Self-Organized Criticality, Complex Networks and Earthquakes

Carmen P. C. Prado

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Erice, Italy

Outlook

1. Part 1: Introduction & Revision

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    1.1 Self-organized Criticality (SOC)

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   1.2 Olami-Feder-Christensen model (OFC)
   1.3 Complex Networks
2. Part 2: Complex Networks & Earthquakes
   2.1 Network of epicenters

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   2.2 Results for OFC model

Outlook

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Collaborators

- Osame Kinouchi
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- Tiago P. Peixoto
- André Timpanaro

Financial Support

- FAPESP and CNPq

Self-organized Criticality

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thus

$$F(c\,x) \propto F(x)$$

The same kind of *'physics'* govern phenomena in all scales.

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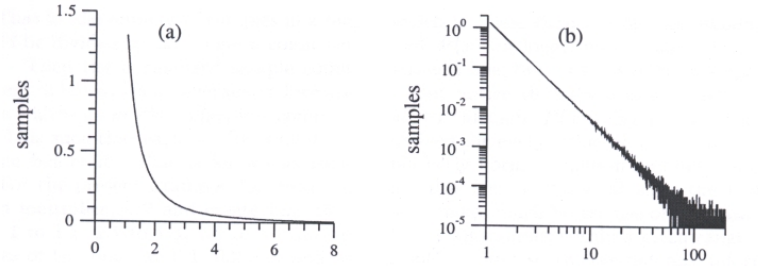
Different phenomena, described by power laws with the same *scaling exponent* share common features in the dynamical processes that generate the power-law;

its dynamics must not depend on details of the system.

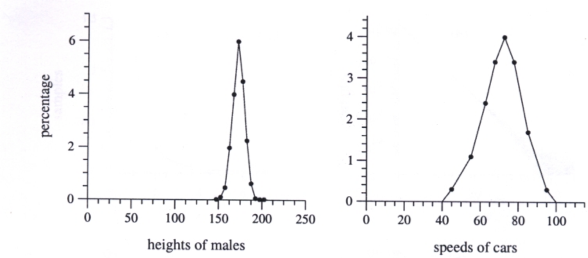
Self-organized Criticality

(b) Power-law distributions decay slowly:

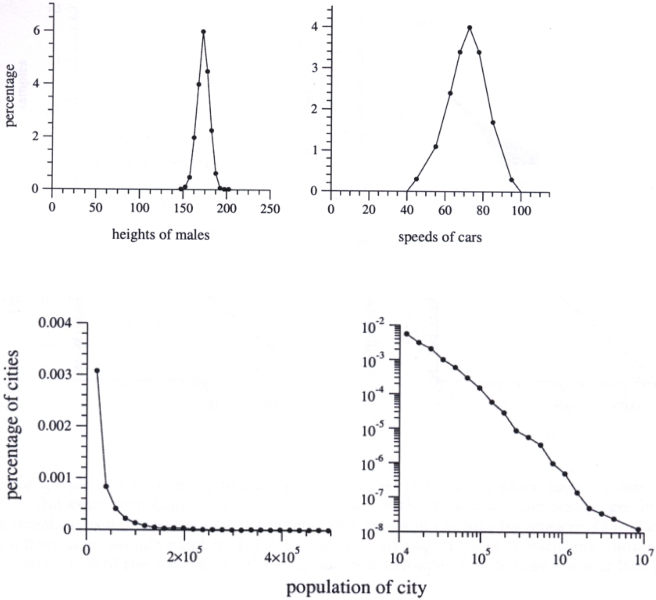
- They have long tails, an “excess” of big events;
- Average = ???



Power-laws


percentage
0
2
4
6
0
50
100
150
200
250
heights of males
0
1
2
3
4
0
20
40
60
80
100
speeds of cars

Power-laws



percentage
6
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0
0
20
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100
speeds of cars
percentage of cities
0.004
0.003
0.002
0.001
0
0
2×10^5
4×10^5
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
10^-8
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population of city

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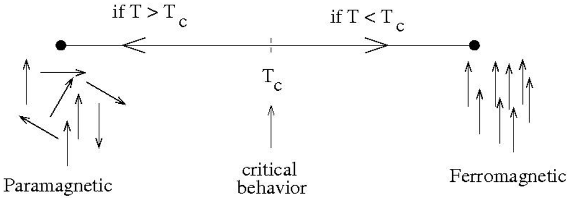
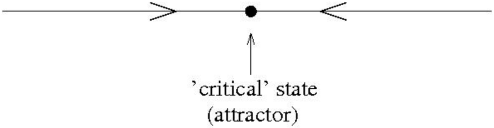


Self-organized Criticality

However, to explain the ubiquity of power-laws, the 'critical' state must be **stable**:



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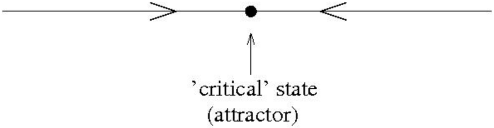

With the concept of SOC, we have a mechanism explaining how non equilibrium extended systems can evolve **naturally** to an stable critical state.

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The behavior of out of equilibrium, extended systems, that, **under a slow drive**, instead of evolving slowly and continuously, **stay static (in an apparent equilibrium) for long periods of time**, and, from time to time, experience **fast relaxation processes**, that led the system to **another (statistically similar) equilibrium state**.

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Distribution functions of quantities describing those relaxation processes exhibits power-law behavior.

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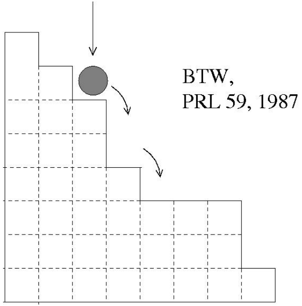
Scaling emerges as an interplay between a **threshold dynamics**, and a **quasi-static driving** (Jensen,1998).



The prototype of SOC is the **sandpile model**, proposed by Bak, Tang and Wisenfeld in 1897.

The addition of a grain of sand can cause an avalanche,

the distribution function of the *size of avalanches* displays a power-law, that is, there is no typical avalanche size; the slope of the pile oscillates around an average value.

Earthquakes are good candidates to SOC:

- Two distinct **time scales**
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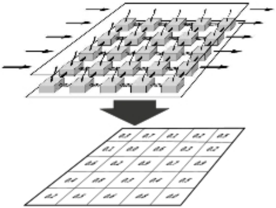
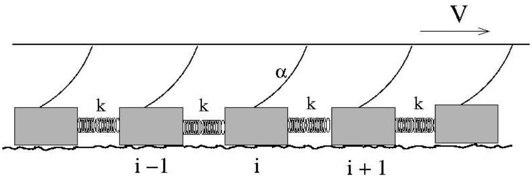
The OFC model is based
on the Burridge and Knopoff stick-slip spring block model

(Burridge, R. and Knopoff, 1967)

- The upper plate moves with V
- Static friction between blocks and lower plate
- Blocks are connected one to another by springs
- Blocks are connected also to the upper plate through springs.



