^2) \quad \mu(I) = I^2 f_2(f_1^{-1}(1/I^2)) \]

And shear at constant volume fraction ??

Bagnold Proc. R. Soc 54
Lois et al PRE 07
Lemaitre PRE 05
Da Cruz et al PRE 05
allows to describe (not perfectly) velocity profiles on inclined plane,

\[ \tau = \mu(I)P \]
\[ \Phi = \Phi(I) \]

Let's go further...
Predicted velocity and volume fraction profiles

Rheology $\mu(I)$ predicts $- V \propto h^{1.5} - (h-z)^{1.5}$

$\Phi = \text{cte}$

- Pb with thin flows and close to free surface

Gdr Midi et al, 2004,
Da cruz et al 2002,
Silbert et al 2001

Gdr Midi et al, 2004,
Da cruz et al 2002,
Rajchenbach 2003
\[ \tau = \mu(I)P \]
\[ \Phi = \Phi(I) \]

allows to describe (not perfectly) velocity profiles on inclined plane, on pile,…

Let’s go further…
3D generalisation: a visco-plastic model (Jop et al Nature 06)

assumptions:

1) \( P \) isotropic

2) \( \dot{\gamma}_{ij} \) and \( \tau_{ij} \) are colinear

(Savage 83, Goddard 86, Schaeffer 87,...)

\[
\tau_{ij} = \mu(I)P \frac{\dot{\gamma}_{ij}}{||\dot{\gamma}||}
\]

\[
\dot{\gamma}_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}
\]

\[
||\dot{\gamma}|| = \sqrt{1/2 \dot{\gamma}_{ij} \dot{\gamma}_{ji}}
\]

Effective viscosity
3D generalisation of the friction law:
granular flows as a viscoplastic fluid
(Jop et al Nature 06)

assumptions:
1) P isotropic
2) $\dot{\gamma}_{ij}$ and $\tau_{ij}$ are co-linear
(Savage 83, Goddard 86, Schaeffer 87, ...)

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} &= 0, \\
\rho_s \phi \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial z} \right) &= \rho_s \phi g \sin \theta - \frac{\partial P}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z}, \\
\rho_s \phi \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial z} \right) &= -\rho_s \phi g \cos \theta - \frac{\partial P}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z},
\end{align*}
\]

\[
\tau_{ij} = \frac{\mu(I)P}{\dot{\gamma}} \dot{\gamma}_{ij}
\]

\[
I = \frac{\dot{\gamma} d}{\sqrt{P/\rho_s}}
\]

Pressure dependent viscosity
flows on a heap: a full 3D problem

(P. Jop et al Nature 06)

$L = 1.5\, \text{m}$

$V(y,z)$
Flow between rough lateral walls:

Jop et al., Nature 2006
Jop et al, Phys. Fluids 2007

Initiation of the flow?
Long wave instability in granular flows
(Y. Forterre, JFM 06)
Experimental Setup: forcing of the instability

Forterre and Pouliquen JFM 02
Instability threshold

Dispersion relation

Forterre, JFM 06
Granular slumping
(Lacaze and Kerswell 08)

Lube et al JFM 2004,
Larrieu et al JFM 2006,
Staron & Hinch JFM 2005,
Lacaze et al Phys. Fluids 2008
...

Lajeunesse et al 05
Lacaze and Kerswell
(preprint 08)
Relative Success of the visco-plastic description.

A starting point to address other configurations...
(simulating the pressure dependent visco-plastic rheology is non trivial...)

But there are problems when approaching the solid...
Limits of the viscoplastic approach:

1) Quasistatic flows (shear band, finite size effect....) 
   A need for non local approach...

2) Transient flows when preparation plays a crucial role
Velocity profile

Exponential tail
Not predicted..
Shear bands in quasi-static flow

(Forterre & Pouliquen ARFM 08, Jop PRE 08)

Howell et al PRL 99
Mueth et al Nature 00
Bocquet et al PRE 02
...

Not captured by the viscoplastic approach
Limit of a local rheology?

Flow threshold
hysteresis
Finite size effects

Not captured by the viscoplastic approach
to go further?

Role of the fluctuations?
Aranson and Tsimring PRE,01,
Louge Phys. Fluids 03,
Josserand et al 06
Lemaitre 02
Bazant 07
Nott 08
Behringer 08...

Role of the correlations?
Pouliquen et al 01,
Ertas and Halsey 03,
Mills et al 08
Jenkins and Chevoir 01,
Jenkins Phys. Fluids 06,
...