OFC model



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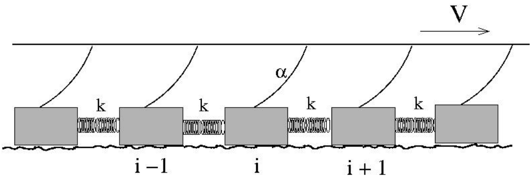

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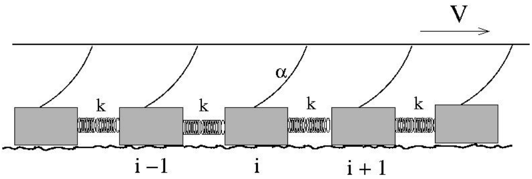

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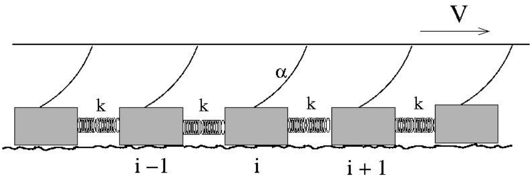

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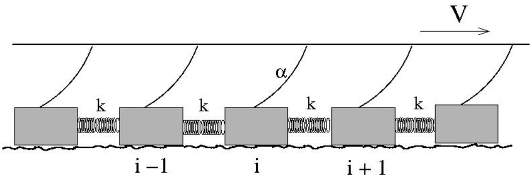

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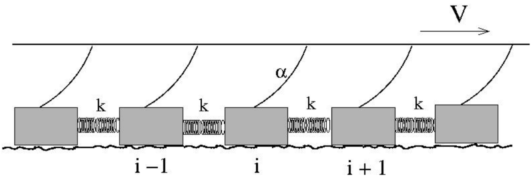

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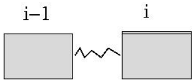
- there is static friction between block i and lower plate
- when a block slides, **it stops at the point where** $\sum F^{\text{elastic}} = 0$
- discretize **time**: in time t, only block i moves;

In two dimensions:

A simple basic physics calculation ...



$$F_i^{\text{left}} = -k\left(x_i - x_{i-1} - \ell_o\right)$$

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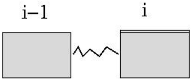

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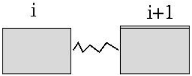

$$F_i^{\text{right}} = -k\,[\ell_o - (x_{i+1} - x_i)]$$

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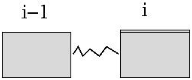

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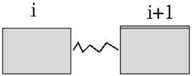

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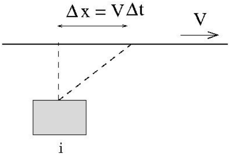

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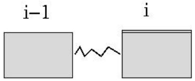

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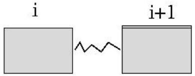

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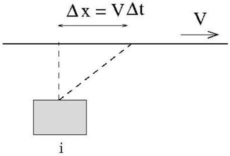

$$F_i^{\text{upper}} = -\lambda\,(V\,\Delta t - x_i)$$

$$F_i = F^{\text{left}} + F^{\text{right}} + F_i^{\text{upper}} = k(x_{i+1} + x_{i-i} - 2x_i) - \lambda\,(V\,\Delta t - x_i)$$

OFC model

Eventually, the total elastic force exceeds the static friction limit, and the block slides, **to a new x' position such that $F_i' = 0$**

$$\Delta F = F_i - F_i' = F_i = (2k + \alpha)\,(x_i' - x_i) \quad \Longrightarrow \quad (x_i' - x_i) = \frac{\Delta F}{(2k + \lambda)}$$

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$$\Downarrow$$

EARTHQUAKE,

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The same can be done for a square lattice...

Summarizing

- (Slow) **driving:**

$$F_{i,j} \quad \longrightarrow \quad F_{i,j} + \delta \quad \text{for all sites;}$$

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- The process goes on until $F_{i,j} < F_{th}\ \forall\ i, j$.

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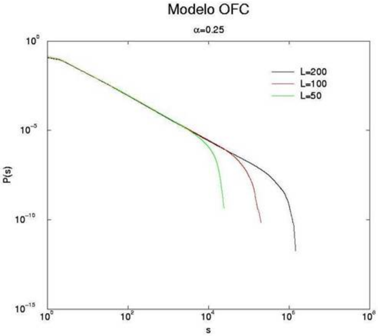
Gutemberg-Richter's:



$$P(s) \sim s^{-b}$$

OFC model

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Omori's law:

(Hergarten & Neugebauer, PRL 88, 2002)

$$N(t) \sim t^{-\alpha}$$

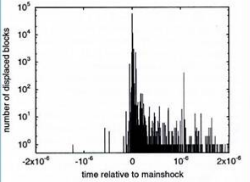

FIG. 2. A sequence of events in the OFC model.

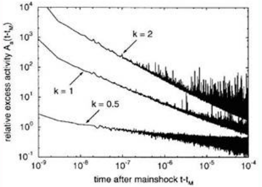

FIG. 3. Seismic activity after large earthquakes.

Hergarten, H. J. Neugebauer, PRL 88, 2002

OFC model

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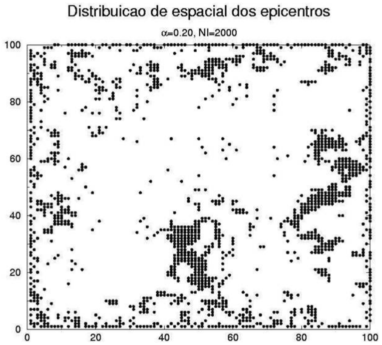
Fractal distribution of epicenters:



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  - Conservative / Non-conservative regimes

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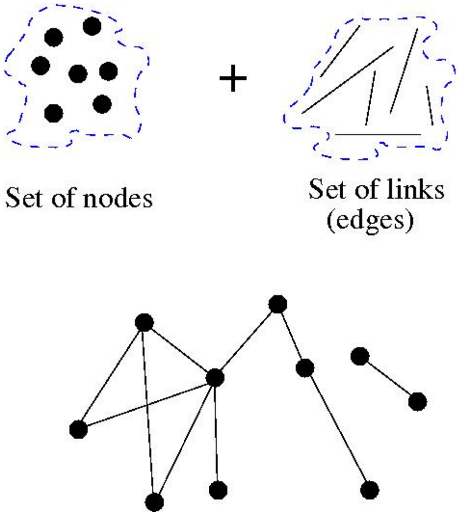
I am going to discuss scale free behavior
related to a network of epicenters, both on OFC and catalog data.

Networks

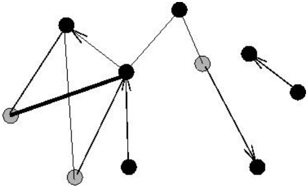
What is a Network?



Networks describe a variety of systems in nature

There are many types of networks...

* Different kinds of nodes,
* links may be directed,
* links may have not the same strength...



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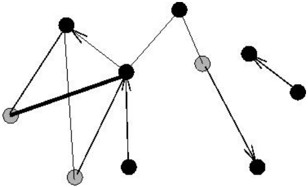

- Regular networks (lattices) have been employed / studied by physicists from way back

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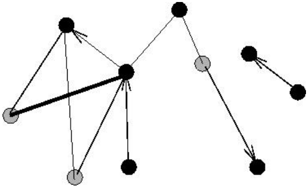

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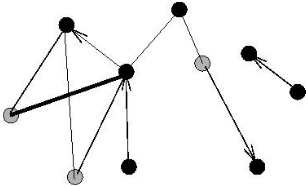

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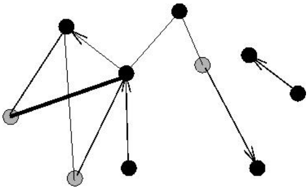

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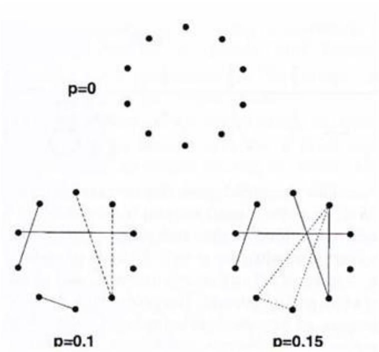
- Computers, growing of interdisciplinarity $\Longrightarrow$ New applications.

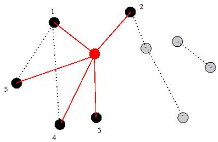
Random network



N nodes connected with probability p

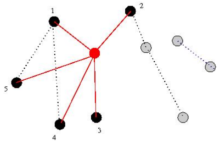
But most (irregular) networks in nature are more complicated than that!

Degree of a node



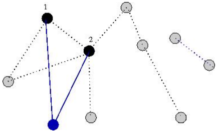
red node degree = 5

Degree of a node



red node degree = 5

Blue node degree = 2



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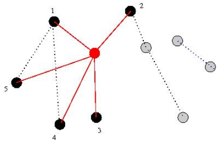

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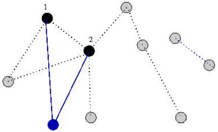
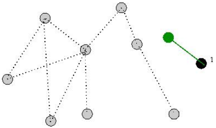
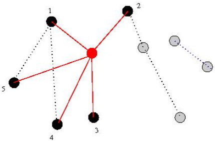




Green node degree = 1

Degree of a node



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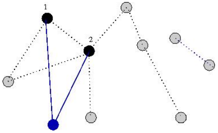
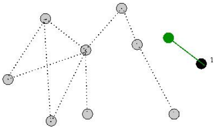
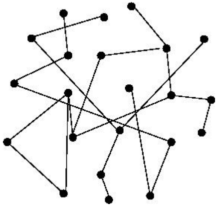


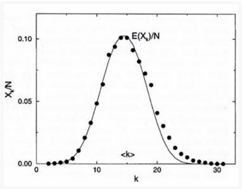


Green node degree = 1

Degree distribution $P(k) =$ **probability that a node has degree** k

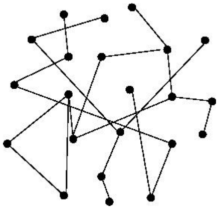
Degree distribution

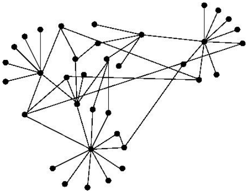


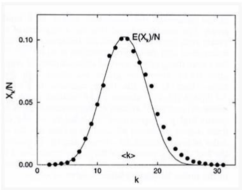


Random graph

Degree distribution







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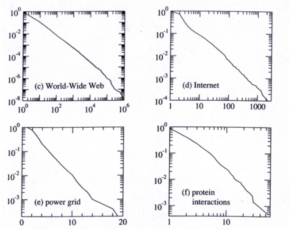
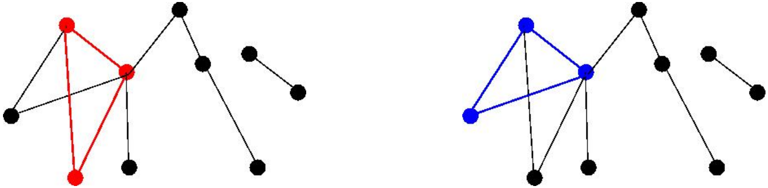


Free scale
(Newman Cont. Physics, 2005)

Clusterization

Averages the number of triangles in a network



Given two nodes A and B that are connected, the clusterization index gives the probability that a third node, connected with A, is also connected with B.

Average distance ℓ

$$\ell = \frac{1}{N(N-1)} \sum_{i,j} D_{i,j}, \qquad D_{i,j} = \text{ smallest dist. between nodes } i \text{ and } j$$

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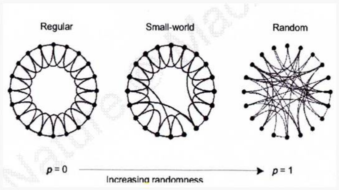
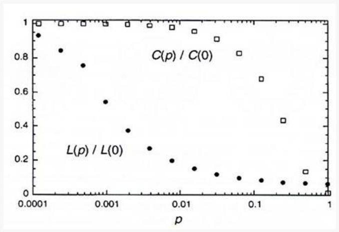
Some topologies have an important property:

In Small world networks ℓ is small even if N is very large.



Watts & Strogatz model



Assortative Mixing

Definition

We say that there are *assortative mixing* when there is a **tendency of connections among nodes with similar characteristics**, as the degree.

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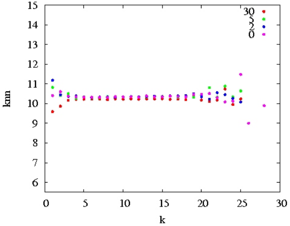
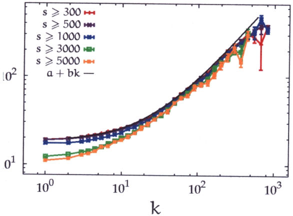
Example

Average degree correlation: the average degree of the neighbors, as a function of the degree of the site.





(b) $\alpha = 0.22$

(of course, dissortative mixing is just the opposite)

Part II: Outlook

- Network of epicenters Abe & Suzuki, EPL 65, (2004)
- Results for OFC model T.P. Peixoto, C. P. C. Prado, PRE (2004,2006)
- Comparisons with LA's catalogue T.P. Peixoto, C. P. C. Prado, PRE (2006)
- Forecasting ??? Kinouchi & Prado, PRE (99) ; de Carvalho & Prado, PRL 2000

Networks and earthquakes

There are at least 3 different ways of building a network from earthquake data:

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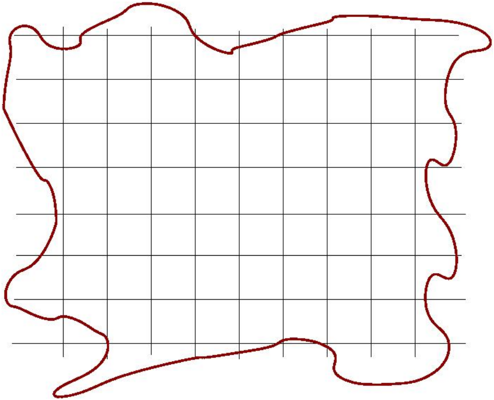
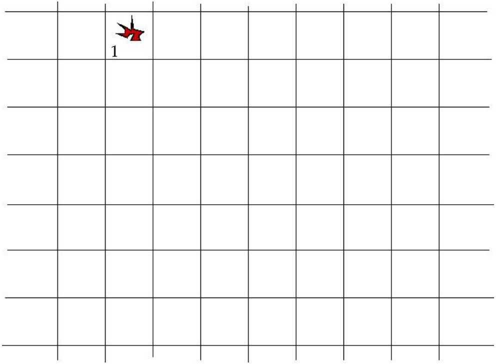
The idea is: networks entangle spatial, temporal, magnitude aspects, unveiling new correlations and structures, that maybe could not be seen in other ways...