Link with plasticity of other amorphous and glassy systems
Evidence for non local effects: Microrheology experiments

M. Van Hecke 2008
Pouliquen, Forterre, Nott

\[ \Omega = 0 \quad \Omega \neq 0 \]
\[ \nu_{\text{creep}} \propto \exp\left(\frac{F - F_c}{F_c}\right) \]
Self activated process

\[ \dot{\gamma}(z) = \sum_{z'} f_{z' \rightarrow z} \left( p_{z' \rightarrow z}^+ - p_{z' \rightarrow z}^- \right), \]

\[ z' \rightarrow z = |\dot{\gamma}(z')| \]

Poulouquin & Forterre, Phil. Trans, 2009
Limits of the viscoplastic approach:

1) Quasistatic flows (shear band, finite size effect...) 
   A need for non local approach...

2) Transient flows when preparation plays a crucial role
Influence of the initial Volume fraction on the Collapse of a pile.

Daerr & Douady 99
Quasi-static case: critical state theory

Coupling Friction- dilatancy
Reynolds Dilatancy
Simple critical state theory

Dilatancy angle

\[ \frac{dY}{dX} = -\frac{1}{\Phi} \frac{d\Phi}{d\gamma} = \tan \Psi \]

assumption: \( \exists \) critical volume fraction \( \phi_c \)

\[ \tan \Psi = K(\Phi - \Phi_c) \]

(Radjai and Roux 98)
Visco plastic theory:
shear rate dependence
but no dilatancy

critical state theory:
dilatancy but
no shear rate
dependence

Shear rate dependent critical state theory
Shear rate dependent critical state theory:

\[
\tau = (\mu(I) + \tan \Psi)P
\]

\[
\tan \Psi = K(\Phi - \Phi_{eq}(I))
\]

\[
\frac{1}{\Phi} \frac{d\Phi}{dt} = \dot{\gamma} \tan \Psi
\]

\[
I = \frac{\dot{\gamma} d}{\sqrt{P/\rho}}
\]
3D generalisation:

\[ \tilde{\gamma}_{ij} = \dot{\gamma}_{ij} - \frac{1}{3} \delta_{ij} \gamma_{kk} \]

\[ I = \frac{\|\dot{\gamma}\|d}{\sqrt{P/\rho_s}} \]

\[ \tan \psi = K(\Phi - \Phi_c(I)) \]

\[ \tau_{ij} = (\mu(I) + \tan \Psi) \frac{\dot{\gamma}_{ij}}{\|\dot{\gamma}\|} \]

\[ \frac{\partial u_i}{\partial x_i} = \tan \Psi \|\dot{\gamma}\| \]
Application to a dry flow:

Initiation of flow on an inclined plane:

\[ \Phi_i \]
Initially dense

Initially loose

Comparison with DEM simulations with N. Taberlet...
Changing time scales... by putting the granular material in water (Cassar et al, Phys. Fluids 06)

\[ 
\text{DV recorder} \rightarrow \text{DV} \\
\text{P.C} \rightarrow \Delta P \\
d=112 \, \mu\text{m} \text{ glass beads} \\
\text{Laser} 
\]
A naive idea:

fluid only plays a role by changing the time scale of rearrangements

\[
\tau = P \mu(I) \quad \text{with} \quad I = \dot{\gamma} t_{\text{micro}}
\]

**viscous:** \[t_{\text{micro}} = \frac{\eta_f}{P}\]

**dry:** \[t_{\text{micro}} = \frac{d}{\sqrt{P/\rho_s}}\]
\[ \mu = \tan \theta \]

Cassar et al. Phys. Fluids 05
Submarine flows on heap

Doppler et al, JFM 07

Flow rate

Velocity profile
And dilatancy ???

And Pore pressure ??

Cf In Faults
Rice JGR 75, Rudnicki JGR 84, ...
How to explain the variety of landslides observed in nature?

Iverson et al., (2000) Science

Large scale experiments in the USGS facility
Dense preparation

Courtesy of Dick Iverson
Loose preparation

Courtesy of Dick Iverson
Pore Pressure feedback argument
(Iverson Rev. Geo. 97, JGR 05)

Loose case

\[ \Phi \uparrow \]
\[ \Rightarrow \text{Fluid expelled} \]
\[ \Rightarrow P_{\text{fluid}} \uparrow \]
\[ \Rightarrow P_{\text{eff}} \downarrow \]
\[ \Rightarrow \text{Friction} \downarrow \]
\[ \Rightarrow \text{Less friction} \]
\[ \text{between grains} \]

Dense case

\[ \Phi \downarrow \]
\[ \Rightarrow \text{Fluid sucked} \]
\[ \Rightarrow P_{\text{fluid}} \downarrow \]
\[ \Rightarrow P_{\text{eff}} \uparrow \]
\[ \Rightarrow \text{Friction} \uparrow \]
\[ \Rightarrow \text{higher friction} \]
\[ \text{between grains} \]

In faults: rice JGR 75, Rudnicki JGR 84, ...
A simple experiment:

Dense sample

Loose sample
Experimental setup
(Pailha et al 08)