The area of the fault is divided in cells:



successive epicenters are registered



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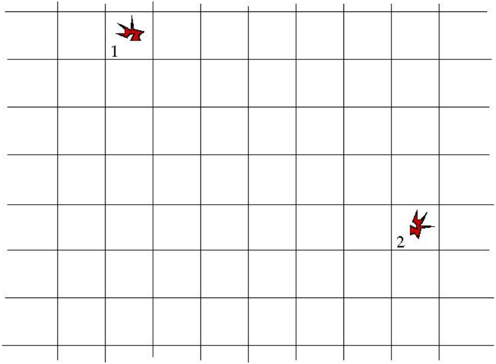

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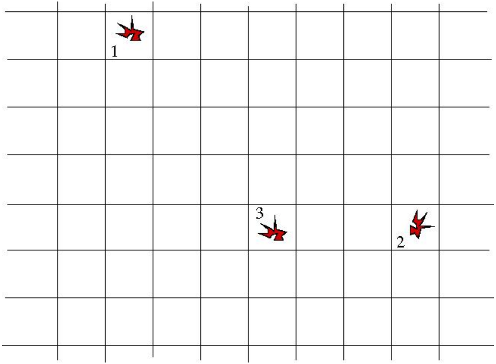

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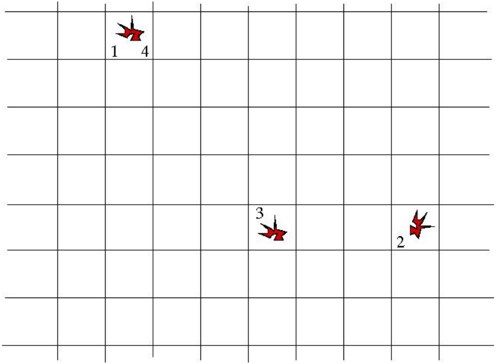

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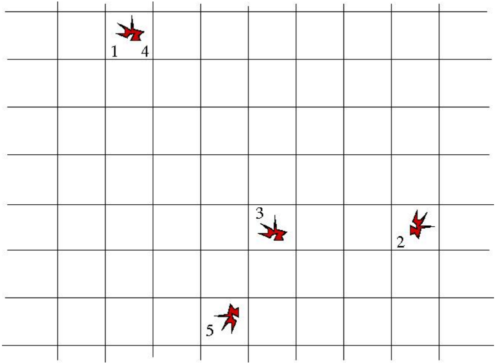

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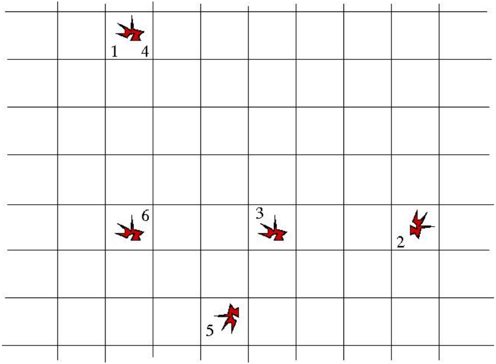

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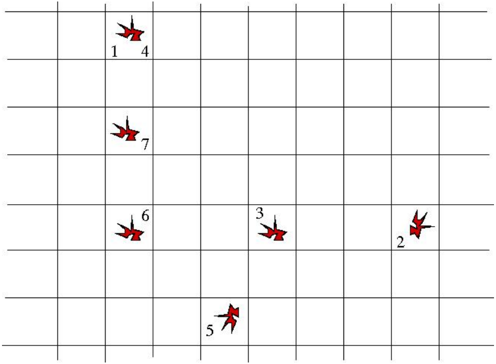

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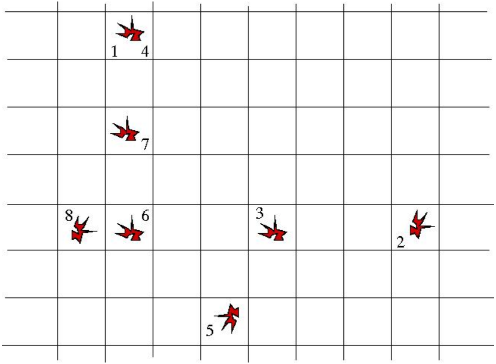

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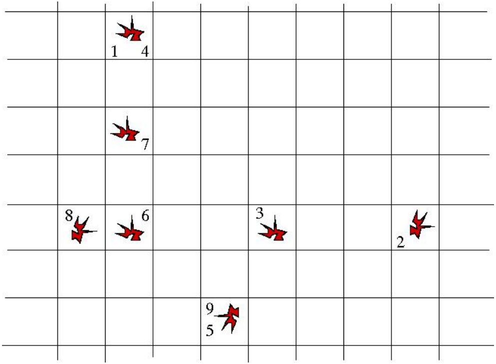

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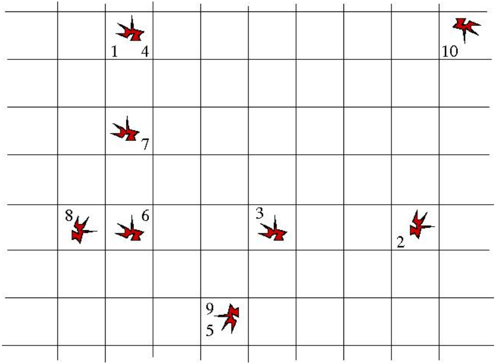

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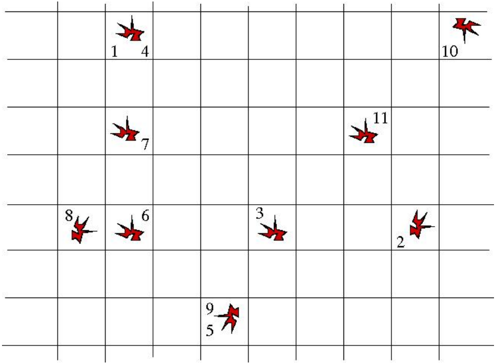

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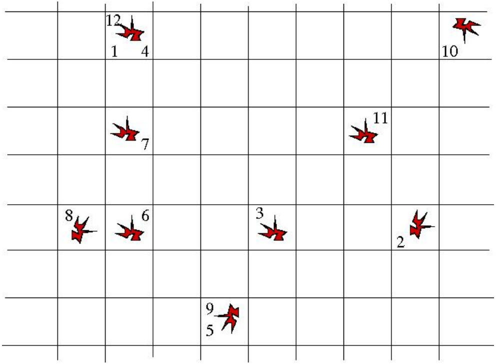
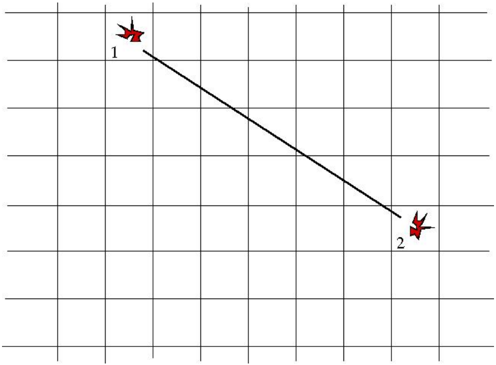


The graph is built:

each cell = vertex; time sequence defines edges



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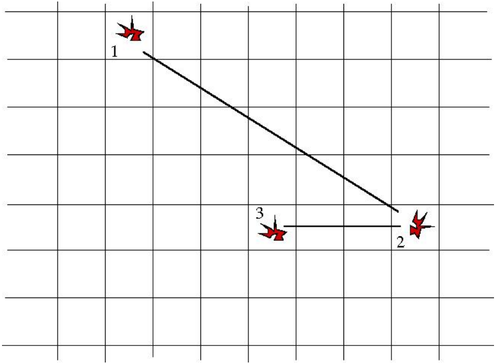

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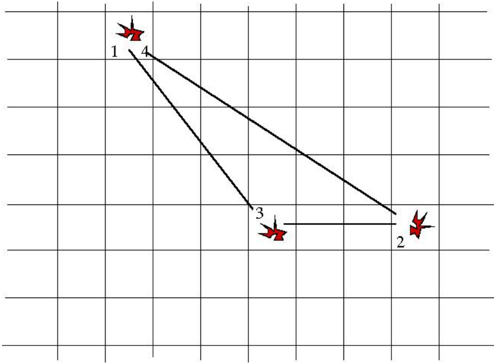

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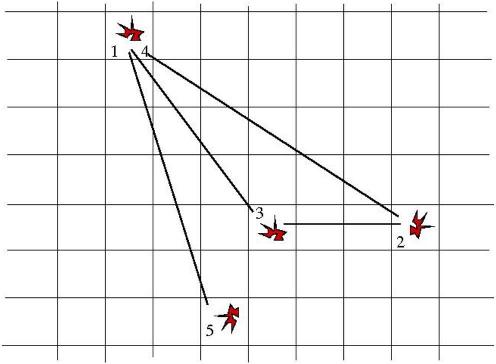

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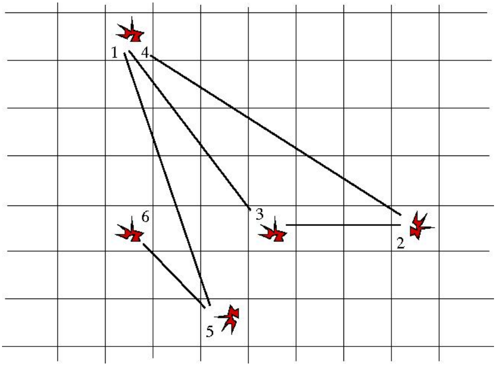

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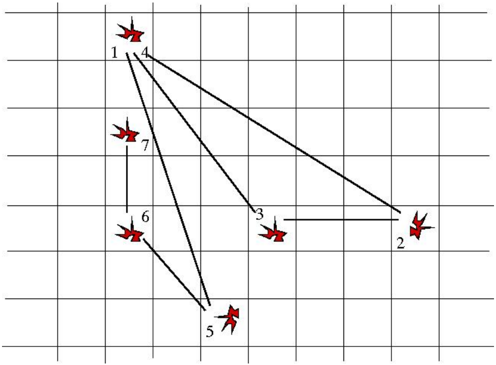

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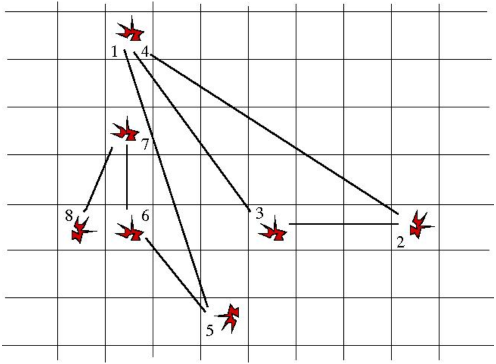

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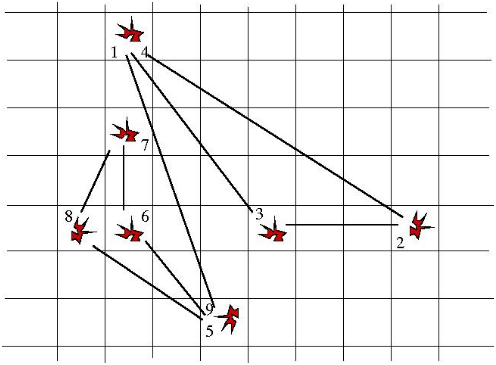

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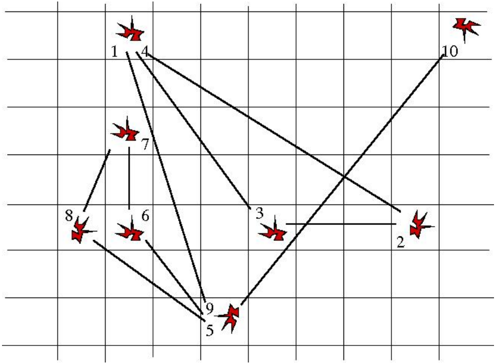

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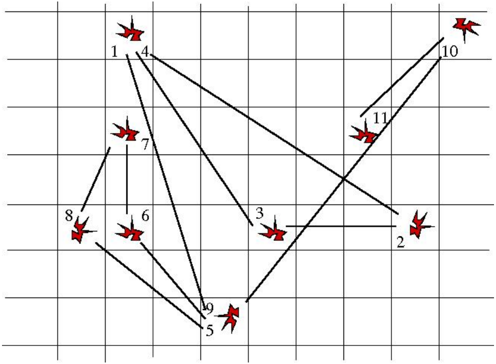

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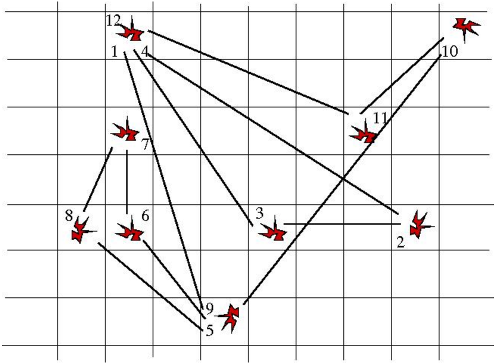

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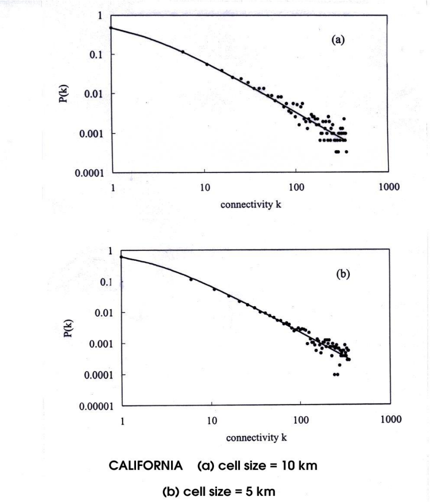
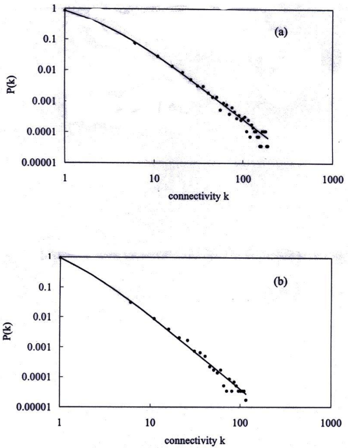
Degree distribution: California



cell of 10 $\times$ 10 km

cell of 5 $\times$ 5 km

Degree distribution: Japan



JAPAN (a) cell size = 10 km

(b) cell size = 5 km

cell of 10 × 10 km

cell of 5 × 5 km

Abe, EPL 65, 2004

Degree distribution: OFC model

Conservative and non conservative versions have very different behavior.

- Conservative: $\sim$ random graph (Poisson distribution);
- Non conservative: $\sim$ scale free (as real data).