Glass beads : 160µm

Liquid: mixture of water and Ucon oil:
\[ \eta = 9.8 \times 10^{-3} \text{ kg/m.s} \]
\[ \eta = 96 \times 10^{-3} \text{ kg/m.s} \]
Experimental procedure

Compaction by taps

Result of sedimentation
Velocity of the Free surface

Typical results

\[\Theta=25^\circ\]
\[h=5\text{mm}\]
\[\eta=96\times10^{-3}\text{ Pa.s}\]

Pressure under The avalanche

(Pailha et al Pof 08)
Velocity of the Free surface

Typical results

$\phi = 25^\circ$
$h = 5\text{mm}$
$\eta = 96 \times 10^{-3} \text{Pa.s}$

Pressure under The avalanche

(Pailha et al Pof 08)
Triggering time in the dense case
**Triggering time**

- $\theta = 25^\circ$
- $\theta = 26.4^\circ$
- $\theta = 28^\circ$
- $\theta = 30^\circ$

- $h = 3.7 \text{ mm}$
- $h = 6.1 \text{ mm}$

$\eta = 9.8 \times 10^{-3} \text{ kg/m/s}$
Deformation

For $\frac{X}{h} = 0.25$

$\frac{t_{\text{trig}} u}{h} = 0.25$

initial preparation erased
Two phase flow model

1. Coupling with the liquid: Two phase equations
   - Fluids mechanics

2. Rheology of the granular phase
   - Granular matter

3. Dilatancy
   - Soil mechanics
submarine avalanches:

Depth averaged approach (Pitman and Le 05):

\[
\begin{align*}
\frac{d\bar{\phi}h}{dt} &= 0 \\
\rho_p \frac{d\bar{\phi}h\bar{u}^p}{dt} &= (\rho_p - \rho_f)g\bar{\phi}h \sin \theta - \tau^p_b + (1 - \bar{\phi})\beta(\bar{u}^f - \bar{u}^p)h \\
\rho_f \frac{d(1 - \bar{\phi})h\bar{u}^f}{dt} &= -(1 - \bar{\phi})\beta(\bar{u}^f - \bar{u}^p)h
\end{align*}
\]
Submarine granular avalanches:

Shear rate critical state theory

\[
\dot{\gamma}_b = 3 \frac{u^p}{h}
\]

Viscous drag due to the vertical displacement

\[
\tan \Psi = K(\Phi - \Phi_{eq}(I))
\]

\[
\frac{1}{\Phi} \frac{d\Phi}{dt} = \dot{\gamma}_b \tan \Psi
\]

\[
\tau^p = (\mu(I) + \tan \Psi)p^p
\]

\[
I = \frac{\dot{\gamma}_b \eta_f}{p^p}
\]

(Cassar et al 05, Doppler et al 07)

Particle-fluid coupling

\[
p^p = \Delta \Phi gh \cos \theta - K_2 \frac{\eta}{d^2} hu^p \tan \Psi
\]

Relative weight

Viscous drag due to the Vertical displacement
Calibration of the model looking at the steady state

$\mu(I_v)$

$\phi_{eq}(I_v)$

$K$
Predictions:

**Velocity**

**Pressure**
Scaling of the triggering time

\[ t_0 = \frac{0.25\eta h}{2\alpha d^2 \Delta \rho g \Phi_i \cos \theta} \]

Different h
Different viscosities
Pore Pressure

Maximum acceleration

(a) $\bar{p}_f^{\min}$ vs. $\tan \theta$ with markers for different $h$ (mm).

(b) $(\frac{d\bar{u}_p}{dt})_{\max}$ vs. $\tan \theta$ for different $h$ (mm).
A simple critical state approach + a viscoplastic rheology + two phase flow equations

⇒ Semi quantitative predictions in the complex dynamics of the flow initiation of submarine avalanches

Beyond the depth averaged approach?
Question of the numerical implementation of such models?
Index matching method

Mickael Pailha
unpublished
Conclusions for constitutive modeling of granular flows

Visco-plastic approach gives the order zero of viscous behavior of granular flows

It can serve as a base for further developments:
- irregular particles?
- cohesive particles?
- polydispersed materials?
- breakable particles?
(dilatancy, underwater granular flows, cohesive flows...)

- link with the microscopic physics?
- how to capture non local effects (role of fluctuations, link with glassy systems,....) ?
Towards more complex granular media:

Polydispersed:
Felix et Thomas PRE 04
Rognon et al 06...

Cohesive granular matter:
Rognon et al 08
Halsey et al 06
Richefeu et al 06 ...

granular matter with fluid interactions
Merci à

Mickael Pailha
Pierre Jop,
Cyril Cassar
Yoël Forterre
Pascale Aussillous
Maxime Nicolas
Prabhu Nott
Jeff Morris
Neil Balmforth
Bruno Andreotti
Olivier Dauchot...