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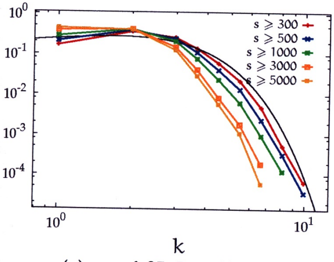
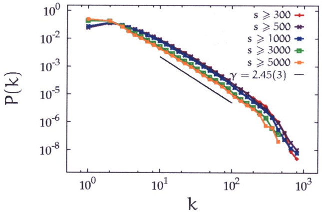


(a) $\alpha = 0.25$, $L = 400$



(b) $\alpha = 0.22$

conservative

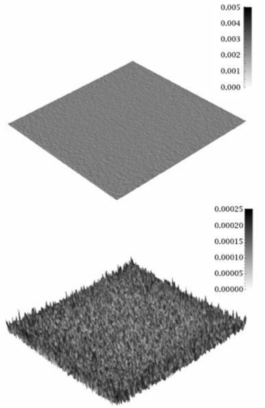
non conservative

Spacial distribution of epicenters

model

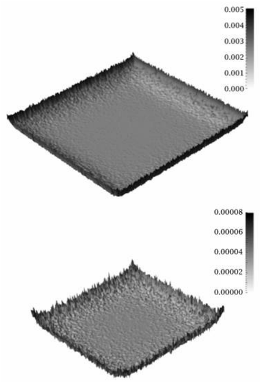
Conservative

Epicenters occur throughout the lattice;



Non conservative

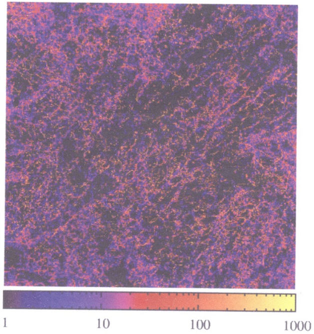
Epicenters are concentrated in the border;



Spacial distribution of degrees

model

Epicenters occur throughout the lattice;



Hubs are well distributed inside the bulk, but aggregated in *stripe-like structures*;

The structure disappears
as only larger earthquakes are taken into account.

In-degree of a vertex placed in the tension lattice,

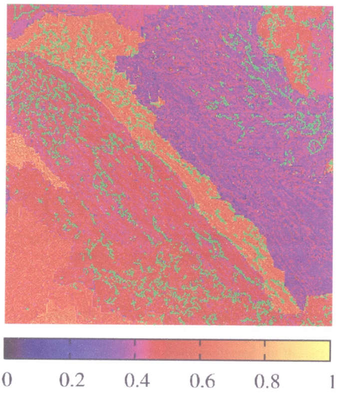
($L = 800, \alpha = 0.18$, $s \geq 2$, whole network).

Relation between epicenters and tension

model

When all events are taken into account, the epicenters seem to happen mostly in the frontier among those synchronized regions, and in *valley-like structures* inside the plateaus

If only larger earthquakes are considered, the epicenters are more and more homogeneously distributed.



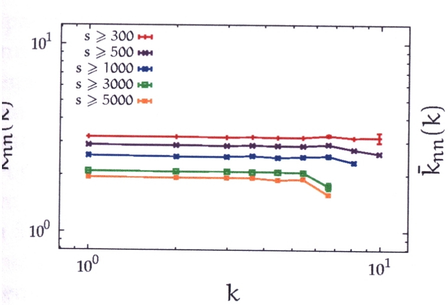
Snapshot of the tension lattice at the stationary state ($L = 800$, $\alpha = 0.18$, $s \geq 2$).

The next 10^4 epicenters are marked in green.

Degree Correlation

conservative model

There is no correlation, as expected in a random graph.



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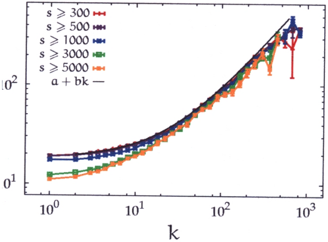
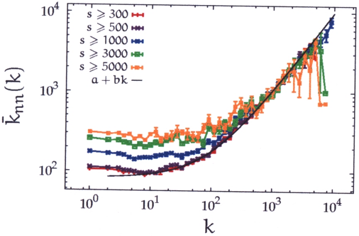
Degree Correlation

non conservative model



(b) $\alpha = 0.22$



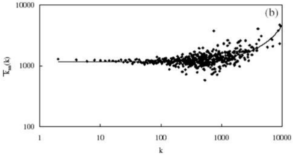
(e) $\alpha = 0.16$

The correlation is linear, $\bar{k}_{nn}(k) = a + b\,k$,

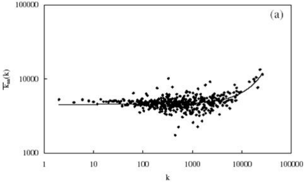
with a, b constants for all values of $\alpha < 1/4$.

Degree correlation

California



cell of 5 $\times$ 5 km

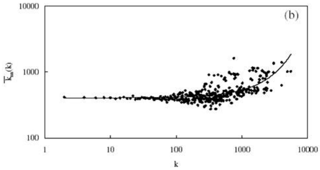


cell of 10 $\times$ 10 km

Abe, ...

Degree correlation

Japan



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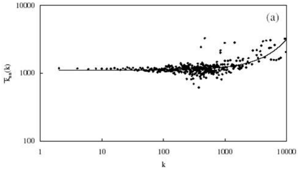

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Abe,

Understanding the dynamics of the model

This linear correlation suggests a basic mechanism:

temporary attractors

Understanding the dynamics of the model

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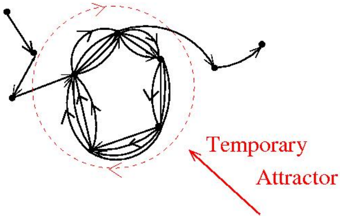
temporary attractors

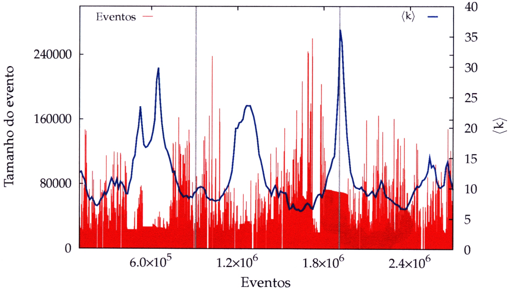
From time to time, epicenters occur in a (small) fraction of possible sites; the dynamics gets trapped in this cycle for a certain amount of time, and after some time, it escapes.



Time evolution of the topology: average degree

This can be confirmed looking at
the time evolution of the average degree:

When dynamics gets trapped, the average degree increases.



Precursors motifs

The emergence of a *temporary attractor* can be related to big events:

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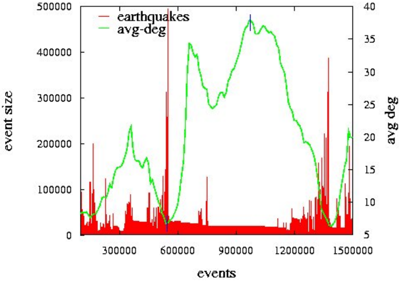
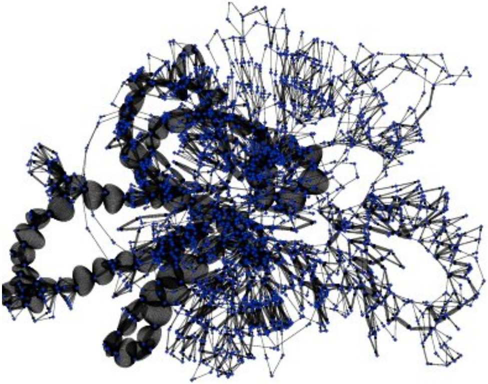


$< k >$ for the subgraph of the last 10^5 events, together with the time series of earthquakes.

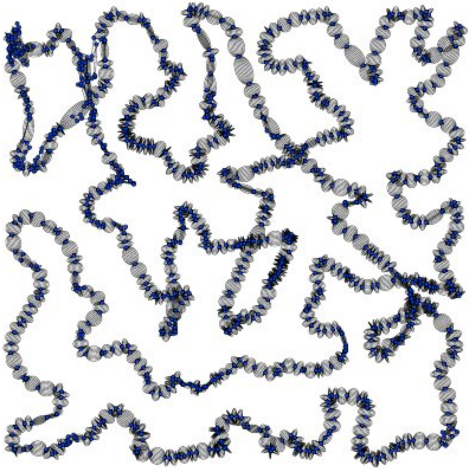
$L = 1000$, $\alpha = 0.18$, and $s \geq 300$.

Region L - out of attractor



Subgraph made of last 10^4 epicenters, from point L

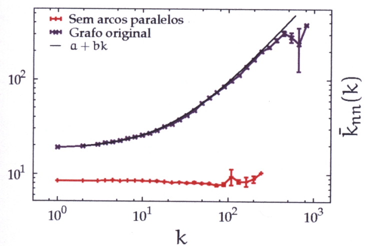
Region R - trapped in the attractor



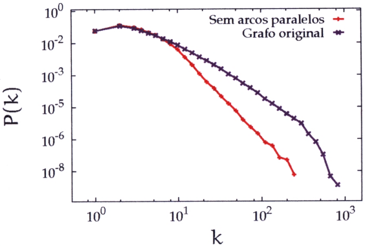
Further evidences

This *marginal synchronization* responds for the degree correlation:
If the multiple edges are removed, correlation disappears!

(But degree distributions are still power laws...)



(a) $\alpha = 0.22$



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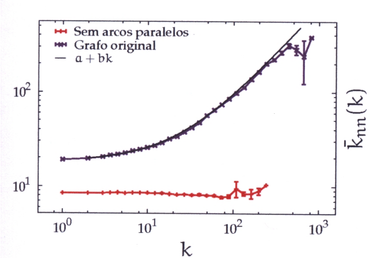

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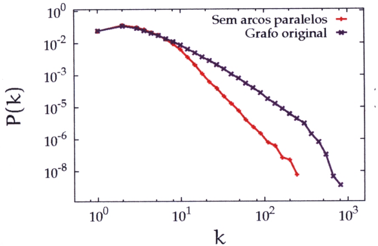


(a) $\alpha = 0.22$

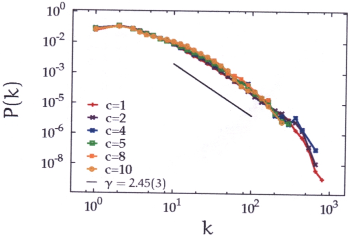
The temporary marginal synchronization was confirmed later by Ramos et al., PRL 96, 2006: (slightly different OFC)

quasi-periodicity $t^* \propto 1 - 4\alpha$.

Influence of the size of the cell

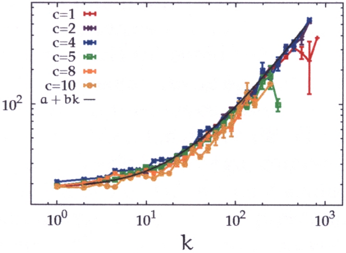
When we divide the area of a fault into cells,
we introduce an arbitrary scale:

What is the influence of this scale?



(b) $\alpha = 0.22$

degree distribution



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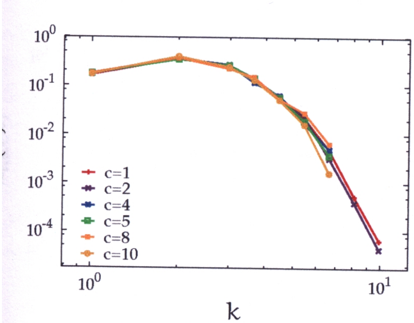
degree correlation

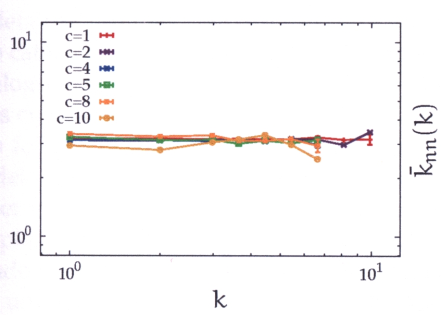
Conservative case

degree distribution



(a) $\alpha = 0.25$, $L = 400$

degree correlation



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What about real data?

(www.data.acec.org/ftp/catalogs/SHLK/)

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We analyze South California Catalog:

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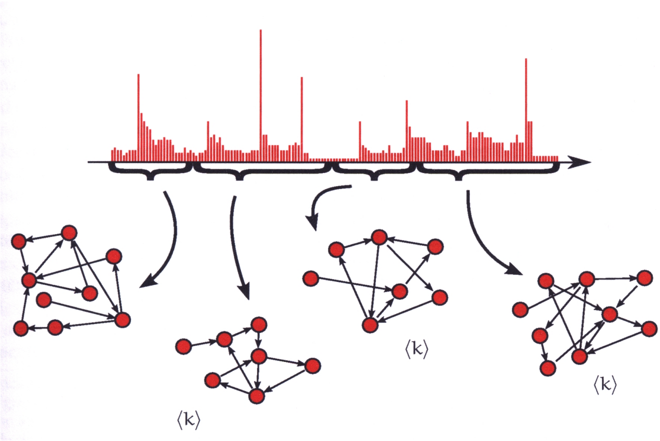
- Look for similar precursor motifs

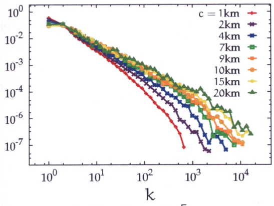
Conclusions

- There is the same linear degree correlation ...
- But the origin is different (no temporary attractors!)

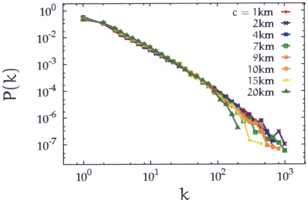
We need networks with same average degree



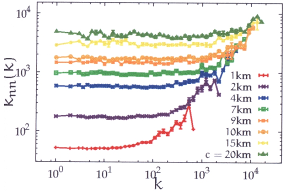
Results for catalog data



(a) E ~ 3.4×10^5



(b) $\langle k \rangle \approx 4.7$



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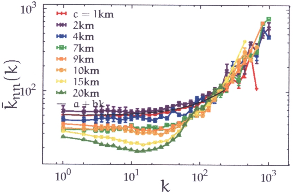

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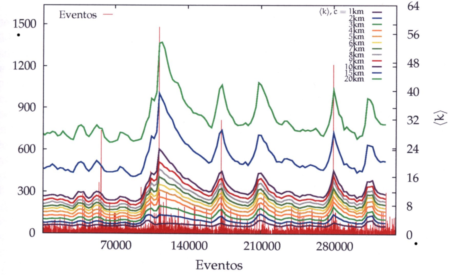
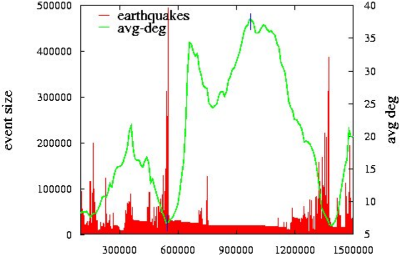
Precursors?

Average degree $\times$ Magnitude

Catalog



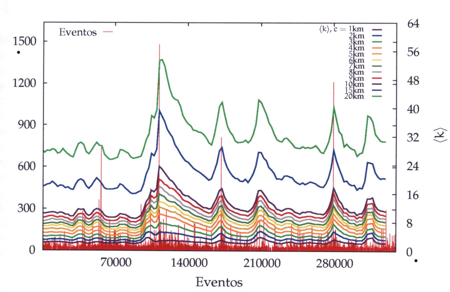
OFC model



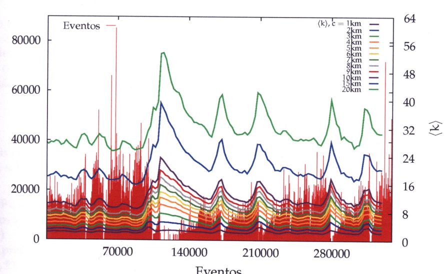
Precursors?

Average degree $\times$ Magnitude

Magnitude



Inter-occurrence time



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  - $\star$ Conservative $\Longrightarrow$ Random graph
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So, what about SOC?

SOC, criticality, OFC model & earthquakes

There have been a lot of discussion about this problem:

- In the context of SOC
- In the context of earthquakes

Why is it important?

- Forecasting

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- In the context of earthquakes

Why is it important?

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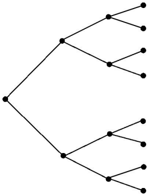
In order to display power-law behavior, it is not necessary to be critical

$$\Downarrow$$

almost-critical

SOC & Branching Processes

There is a connection between SOC and branching processes.



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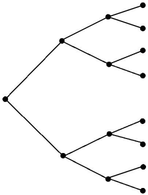

Branching Processes

- Are characterized by the branching rate σ;
- $\sigma =$ constant
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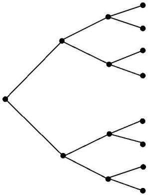

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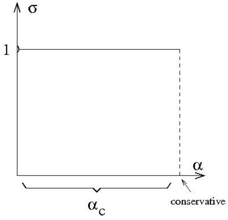
Self-organized Criticality

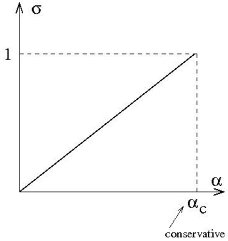
- Branching rate evolves with time: $\sigma = \sigma(t)$
- In the (statistically) stationary state: $\sigma \rightarrow \sigma_\infty$ (critical if $\sigma_\infty = 1$)

What are the possibilities for SOC?

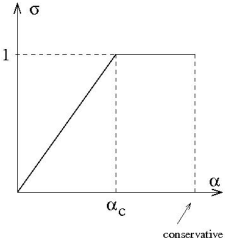
In the OFC model, the branching rate σ may depend on α:



Critical $\forall\ \sigma > 0$



Critical only if conservative



Critical for $\alpha \geq \alpha_c$

Branching Rate

Knowing the distribution of energy $p(E)$, we can calculate σ:

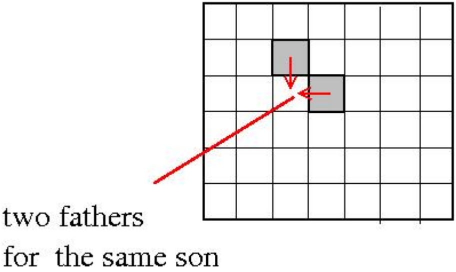
$$\sigma = \frac{\int_{E_c}^{\infty} P^{+}(E^{+})\, p(E^{+})\, dE^{+}}{\int_{E_c}^{\infty} p(E^{+})\, dE^{+}}$$

$p(E^{+}) =$ Energy distribution of unstable sites ($E > E_c$);

$P^{+}(E^{+}) =$ Probability that a stable site becomes unstable if it receives $E = \alpha\, E^{+}$.

How can we calculate p(E^+)?

In lattice models there are *correlations*, and it is not easy to calculate σ.



But in a similar model...

Extremal Feder & Feder model

OFC

- Driving: global
  $E_{i,j} \to E_{i,j} + \delta$,
  until $E_{i,j} = E^*_{i,j} > E_c$

- Relaxation rule:
  $E_{i,j} \to 0$
  $E_{\mathrm{nn}} \to E_{\mathrm{nn}} + \alpha\, E_{i,j}$

EFF

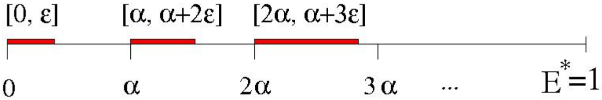
- Driving: extremal dynamics
  $E^*_{i,j} = \max E_{i,j}$

- Relaxation rule:
  $E_{i,j} \to 0 + \eta_1$
  $E_{\mathrm{rn}} \to E_{\mathrm{rn}} + \alpha + \eta_2$

For EFF model it is possible to calculate p(E) and σ

$P_t(E) \neq 0$ only if E belongs to one of the intervals I_n:

$$I_n \equiv [(n-1)\,\alpha,\ (n-1)\,\alpha + n\,\epsilon], \qquad n = 1, \ldots\, n_{max}$$



$$p_n = \int_{(n-1)\alpha}^{(n-1)\alpha+n\epsilon} p(E^+)\, dE \qquad \Longrightarrow$$

The process
can be thought as a
jumping of sites
among intervals I_n.

At every time step

(upgrade of the critical site plus k random neighbors)

- A site from the last interval is transfered to I_1;
- A site is removed from I_1 with probability $k\,p_1$
- ... so on so forth ...

$$\Downarrow$$

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$$\Downarrow$$

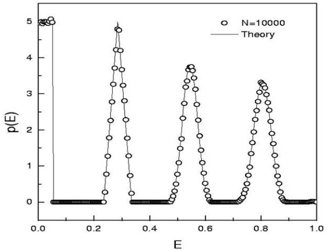
Master equation $\Longrightarrow$

p(E) in the stationary state $\Longrightarrow$

the branching rate σ.

Finally...

Energy distribution in the steady state:

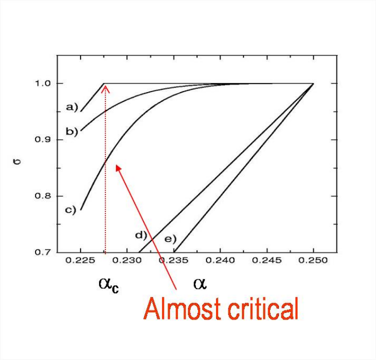


Kinouchi & Prado, PRE 59, 1999

Branching rate:

$$\sigma_\infty = \begin{cases} 1 - \frac{C}{k\,(k+1)}\,\left(\frac{1-k\,\alpha}{\epsilon}\right)^{k+1} & \text{if} \quad \eta_2 \neq 0 \\ 1 - \frac{1-k\,\alpha}{\epsilon} & \text{if} \quad \eta_2 = 0 \end{cases}$$

Branching rate



Theoretical results for EEF

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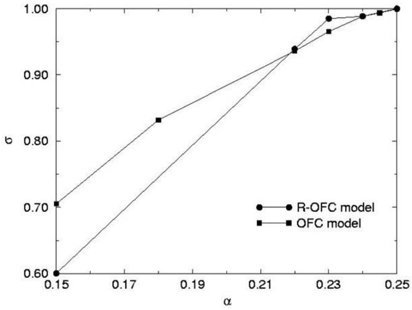
with noise

$$\eta_2 \neq 0 \begin{cases} \text{(b)}\ \epsilon = 0.0625; \\ \text{(c)}\ \epsilon = 0.0500; \end{cases}$$

without noise

$$\eta_2 = 0 \begin{cases} \text{(d)}\ \epsilon = 0.25; \\ \text{(e)}\ \epsilon = 0.20. \end{cases}$$

For OFC and RN-OFC models



(de Carvalho, Prado, PRL 84, 2000)

'Almost-criticality' may be much more frequent...

Besides earthquakes,
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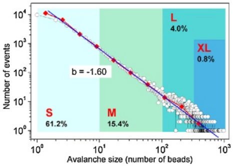

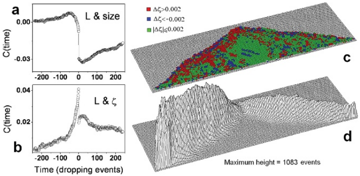

Ramos, Altshuler, Maloy, PRL 102, 2009

The End